13621 120 MINUTES

1. The curve represented by the equation Re $(\bar{z}^2) = C$, where $C > 0$ is a
   A) Circle B) Ellipse
   C) Parabola D) Hyperbola

2. The value of $\sum_{k=0}^{24}\left(\sin\left(\frac{2\pi k}{25}\right) - i\cos\left(\frac{2\pi k}{25}\right)\right)$ is
   A) 0 B) $-i$
   C) 25 D) 24

3. Product of the roots of the equation $|x|^2 - |x| - 12 = 0$ is
   A) -12 B) -16
   C) 9 D) 12

4. If a, b, c ε R, $a \neq 0$, $c > o$ and the quadratic equation $ax^2 + bx - c = 0$ has no real roots, then
   A) $(a + b - c)\, c = 0$ B) $a + b - c > 0$
   C) $(a + b - c)\, c < 0$ D) $(a + b - c)\, c > 0$

5. The sum of the series $\sum_{r=0}^{10}(-1)^r\, 10C_r\left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \text{---}\right)$ is
   A) $\frac{1}{2^{10}}$ B) $\frac{1}{2^{10} - 1}$
   C) $\frac{1}{2^{10} + 1}$ D) $\propto$

6. If A = $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ then $A^{101} + I$ is
   A) I B) A
   C) 0 D) A + I

7. Which one of the following is true for the system $2x + 3y = 1$?
   A) The system is non-consistent
   B) The system is consistent and has a unique solution
   C) The system is consistent and has two solutions
   D) The system is consistent and has infinite number of solutions

8. If the circle $x^2 + y^2 + 2gy + c = 0$ contains the point (g, o), then
   A) $c = 0$ B) $c < 0$
   C) $c > 0$ D) $c = g/2$

9. If the two circles $x^2 + y^2 + 8x - 6y + 4 = 0$ and $x^2 + y^2 + 10x + c(y + 1) = 0$ cut orthogonally, then the value of c is

A) 9 B) 8 C) 10 D) 5

10. If e is the eccentricity of the hyperbola conjugate to the hyperbola $9x^2 - 16y^2 + 72x - 32y - 16 = 0$, then e is

A) 3/5 B) 4/5 C) 5/3 D) 5/4

11. The angle between the plane $x - 2y - 4z + 7 = 0$ and the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is

A) 0 B) $\pi/3$ C) $\pi/4$ D) $\pi/2$

12. The number of mappings which are not onto on a set A = {1, 2, 3, 4} is

A) 256 B) 232 C) 16 D) 24

13. The domain of $f(x) = \sin^{-1}\left(\frac{x-2}{2}\right) - \log_{10}(3-x)$ is

A) $(-\infty, 3)$ B) $(-\infty, 2)$ C) $[0, 4]$ D) $[0,3)$

14. The graph of the function $\cos x.\cos(x + 2) - \cos^2(x + 1)$ is

A) a straight line passing through $\left(\pi/2, -\sin^2 1\right)$ with slope 2

B) a straight line passing through $\left(\pi/2, -\sin^2 1\right)$ with slope 0

C) a straight line passing through $\left(\pi/2, -\sin^2 1\right)$ with slope 1

D) a parabola with vertex $(1, -\sin^2 1)$

15. Which one of the following sets is not convex?

A) $\{(x, y)\ \varepsilon\ R^2 : -1 \le x \le 4, -1 \le y \le 5\} \cup \{(x, y)\ \varepsilon\ R^2 : x^2 + y^2 = 1\}$

B) $\{(x, y)\ \varepsilon\ R^2 : -1 \le x \le 4, -1 \le y \le 5\} \cap \{(x, y)\ \varepsilon\ R^2 : x^2 + y^2 = 1\}$

C) $\{(x, y) : R^2 : x^2 + y^2 = 4x\} \cap \{(x, y)\ \varepsilon\ R^2 : y = +\sqrt{4 - x^2}\}$

D) $\{(x, y)\ \varepsilon\ R^2 : x^2 + y^2 = 4x\} \cup \{(x, y)\ \varepsilon\ R^2 : y = +\sqrt{4 - x^2}\}$

16. Let $f(3) = 9$ and $f'(3) = 9$. Then $\lim_{x \to 3} \frac{x f(3) - 3 f(x)}{x - 3}$ is

A) 18 B) –18 C) 9 D) –9

17. Let F(x) = f (x) g(x) h(x) for all real x, where f(x), g(x) and h(x) are differentiable functions. At some point x_0, $F'(x_0) = 14F(x_0)$, $f'(x_0) = 7f(x_0)$, $g'(x_0) = 2g(x_0)$ and $h'(x_0) = kh(x_0)$. Then the value of k is

A) 2 B) 7 C) 5 D) 14

18. The value of $\int_{-\pi/3}^{\pi/3} \log\left(\frac{4+3\sin\theta}{4-3\sin\theta}\right) d\theta$ is

A) 0 B) 1 C) 2 D) $\pi / 3$

19. The general solution of the equation $\frac{d^2y}{dx^2} = xe^x$ is

A) $xe^x - 2e^x$
B) $c_1 x + c_2$, where c_1 and c_2 are arbitrary
C) $xe^x - 2e^x + c_1x + c_2$, where c_1 and c_2 are arbitrary constants
D) $xe^x + c_1e^x + c_2x$, where c_1 and c_2 are arbitrary constants

20. The slope of the tangent to the curve $y = \int_{x}^{x^2} \log t\,dt$ at x = 2 is

A) 7 log 2 B) 5 log 2
C) 3 log 2 D) 8 log 2

21. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n}{n!} z^n$ is

A) ½ B) 2
C) ∞ D) 1

22. Consider the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$, $x \in R$. Let (i) and (ii) be two statements about this series given by
(i) The series is uniformly convergent on R
(ii) The series is absolutely convergent for each x in R

Then

A) (i) and (ii) are both true B) (i) is true, but (ii) is not true
C) (ii) is true, but (i) is not true D) (i) and (ii) are both false

23. $\lim_{n \to \infty} \left(\sqrt{n^2 + n} - n\right)$ is

A) 1 B) ½
C) ∞ D) 0

24. Let E be a subset of R and let m* denote the Lebesque outer measure. Then which of the following statements is not true?
A) m*(E) = 0, if E is countable
B) If m*(E) = 0, then E is countable
C) m*(E) = 1, if E = [0, 1]
D) If m*(E) = 0, then E is measurable

25. Let f be the function defined on R by $f(0) = 0$, $f(x) = x^2 \sin 1/x$ for $x \neq 0$. The value of $D_{-} f(0)$, the lower left hand derivative of f at 0 is

A) 1 B) 0 C) – 1 D) ½

26. Let f and $\propto$ be defined on [0, 1] by $f(x) = 2x^2 + 3x$ and $\propto(x) = x^2$. Then the value of $\int_0^1 f \, d\propto$

A) 2 B) 3 C) ½ D) ¼

27. For m, n = 1, 2, 3, ---, let $S_{m,n} = \frac{m}{m+n}$ Let $a = \lim_{n \to \infty} \lim_{m \to \infty} S_{m,n}$;
$b = \lim_{m \to \infty} \lim_{n \to \infty} S_{m,n}$

Then
A) a = 0, b = 1 B) a = 1, b = 0 C) a = b = 1 D) a = b = 0

28. Let $G = \{z \in C : |z+1| + |z-1| < 4\}$. Then
A) G is connected, but not simply connected
B) G is simply connected, but not convex
C) G is convex
D) G is not convex

29. The singularity of the function $\frac{\sin z}{z^2}$ at z = 0 is
A) Pole of order 2
B) A removable singularity
C) An essential singularity
D) A simple pole

30. The number of fixed points of the Mobius-transformation $S(z) = az + b$, $a \neq 0$ is
A) 2 B) 1 C) 0 D) 3

31. If the series $\sum_{n=1}^{\infty} a_n z^n$ has radius of convergence R, then the series $\sum_{n=1}^{\infty} \frac{a_n}{n} z^n$ has radius of convergence

A) 2R B) R^2 C) R D) 1

32. The value of the integral $\int_r \frac{\sin z}{z^4}\, dz$, where r is the circle defined by $r(t) = e^{it}, 0 \le t \le 2\pi$ is

A) $2\pi i$ B) $\frac{\pi i}{4}$ C) $-\frac{\pi i}{3}$ D) 0

33. Let r be the positively oriented rectangular path with vertices 0, 1, 1 + i, i. Then $\int_r (z^2 + 1)\, dz$ is

A) ¼ B) 1/3 C) 0 D) 1

34. Let f be an analytic function with an isolated singularity at z = 0. If this is a simple pole of f, then which of the following statements is not true for f ?

A) $\lim_{z \to 0} |f(z)| = \infty$ B) $\lim_{z \to 0} z f(z)$ exists and is finite

C) $\lim_{z \to 0} z f(z) = 0$ D) $\lim_{z \to 0} z^2 f(z) = 0$

35. The value of the cross ratio (0, 1, i, -1) is

A) 1 B) i

C) $1 - i$ D) $1 + i$

36. Consider the harmonic function **U** (x, y) = (x cos y – y sin y) e^x, where $x + iy \in \mathbb{C}$. Its harmonic conjugate V(x, y) is given by

A) (- y cos y + x sin y) e^x B) (y cos y – x sin y) e^x

C) (-y cos y – x sin y) e^x D) (y cos y + x sin y) e^x

37. Let f be a function analytic on a region G. Suppose there exists a point z_0 in G such that $f^{(n)}(z_0) = 0$ for n = 1, 2, 3, --- . Then which of the following statements is not necessarily true?

A) $f \equiv 0$ in G

B) $f' \equiv 0$ in G

C) $|f|$ attains its maximum on G

D) f is a constant on G

38. The value of the integral $\frac{1}{2\pi i} \int_{r} \frac{dz}{z(2-z)}$ where r is the circle given by $r(t) = 2 + e^{it}, 0 \le t \le 2\pi$ is

A) 1 B) ½ C) – 1 D) – ½

39. Suppose f is analytic in the annulus $G = \{z : 0 < |z| < 1\}$. Then which of the following statements is not necessarily true?

A) f is infinitely many times differentiable on G

B) Real and imaginary parts of f are harmonic in G

C) f has a Taylor series expansion in G about z = 0

D) f has a Laurent series expansion in G about z = 0

40. The residue of $1/_{z^2+1}$ at $z = i$ is

A) i B) $i/2$ C) $-i/2$ D) 1

41. Which of the following is a unit in the ring **Z** $[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in Z\}$ where **Z** is the ring of integers?

A) $1+\sqrt{2}$ B) $2+\sqrt{2}$ C) $3+\sqrt{2}$ D) $1+2\sqrt{2}$

42. Which of the following is a zero divisor in the ring $\mathbf{Z}_{15}$?

A) 2 B) 4 C) 6 D) 8

43. Which of the following is an irreducible polynomial in $\mathbf{Z}_3[x]$?

A) $x^3 + 2$ B) $x^3 + 2x + 1$

C) $x^3 + 2x^2 + 2$ D) $2x^3 + 2x + 1$

44. The degree of the extension $\left[Q(\sqrt{2+\sqrt{3}} : Q\right]$ is

A) 1 B) 2 C) 3 D) 4

45. Let $\propto$ be a real cube root of 2 and let K = Q ($\propto$). Then the order of the automorphism group Aut (K / Q) is

A) 1 B) 2 C) 3 D) 6

46. Let F be the splitting field of $(x – 1)(x^2 – 2)$ over Q. Then [F : Q] is

A) 1 B) 2 C) 3 D) 6

47. Which of the following is not the order of a finite field?

A) 3 B) 5 C) 15 D) 25

48. Let V be the vector space of all polynomials of degree ≤ 2 over the reals. Which of the following is not a basis of V?

A) $\{ 1 + x, 2 + x, 1 + x^2\}$ B) $\{ 1 + x, (1 + x)^2, 1 - x \}$
C) $\{ 1 + x, 1 - x, 1 - x^2\}$ D) $\{ 1 + x, x^2 - 1, x + x^2 \}$

49. Consider the vector space $\mathbf{R}^3$ over $\mathbf{R}$. Which of the following is a subspace of $\mathbf{R}^3$?

A) $\{ (x, y, z) : x + y = 2x + z \}$ B) $\{ (x, y, z) : x + y = x +2 \}$
C) $\{ (x, y, z) : x + y = y + 1 \}$ D) $\{ (x, y, z) : x + y = z + 1 \}$

50. Let $f : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be defined by $f(x, y, z) = (x - 2y, 2y + z, x + z)$. Then the dimension of the null space of f is

A) 0 B) 1 C) 2 D) 3

51. Which of the following is an eigen value of

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A) 2 B) 3 C) 4 D) -1

52. Which of the following is an eigen vector of

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A) (1, 1, 0) B) (0, 1, 0) C) (1, 0, 2) D) (1,– 1,1)

53. The set $\{ (1, 1, 0), (1, -1, 1), (x, 0, 1) \}$ is linearly dependent in $\mathbf{R}^3$ for x =

A) 0 B) 1 C) 2 D) -1

54. Which of the following is not an associative binary operation on the set N of natural numbers. For all $a, b \in N$,

A) $a * b = a$ B) $a * b = b$
C) $a * b = a + b + ab$ D) $a * b = a + a^2b$

55. Which of the following is not a subgroup of the cyclic group $\mathbf{Z}_{12}$?

A) { 0, 6 } B) { 0, 4, 8 }
C) { 0, 5, 10 } D) { 0, 3, 6, 9 }

56. Which of the following is a generator of the cyclic group $\mathbf{Z}_{100}$?

A) 5 B) 12
C) 21 D) 24

57. The permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 3 & 5 & 7 & 6 & 4 \end{pmatrix}$ is the same as

A) (1 2) (4 5 6 7) B) (1 2) (4 5 7)
C) (1 2 3) (4 5 6 7) D) (1 2 3) (4 5) (6 7)

58. Which of the following is a left coset of $\mathbf{Z}_{20}$ with respect to some subgroup?

A) {1, 6, 11, 16} B) {0, 6, 12, 18}
C) {4, 8, 12, 16} D) {2, 8, 13, 18}

59. The number of mutually non-isomorphic abelian groups of order 72 is

A) 1 B) 2
C) 3 D) 6

60. Let G be a group of order 30. Then G is

A) Abelian B) Cyclic
C) Simple D) Solvable

61. Let τ be the topology on **R** consisting of **R**, ϕ and all open intervals of the form (a, ∞) where a $\in$ **R**. Then the closure of the interval A = [0, 1] is

A) [0, 1] B) $(-\infty, 1]$
C) $[0, \infty)$ D) **R**

62. The connected subsets of the real line with usual topology are

A) All intervals
B) Only bounded intervals
C) Only finite intervals
D) Only semi-infinite intervals

63. Let X and Y be topological spaces and let $f : X \to X$ be a one-one, onto and continuous map. Then f is a homeomorphism if

A) X and Y are compact
B) X is Hausdorff and Y is compact
C) X is compact and Y is Hausdorff
D) X and Y are Hausdorff

64. The relative topology inherited by the set of integers as a subspace of R, the set of real numbers with usual topology is

A) The usual topology B) The indiscrete topology
C) The discrete topology D) None of these

65. Consider the class B of all open equilateral triangles and the class B' of open squares with horizontal and vertical sides. Then for the usual topology on $\mathbf{R}^2$

A) Both B and B' are bases B) Only B is a base
C) Only B' is a base D) Neither B nor B' is a base

66. Which of the following subsets of the real line **R** with usual topology is compact?
A) $[0, 1] \cup [2, 3]$
B) $(0, 1)$
C) The set of all rational numbers
D) $(1, \infty)$

67. Let C(I) denote the set of all continuous real valued functions on the closed unit interval I = [0,1] and let x_0 be a fixed point of I
For f, g $\in$ C (I), let d_1, d_2 be defined by
$d_1(f, g) = \sup_{x \in I} |f(x) - g(x)|$ and $d_2(f, g) = |f(x_0) - g(x_0)|$

Then

A) Both d_1 and d_2 are metrics on C(I)
B) Only d_1 is a metric on C(I)
C) Only d_2 is a metric on C(I)
D) Neither d_1 nor d_2 is a metric on C(I)

68. The boundary of the open unit disc $\{z : |z| < 1\}$ in the complex plane with usual topology is
A) $\{z : |z| \leq 1\}$ B) $\{z : |z| \geq 1\}$
C) $\{z : |z| = 1\}$ D) $\{z : |z| > 1\}$

69. Which of the following is not a complete metric space?
A) The Real line **R** with the usual metric
B) **R** with the discrete metric
C) **C** [0, 1], the space of all continuous real valued functions on [0, 1] with the metric $d(f, g) = \int_0^1 |f(x) - g(x)| dx$
D) **C** [0, 1] with the metric $d(f, g) = \sup |f(x) - g(x)|$

70. Which of the following is always true?
A) A subspace of a compact space is compact
B) A subspace of a connected space is connected
C) A subspace of a normal space is normal
D) A subspace of a Hausdorff space is Hausdorff

71. Let **X** be the real normed space $\mathbf{R}^3$ with norm $\| \ \|_2$ .Then $\mathbf{R}^3$ is homeomorphic to
A) $\{(x(1), x(2), x(3)) \varepsilon R^3 : |x(1)| + |x(2)| + |x(3)| = 1\}$
B) $\{(x(1), x(2), x(3)) \varepsilon R^3 : |x(1)|^2 + |x(2)|^2 + |x(3)|^2 < 1\}$
C) $\{(x(1), x(2), x(3)) \varepsilon R^3 : |x(1)|^2 + |x(2)|^2 + |x(3)|^2 \leq 1\}$
D) $\{(x(1), x(2), x(3)) \varepsilon R^3 : |x(1)|^2 + |x(2)|^2 + |x(3)|^2 = 1\}$

72. Consider the norms $\| \|_1, \| \|_2$ and $\| \|_\infty$ on $\mathbf{R}^3$, where $\mathbf{R}$ is the space of all real numbers. Then for $x \varepsilon \mathbf{R}^3$, which one of the following is not true

A) $\|x\|_2 \le 2\|x\|_1$
B) $\sqrt{2}\,\|x\|_2 \le 2\|x\|_\infty$
C) $\sqrt{3}\,\|x\|_2 \le 3\|x\|_\infty$
D) $\|x\|_1 \le 3\|x\|_\infty$

73. Let $X = R^4$ be the normed space with norm $\| \|_p$, $1 \le p \le \infty$. Then the Hahn-Banach extension to X is unique if

A) $P = \infty$
B) $P = 1$
C) $P = 2$
D) $P = 4$

74. Let X be the normed space ℓ^1 of all summable complex numbers and Y be the subspace spanned by the set { (1,0,0,…), (0,1,0,…), (0,0,1,0,0,…) }. Then X/Y is

A) Separable and a Banach space
B) Not separable but a Banach space
C) Neither separable nor a Banach space
D) Separable but not a Banach space

75. Which one of the following is a Banach Space?

A) C_{00} with the norm $\| \|_\infty$
B) P [-1, 1], the normed linear space of all polynomials defined on [-1, 1] with norm $\| \|_\infty$
C) $C_c(R)$, where R is the metric space with usual metric
D) C_0([-1, 1])

76. Let X be the real normed space R^2 with norm $\| \|_2$. Then the linear transformation on R^2 to itself represented by the matrix

$$\begin{bmatrix} \cos \pi/6 & \sin \pi/6 \\ -\sin \pi/6 & \cos \pi/6 \end{bmatrix} \text{is}$$

A) Bounded but not isometry
B) Not bounded but isometry
C) Bounded and isometry
D) Neither bounded nor isometry

77. Let x and y be two elements in a real Hilbert space H with $\|x\| = 4, \|y\| = 3$ and $\|x - y\| = 3$. Then $\langle x, y \rangle$ is

A) 8
B) 6
C) 4
D) 10

78. For every element x in the Hilbert space $L^2([0, 2\pi])$, which one of the following is not true?

A) $\int_0^{2\pi} x(t)\, e^{int}\, dt \to 0$

B) $\int_0^{2\pi} x(t) \sin(nt)\, dt \to 0$

C) $\int_0^{2\pi} x(t) \cos(nt)\, dt \to 0$

D) None of the above is not true

79. Let X be the complex Hilbert space C^2 and let $A : X \to X$ be defined by $A(x(1), x(2)) = (-x(1), i\, x(2))$ for $(x(1), x(2)) \varepsilon C^2$. Then A is

A) Normal but not unitary

B) Self-adjoint but not normal

C) Unitary but not self-adjoint

D) Normal, self adjoint and unitary

80. Consider the complex Hilbert space $L^2([-\pi, \pi])$. For n ε Z, let $u_n(t) = e^{in(\pi + t)}$.

Then $\left\| \sum_{n=1}^{10} U_n \right\|^2$ is

A) $\sqrt{10\,\pi}$

B) $10\,\pi$

C) $\sqrt{20\,\pi}$

D) $20\,\pi$

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