15621 120 MINUTES

1. If (a – ib) (x – iy) = – ($a^2 + b^2$), then
A) x = a, y = b B) x = – a, y = b
C) x = a, y = – b D) x = – a, y = – b

2. If $1, \alpha_1, \alpha_2$, ---, α_{49} are the 50th roots of unity, then $(1 + \alpha_1)(1 + \alpha_2)$---$(1 + \alpha_{49})$ is
A) 0 B) 1
C) –1 D) 50

3. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$ where $i = \sqrt{-1}$
A) is a real number only for $n_1 = n_2 + 1$
B) is a real number only for $n_1 = n_2$
C) is a non real complex number for all n_1 and n_2
D) is a real number for all n_1 and n_2

4. The sum of the roots of the equation $|x|^2 - 5|x| + 6 = 0$ is
A) 5 B) –5 C) 0 D) 1

5. The coefficient of x^5 in the expansion of $E = 1 + (1+x) + (1+x)^2 + \text{---} + (1+x)^{10}$ is
A) $10C_5$ B) $10C_6$ C) $11C_6$ D) $11C_5$

6. A box contains 2 white balls, 3 black balls and 3 red balls. The number of ways in which we can select three balls from the box if at least one red ball included is
A) 46 B) 56 C) 66 D) 10

7. Which one of the following matrix A satisfies the equation $x^2 – 1 = 0$?
A) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

8. The system of equation 2x + 3y = 5 is
A) inconsistent
B) consistent and has a unique solution
C) consistent and has two solutions
D) consistent and has infinitely many solutions

9. The equation of the diagonal through the origin of the quadrilateral formed by the lines x = 0, y = 0, x+y = 1 and 6x+y = 3 is given by
A) 3x = y B) 3x = 2y C) x = y D) 3x = 4y

10. If the centroid of the triangle with vertices at (0,0), (2sin t, cos t) and (-sin t, cos t) lies on the line y = 2x, then t is
A) 0 B) $\Pi/4$ C) $\Pi/3$ D) $\Pi/2$

11. If the two circles $x^2+y^2+ay = 0$ and $x^2+y^2 = c^2$ (with $c > 0$) touch each other, then which one of the following is not true?

A) $a^2 - c^2 = 0$ B) $\frac{c}{2} = \left|\frac{|a|}{2} - c\right|$

C) $a^2 + c^2 = 0$ D) $\frac{|a|}{2} = \left|\frac{|a|}{2} - c\right|$

12. The domain of the function $\log_x\left(\frac{\cos^{-1}(x)}{\sqrt{3} - 2\sin x}\right)$, where $x > 0$, is

A) (0, ½) B) (0, 1) C) (0, 2) D) (½, 1)

13 The number of functions which are not one-one on a set A = {1,2,3,4,5} is

A) 625 B) 3125 C) 3005 D) 2500

14. The period of the function $y = |\sin x| + |\cos x|$ is

A) $\Pi/2$ B) Π C) $\frac{3\Pi}{2}$ D) 2Π

15. $\lim_{x\to\infty} x\left(e^{\frac{1}{2x}} - 1\right)$ is

A) 0 B) 1 C) –1 D) $\frac{1}{2}$

16. If $y = \log_{e^x}(x-3)^2$ for $x \neq 0, 3$. Then $y'(4)$ is

A) $\frac{1}{2}$ B) $2/3$ C) $3/4$ D) 1

17. The function $f(x) = x^x$ decreases on the interval

A) (0, 1) B) (0, e) C) (0, 1/e) D) (1, e)

18. Let P be a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2, then the maximum value of A is

A) ab B) $b\sqrt{a^2 - b^2}$

C) $a\sqrt{a^2 - b^2}$ D) $ab\sqrt{a^2 - b^2}$

19. The locus of a point p(x,y,z) which moves such that y = 5, is a plane parallel to

A) xy plane B) xz plane

C) yz plane D) None of these

20. If l, m, n are the direction cosines of a line, then

A) $l^2 + m^2 + n^2 = 1$ B) $l^2 - m^2 + n^2 = 1$

C) $l^2 + m^2 + n^2 = -1$ D) $l^2 + m^2 + n^2 = 0$

21. From the following functions, pick the function which is uniformly continuous on (0,1)

A) $f(x) = \frac{1}{x}$ B) $f(x) = \frac{1}{x^2}$

C) $f(x) = \frac{\sin x}{x^2}$ D) $f(x) = \frac{1-\cos x}{x^2}$

22. If $\sum_{n=0}^{\infty} a_n$ is an absolutely convergent series, then which of the following series is divergent

A) $\sum_{n=0}^{\infty} a_n^2$ B) $\sum_{n=0}^{\infty} a_n (1+a_n)$

C) $\sum_{n=0}^{\infty} a_n (1+a_n)^2$ D) $\sum_{n=0}^{\infty} \cos a_n$

23. The series $\sum_{n=1}^{\infty} \frac{x^n}{n}$

A) converges for all x in [–1, 1]
B) converges for all x in (–1, 1), but the convergence is not uniform on (–1,1)
C) is not uniformly convergent on [–r, r], for some r, $0 < r < 1$
D) is not absolutely convergent on (–1, 1)

24. Let C denote the set of all points in [0,1] that can be represented in the form $n = \frac{a_1}{3} + \frac{a_2}{3^2} + ---$, where each $a_n = 0$ or 2. Then the Lebesgue measure of C is

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 0

25. Let f be defined on [0,1] by

$$f(x) = \begin{cases} 1, \text{if } x \text{ is rational} \\ -1, \text{if } x \text{ is irrational} \end{cases}$$

Then

A) f and f^2 are both Riemann integrable on[0,1]
B) f and f^2 are both Lebesgue integrable on[0,1]
C) f is not Lebesgue integrable on[0,1]
D) f^2 is not Riemann integrable on[0,1]

26. Let $\{a_n\}$ be a sequence of complex numbers such that $\sum_{n=0}^{\infty} a_n$ converges, but $\sum_{n=0}^{\infty} |a_n|$ divergent. If R is the radius of convergence of the series $\sum_{n=0}^{\infty} a_n z^n$, then

A) $0 < R < 1$ B) $R = 1$ C) $1 < R < \infty$ D) $R = \infty$

27. The power series $\sum_{n=0}^{\infty} 4^{-n} (z-1)^{2n}$ converges if

A) $|z| < 4$ B) $|z| \le 2$ C) $|z-1| \le 2$ D) $|z-1| < 2$

28. The function $\frac{2z-1}{2-z}$ maps the disc D = $\{z : |z| < 1\}$ onto

A) the complex plane B) D
C) $\{z \in C : \text{Im} z \ge 0\}$ D) $\{z \in C : \text{Im} z \le 0\}$

29. Let γ be the curve defined by $\gamma(\theta) = 3e^{2i\theta}$, $0 \le \theta \le 2\pi$. Then the value of the integral $\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z}$ is

A) 0 B) 2 C) 1 D) 3

30. Consider the function f defined on the complex plane by $f(z) = e^{iz}$. On which of the following sets is the function f bounded?

A) $\{z \in C : \text{Re}\, z \ge 0\}$ B) $\{z \in C : \text{Re}\, z \le 0\}$
C) $\{z \in C : \text{Im}\, z \ge 0\}$ D) $\{z \in C : \text{Im}\, z \le 0\}$

31. Let $U(x+iy) = x^3 - 3xy^2$. For which of the following functions V, is U+iV holomorphic in C?

A) $V(x+iy) = 3x^2y - y^3$ B) $V(x+iy) = y^3 - 3x^2y$
C) $V(x+iy) = x^3 - 3xy^2$ D) $V(x+iy) = 3xy^2 - x^3$

32. Let $f(z) = z^2(1 - \cos z)$, $z \in C$. Then at $z = 0$, f has a zero of order

A) 1 B) 2
C) 3 D) 4

33. Let $f(z) = \frac{z}{\sin \pi z}$, $z \in C$, $z \neq 0, \pm 1, \pm 2$, ---. Then

A) $z = 0$ is a simple pole of f
B) $z = 0$ is a removable singularity of f
C) all singularities of f are simple poles
D) all singularities of f are removable

34. Let f be a meromorphic function on C - {0}. Then in which of the following cases is z = 0 <u>not</u> an essential singularity of f

A) $z = 0$ is a limit point of zeros of f
B) $z = 0$ is a limit point of poles of f
C) f is bounded in a deleted neighborhood of $z = 0$
D) For any $\delta > 0$, $\{f(z) : 0 < |z| < \delta\}$ is dense in C

35. Let f be an entire function. If $\text{Re}\, f(z) \ge 0$ for all $z \in C$, then

A) f is not bounded on C B) Re f is not bounded on C
C) Im f is not bounded on C D) f reduces to a constant function on C

36. Let $\gamma(t) = 2e^{it}$, $0 \leq t \leq 2\pi$. The value of the integral

$$\frac{1}{2\pi i} \int_{\gamma} \frac{z^3}{z-1}\, dz \quad \text{is}$$

A) 0 B) 1 C) 3 D) 8

37. Let f and g be defined on C by $f(z) = e^z - 1$; $g(z) = \cos z - 1$. Then
A) Both f and g have simple zeros at $z = 0$
B) Both f and g have zero of order 2 at $z = 0$
C) f has a simple zero and g has a double zero at $z = 0$
D) f has a double zero and g has a simple zero at $z = 0$

38. Let f be a non constant entire function. Then which of the following statements is incorrect?
A) f maps open sets in C onto open sets
B) About each point z_0 in C, f has a power series expansion with radius of convergence infinity
C) There exists a point $z_0 \in C$ at which $|f(z)|$ attains its maximum.
D) f is infinitely many times differentiable on C and each $f^{(n)}$ is an entire function

39. Let f be an analytic function on the unit disk D such that $|f(z)| \leq 1$ on D and $f(0) = 0$. Then from the following pick the incorrect statement
A) $|f(z)| \leq |z|^2$ for all $z \in D$ B) $|f(z)| \leq |z|$ for all $z \in D$
C) $|f'(0)| \leq 1$ D) If $f'(0) = 1$ then $f(z) = z$ for all $z \in D$

40. From the following statements, pick the incorrect one; that completes the following sentence. There exists a one-to-one analytic function that maps the unit disk D onto
A) C B) D
C) $\{ z : \text{Re } z > 0\}$ D) $\{ z : \text{Im } z < 0\}$

41. Let Z be the ring of all integers. Which of the following is the sub ring generated by 4 and 10?
A) $\{ 4n : n \in Z\}$ B) $\{ 10n : n \in Z\}$
C) $\{ 5n : n \in Z\}$ D) $\{ 2n : n \in Z\}$

42. Let Z_{10} be the ring of integers mod 10. Then the number of solutions of the equation $x^2 + x = 0$ in Z_{10} is
A) 4 B) 3 C) 2 D) 1

43. Let f(x) be a polynomial of degree 4 and g(x) be a polynomial of degree 3 over the reals. Then the degree of $f(x) - g(x)$ is
A) 7 B) 4 C) 3 D) 1

44. Which of the following is an ideal in the ring of polynomials over the reals?
A) $\{f(x) : f(0) = 1\}$ B) $\{f(x) : f(1) = 0\}$
C) $\{f(x) : f(1) = 2\}$ D) $\{f(x) : f(2) = 1\}$

45. Which of the following is a maximal ideal of the ring Z of integers?
A) 8Z B) 6Z C) 4Z D) 3Z

46. Let $\varphi : F(x) \to F$ be a homomorphism given by $\varphi : f(x) \mapsto f(2)$. Then which of the following is a generator of the Kernel of φ?
A) $x^2 + 1$ B) $x^2 + 2$ C) $x + 2$ D) $x - 2$

47. Which of the following is a unit in the ring $Z\left(\sqrt{2}\right)$?
A) $1+\sqrt{2}$ B) $2+\sqrt{2}$ C) $1+2\sqrt{2}$ D) $1+3\sqrt{2}$

48. In the field Z_{17} of integers mod 17 what is 10^8?
A) 1 B) 8 C) 10 D) 16

49. Which of the following is an irreducible polynomial over the rationals?
A) $x^2 + x + 1$ B) $4x^2 + 8x - 5$
C) $4x^2 + 4x - 3$ D) $x^2 + 4x - 5$

50. Let K be an algebraic extension of a field F. Which of the following is true?
A) K is a finite extension of F
B) For each $a \in K$, F(a) is a finite extension of F
C) For each $a \in K$, the irreducible polynomial of a over F has degree 2
D) For each $a, b \in K$, F(a) and F(b) are isomorphic

51. Let K be a transcendental extension of F. Which of the following is true?
A) For every $a \in K$, F(a) is an infinite extension of F.
B) For every $a \in K$, with $a \notin F$, F(a) is isomorphic to K.
C) For every $a \in K$, there is a $b \in K$ such that a is algebraic over F(b).
D) For every $a, b \in K$, F(a) is isomorphic to F(b)

52. Which of the following is not a splitting field over Q?
A) $Q\left(\sqrt{2}+\sqrt{3}\right)$ B) $Q\left(\sqrt{2}+i\right)$
C) $Q\left(\sqrt[3]{2}\right)$ D) $Q\left(\sqrt{3}, \sqrt{5}\right)$

53. Which of the following is a basis for $\mathbf{R}^3$ over $\mathbf{R}$?
A) {(1,2,1), (1,3,1), (1,4,1)} B) {(1,1,2), (1,1,3), (1,1,4)}
C) {(1,1,1), (2,1,1), (3,1,3)} D) {(1,1,1), (2,1,1), (3,1,1)}

54. Consider the system of equations

$$3x + 4y + z = 0$$
$$2x + 3y + 2z = 0$$
$$x + 2y + 3z = 0$$

The dimension of the space of solutions is

A) 0 B) 1 C) 2 D) 3

55. Which of the following pairs of vectors is orthogonal in $\mathbf{R}^2$?

A) (1, 1) and (2, – 1) B) (1, 2) and (3, – 1)
C) (1, 3) and (3, 1) D) (1, 4) and (4, – 1)

56. Which of the following is an eigen vector of T: $\mathbf{R}^3 \to \mathbf{R}^3$ given by T (x, y, z) = (2x + y, y, z)

A) (1,1,0) B) (1,0,0) C) (0,1,1) D) (0,1,0)

57. Let α = (1 2 3) (4 5) be a permutation on 5 symbols. Then the order of α is

A) 2 B) 3 C) 5 D) 6

58. The number of non isomorphic abelian groups of order 40 is

A) 1 B) 2 C) 3 D) 4

59. Which of the following is not a cyclic group?

A) $Z_4 \oplus Z_5$ B) $Z_3 \oplus Z_{10}$ C) $Z_4 \oplus Z_6$ D) $Z_3 \oplus Z_8$

60. Let H be a normal subgroup of a finite group G and $a \in G$. Let $aH \in G/H$ be an element of order 3 in G/H. Then which of the following holds?

A) $a \in H$ B) $a^2 \in H$ C) $a^5 \in H$ D) $a^6 \in H$

61. Which of the following is a metric on the set R of reals?

A) $d(x, y) = |x + y|$ B) $d(x, y) = \max\{|x + y|, |x - y|\}$
C) $d(x, y) = |x| + |y|$ D) $d(x, y) = \max\{x - y, y - x\}$

62. Let $d(f,g) = \sup |f(x) - g(x)|$ be a metric on the set of functions from [0, 1] to **R**. For $f(x) = x$ and $g(x) = x^2$, d(f, g) =

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{\sqrt{2}}$ D) 1

63. Let $\mathbf{R}^2$ be the metric space with $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ for $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Then which of the following points is in the closed ball $S_1(0, 0)$.

A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ B) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ C) (1, 1) D) $\left(1, \frac{1}{2}\right)$

64. Let (x_n) be a sequence in the metric space of non negative reals with usual metric. Which of the following conditions ensures that (x_n) is a Cauchy sequence?

A) $\left|x_n^2 - 2\right| < 1$ for all n B) $\left|x_n^2 - 2\right| < \frac{1}{n}$ for all n

C) $\left|x_n^2 - 2\right| = 1$ for all n D) $\left|x_n^2 - 2\right| > 1$ for all n

65. Which of the following is a subbase for the usual topology of **R**?

A) $\{(a, \infty) : a \in \mathbf{R}\}$ B) $\{(a, a+1) : a \in \mathbf{R}\}$

C) $\{(n, n+1) :$ n is an integer$\}$ D) $\{(n, n+2) :$ n is an integer$\}$

66. Let X = {1,2,3,4,5} and τ = {X, ϕ, {1,2}, {1,2,3}} be a topology on X. Then the closure of {3, 4} is

A) {3,4} B) {2,3,4} C) {1,2,3,4} D) {3,4,5}

67. Which of the following is a compact subset of the real line?

A) the set N of naturals B) the set Z of integers

C) $\left\{1, 1+\frac{1}{2}, 1+\frac{1}{3}, ---, 1+\frac{1}{n}, ---\right\}$ D) $\left\{1, 1+\frac{1}{2}, 2+\frac{1}{3}, ---, n+\frac{1}{n+1}, ---\right\}$

68. Let f : **R** $\rightarrow$ Q be a continuous function from the real line to the subspace of rationals. Then which of the following is true?

A) f(1) = f(2) B) f(x+y) = f(x) + f(y) for all x, y

C) f is one –to-one D) f is onto

69. Let X be the two point discrete space and X^N be the product space where N is the set of naturals. Then which of the following is not true about X^N?

A) Discrete space B) Hausdorff space

C) Compact space D) Normal space

70. Which of the following subspaces of the real line **R** are homeomorphic?

A) Q andQ' where Q is the set of rationals and Q' is the set of irrationals

B) Q and N where N is the set of naturals

C) Q and Z where Z is the set of integers

D) N and Z

71. Let $\mathbf{R}^2$ be the normed linear space with usual norm. For which of the following x and y, $\|x+y\| = \|x\| + \|y\|$ holds

A) x = (1,1), y = (1,0) B) x = (2,1), y = (1,2)

C) x = (2,4), y = (3,6) D) x = (2,5), y = (3,6)

72. Which of the following is not a norm on $\mathbf{R}^2$?

A) $N(x,y) = |x| + |y|$ B) $N(x,y) = \max\{|x|, |y|\}$

C) $N(x,y) = \max\{|x+y|^2, |x-y|^2\}$ D) $N(x,y) = \sqrt{x^2+y^2}$

73. Let X be an inner product space and x, y $\in$ X with $\langle x,y \rangle = 0$. Which of the following is not necessarily true?

A) $\|x+y\| = \|x\| + \|y\|$ B) $\|x+y\|^2 = \|x\|^2 + \|y\|^2$

C) $\|x+y\| = \|x - y\|$ D) $\|x+2y\| = \|x - 2y\|$

74. Let R be a subspace and S be a subset of a finite dimensional inner product space. Then which of the following is true?

A) If $R^{\perp} = S^{\perp}$ then R = S B) If $R^{\perp} \subseteq S^{\perp}$ then R $\subseteq$ S

C) If $R^{\perp} = S^{\perp}$ then R $\subseteq$ S D) If $R^{\perp} = S^{\perp}$ then S $\subseteq$ R

75. Let $\mathbf{R}^2$ be the usual inner product space and u = (1,1). Define f_u: $\mathbf{R}^2 \to \mathbf{R}$ by $f_u(x) = <x, u>$. Then $\| f_u \|$ =

A) 1 B) 2 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$

76. Let E be a countable orthonormal set in an inner product space X and x $\in$ X. Then which of the following is true?

A) $\sum_{u \in E} |\langle x, u\rangle|^2 = \|x\|^2$ B) $\sum_{u \in E} |\langle x, u\rangle|^2 \le \|x\|^2$

C) $\sum_{u \in E} |\langle x, u\rangle| = \|x\|$ D) $\sum_{u \in E} |\langle x, u\rangle| > \|x\|$

77. Let X be a Hilbert space and E be a finite orthonormal set in X. For each x $\in$ X let $p(x) = \sum_{u \in E} \langle x, u\rangle u$. Then which of the following is true about p?

A) p (x) = x for all x $\in$ X
B) p (x) = x for all x $\in$ span E
C) p (x + y) = p(x) for all x $\in$ span E
D) p (x) = p(y) for all x, y $\in$ span E

78. Let X = $\mathbf{R}^2$ and X_0 = {(x,0): x $\in$ $\mathbf{R}$}. Define g(x,0) = x for all (x,0) $\in$ X_0. Then which of the following is a Hahn Banach extension of g?

A) f (x, y) = x + y B) f (x, y) = x + 2y

C) f (x, y) = 2x + y D) f (x, y) = $|x + y|$

79. Let $\ell^2 \to \ell^2$ be the right shift operator given by $(x_1, x_2, ---) \mapsto (0, x_1, x_2, ---)$. Then which of the following is true about the adjoint A*?

A) N (A^*) is of dimension 1 B) R (A^*) is of dimension 1

C) N $(A^*)^{\perp}$ is of dimension 1 D) R $(A^*)^{\perp}$ is of dimension 1

80. Let X = C^2 be the usual inner product space. Let A be the linear operator on X defined by A $(x_1, x_2) = (\alpha x_1, \beta x_2)$ for $\alpha, \beta \in$ C. Then which of the following is true about A?

A) Self adjoint if $\alpha = \beta$ B) Unitary if $\alpha = \beta$

C) Self adjoint if $|\alpha| = |\beta| = 1$ D) Unitary if $|\alpha| = |\beta| = 1$
