A

19721 **120 MINUTES**

1. If a, b, c are integers and a + c = b, then equation $ax^2 - bx + c = 0$ *has*
   A) equal roots B) irrational roots
   C) rational roots D) imaginary roots

2. Let G be the graph of $y = 2^x$. Then which one of the following is not true?
   A) G does not pass through the origin
   B) G cuts both x-axis and y-axis
   C) G lies only in the first and second quadrants
   D) y is an increasing function.

3. If (1, 2) is the midpoint of the segment of a straightline intercepted between the axes then the equation of the line is
   A) $2x + y = 4$ B) $x + 2y = 4$ C) $2x + y = 2$ D) $x + 2y = 2$.

4. The equation of the circle which touches the lines x = 0, y = 0, x = a, y = a is
   A) $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$
   B) $2x^2 + 2y^2 - 2ax - 2ay + a^2 = 0$
   C) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$
   D) $x^2 + y^2 - ax - ay + a^2 = 0$

5. If the focus, centre and eccentricity of an ellipse are respectively (1, 2), (2, 3) and $^1/_2$, then the equation of the minor axis is:
   A) $x + y - 3=0$ B) $x - y + 1=0$ C) $x + y + 1=0$ D) $x + y - 5 = 0$.

6. The distance of the origin from the plane $3x - 6y + 2z - 14=0$ is
   A) $\frac{1}{2}$ B) 2 C) 7 D) 14

7. $\int \cos x \ \mathrm{cosec}^2 x dx$ is equal to
   A) cosec x + c B) −cosec x + c C) cot x + c D) − cot x + c

8. Find the area bounded by the curves $y = |x + 2|$, $x = -3$, $x = 2$ and the x-axis.

   A) $\frac{49}{2}$ sq. units B) $\frac{33}{2}$ sq. units C) $\frac{17}{2}$ sq. units D) $\frac{15}{2}$ sq. units

9. A bag contains 10 tickets numbered 1, 2,..., 10 of which 4 are drawn at random and arranged in ascending order $x_1 < x_2 < x_3 < x_4$. What is the probability that $x_3 = 7$.

   A) $^1/_{10}$ B) $^3/_{10}$ C) $^3/_{35}$ D) $^9/_{42}$

10. Let $f_n(x) = x^n$ be a sequence of functions defined on [0, 1]. Let $f(x) = \begin{cases} 0 & if\ x < 1 \\ 1 & if\ x = 1 \end{cases}$ and $g(x) = 0$ for all x. Then which of the following is true?

A) f_n converges to f pointwise and not uniformly
B) f_n converges to g pointwise and not uniformly
C) f_n converges to f uniformly
D) f_n converges to g uniformly

11. Consider f(x) defined on [0, 1] as follows. $f(x) = \begin{cases} 0 & if\ x\ is\ rational \\ 1 & otherwise \end{cases}$
Then which of the following is true?

A) f is Riemann integrable and $\int f = 1$
B) f is Riemann integrable and $\int f = 0$
C) f is Lebesgue integrable and $\int f = 1$
D) f is Lebesgue integrable and $\int f = 0$

12. Consider the following statements about a non measurable subset A of $\mathbb{R}$.
(i) $A \cup B$ is non measurable for all $B \subseteq \mathbb{R}$.
(ii) $A \cap B$ is measurable for some $B \subseteq \mathbb{R}$.
(iii) $A + x$ is measurable for all $x \in \mathbb{R}$.
Then which of the following holds.

A) (i) and (ii)
B) (i) and (iii)
C) (i) holds and (iii) does not hold
D) (ii) holds and (iii) does not hold

13. The real part of $\frac{1+2i}{1-i}$ is

A) 1 B) 2 C) −1 D) −2

14. If the radius of convergence of the power series $\sum a_n z^n$ is 2 then the radius of convergence of $\sum a_n z^{2n}$ is

A) 2 B) $\sqrt{2}$ C) 4 D) $\frac{1}{2}$

15. Which of the following is a harmonic conjugate of $u(x) = x^2 - y^2 + x$.
A) $x + 2xy$ B) $y + 2xy$ C) $y^2 - 2xy$ D) $x^2 + 2xy$

16. The residue of $f(z) = \frac{e^z}{(z+1)^2}$ at $z = -1$ is

A) e B) $^1/_e$ C) $^e/_2$ D) $e^2/2$

17. The order of the subgroup generated by (12) and (34) in the symmetric group S_4 is

A) 2 B) 4 C) 6 D) 12

18. Let f be a non trivial homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{15}$. Then which of the following holds?

A) Im f is of order 10. B) Ker f is of order 5.

C) Ker f is of order 2. D) f is a one to one map.

19. Let G be a group of order 70. Then the number of 5-Sylow subgroups of G is

A) 1 B) 3 C) 5 D) 7

20. Which of the following is a zero divisor in the polynomial ring $\mathbb{Z}_{12}[x]$?

A) 1+ x B) 2+ x C) 3+2x D) 4+2x

21. Which of the following is an irreducible polynomial over the rationals?

A) $x^3 +2x +3$ B) $x^3 +3x^2 +6$ C) $2x^3 + x^2 +1$ D) $x^3 - 2x +1$

22. Let α be the real cube root of 2 and let $\mathbb{Q}$ be the field of rationals. Then the degree $[\mathbb{Q}_{(\alpha)} : \mathbb{Q}]$ equals:

A) 1 B) 2 C) 3 D) 4

23. Let A be a 3 × 3 matrix such that $A^3 - 2A^2 - I = 0$ where I is the identity matrix. Then A^{-1} equals:

A) A B) A^2 C) $A^2 - 2A$ D) $A^2 +2A$

24. Consider the following system of linear equations.

$2x +3y + z = 1$

$3x +2y +4z = 4$

$x + y +z = 2$

Then which of the following is true about the system?

A) It has a unique solution.

B) It has exactly two solutions.

C) If has infinitely many solutions.

D) It has no solution.

25. Let S be the subspace of $\mathbb{R}^3$ spanned by (1, 0, 1). Then which of the following subspace W has the property that $\mathbb{R}^3 = S \oplus W$?

A) W = span of {(1, 1, 1), (2, 1, 1)}

B) W = span of {(1, 1, 1), (1, 2, 1)}

C) W = span of {(1, 1, 1), (0, 1, 0)}

D) W = span of {(1, 1, 1), (2, 0, 2)}

26. Let $f: \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation given by $f(x_1, x_2, x_3, x_4) = (x_1, x_1, x_1, x_4 - x_1)$. Then dimension of null space of f is

A) 0 B) 1 C) 2 D) 3

27. Which of the following is a diagonalizable matrix?

A) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

28. Let $(x-1)^2 (x-2)^3$ be the characteristic polynomial of a diagonalizable matrix. Then its minimal polynomial is
A) $(x-1)(x-2)$ B) $(x-1)(x-2)^3$
C) $(x-1)^2(x-2)$ D) $(x-1)^2(x-2)^3$

29. Which of the following is not true in the case of divisibility and gcd.
A) If $a \mid bc$ and if $(a,b) = 1$, then $a \mid c$
B) If $(a,b)=(a,c)=1$, then $(a,bc)=1$
C) If $(a,b)=1$, then $(a+b, a-b)$ is either 1 or 3
D) If $(a,b)=1$ and if $d \mid (a+b)$, then $(a,d)=(b,d)=1$.

30. The number of integers n, $1 \le n \le 10$ such that $\varphi(n)=\varphi(2n)$, where $\varphi(n)$ is the Euler totient function, is
A) 1 B) 2 C) 3 D) 4

31. If the solution of the linear congruence equation $7x \equiv 6 \pmod{15}$ is of the form $x \equiv 6 \cdot 7^n \pmod{15}$, then n equals
A) 3 B) 6 C) 7 D) 8

32. The differential equation of the family of circles touching the y-axis at the origin is
A) $2xy\frac{dy}{dx} + x^2 = y^2$ B) $x^2 - 2xy\frac{dy}{dx} = y^2$
C) $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$ D) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0.$

33. The particular solution of the equation $y'' + y = \tan x$ is
A) $y = \sin x \cos x - \cos x \int \sin x \tan x \, dx$
B) $y = -\sin x \cos x - \cos x \int \sin x \tan x \, dx$
C) $y = \sin x \cos x + \cos x \int \sin x \tan x \, dx$
D) $y = -\sin x \cos x + \cos x \int \sin x \tan x \, dx$

34. If $P_n(x)$ denotes the n^{th} degree Legendre polynomial then find the value of $\int_{-1}^{1} P_3^2(x)dx$
A) $^2/_5$ B) $^2/_7$ C) $^2/_3$ D) $^2/_9$

35. The integral of the equation $(4x + yz)dx + (xz - 2y)dy + (xy - 2z)dz = 0$ is
A) $2x^2 + y^2 + z^2 - xyz = c$ B) $4x^2 - 2y^2 - 2z^2 + xyz = c$
C) $2x^2 - y^2 - z^2 - xyz = c$ D) $2x^2 - y^2 - z^2 + xyz = c$

36. The auxiliary equations for finding a complete integral of the equation $p + q + pq = 0$ by Charpit's method are
A) $\frac{dx}{1+q} = \frac{dy}{1+p} = \frac{dz}{p+q+2pq} = \frac{dp}{0} = \frac{dq}{0}$ B) $\frac{dx}{1+p} = \frac{dy}{1+q} = \frac{dz}{p+q+2pq} = \frac{dp}{0} = \frac{dq}{0}$

C) $\frac{dx}{p+1} = \frac{dy}{q+1} = \frac{dz}{p+q+2pq} = \frac{dp}{p} = \frac{dq}{q}$ D) $\frac{dx}{1+q} = \frac{dy}{1+p} = \frac{dz}{p+q+2pq} = \frac{dp}{p} = \frac{dq}{q}$

37. The value of m such that the equation $xu_{xx} + mu_{xy} + yu_{yy} - 2u_x = 0$ is parabolic is

A) xy B) $\sqrt{xy}$ C) 2xy D) $-2\sqrt{xy}$

38. Let d be a metric on the set $\mathbb{N}$ of all natural numbers defined by $d(x, y) = |x - y|$. Then which of the following is not true in this space.
A) {1} is an open set. B) {1} is a closed set.
C) {1, 2} is an open set. D) every open ball is a closed ball

39. Let $\mathbb{R}$ be the set of all reals. Then which of the following is a metric on $\mathbb{R}$.
A) $d(x,y) = \max\{|x|, |y|\}$ B) $d(x,y)= x^2 + y^2$

C) $d(x,y)= \frac{|x-y|}{1+|x-y|}$ D) $d(x,y)= 1 + |x - y|$

40. Let $\mathbb{R}$ be a topological space with base $\{(a, \infty): a< 0\}$. Then which of the following is a limit of the sequence $x_n =(-1)^n$.
A) 0 B) 1 C) -1 D) 2

41. Let X be the normed linear space $\mathbb{R}^2$ with norm $\|\|_p$. Then the value of p for which X is strictly convex is
A) 1 B) 2 C) 3 D) ∞

42. Let $X = \mathbb{R}^2$ with norm $\|\|_1$ and $A \in BL(X)$ be represented by the matrix $M = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$
Then $\|A\|$ is equal to
A) 3 B) 4 C) 5 D) 6

43. Let H be the Hilbert space l^2 and $S = \{(0, 1, 1, 0,...) , (1, 1, 1, 0,...)\}$. Then the set whose linear span is equal to the linear span of S is

A) $\left\{(1,0,1,0,\ldots),(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,\ldots)\right\}$
B) $\left\{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,\ldots\right),(1,0,1,0,\ldots)\right\}$
C) $\left\{(1,0,0,0,\ldots),(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,\ldots)\right\}$
D) $\left\{(1,0,0,0,\ldots),(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,\ldots)\right\}$

44. Let R be a relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$ such that $((a,b),(c,d)) \in R$ iff a-d=b-c. Which one of the following is true about R?
A) Reflexive but not symmetric
B) Symmetric but not reflexive
C) Both reflexive and symmetric
D) Neither reflexive nor symmetric

45. If □, □, □ are the roots of $2x^3 + x^2 - 2x - 1 = 0$, then the value of $\alpha^2 + \beta^2 + \gamma^2$.

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) $\frac{9}{4}$

46. If $\alpha_1, \alpha_2, \cdots, \alpha_{2019}$ are the roots of $x^{2019} + 1 = 0$. Then the value of the product $(1 + \alpha_1)(1 + \alpha_2) \cdots (1 + \alpha_{2019})$ is

A) 0 B) -1 C) 1 D) 2019

47. If $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = L$ and $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = M$, then

A) L exists but M does not B) L does not exist but M exists

C) Both L and M exist D) Both L and M do not exist

48. The domain of the functions f defined by f(x) $= \frac{\sqrt{-x}}{(x-3)(x+5)}$ is

A) $(-\infty,-5)$ U $(-5,3)$ U $(3,\infty)$ B) $(-\infty,5]$ U $(3,\infty]$

C) $(-\infty,-5)$ U $(-5,0]$ D) $(-\infty,3)$ U $(3,\infty)$

49. Which of the following sets of functions is countable?

i) $\{ f \mid f : \mathbf{N} \to \{0,1\}\}$

ii) $\{ f \mid f : \{0,1\} \to \mathbf{N} \}$

iii) $\{ f \mid f : \mathbf{N} \to \{0,1\},\ f(1) \leq f(2)\}$

iv) $\{ f \mid f : \{0,1\} \to \mathbf{N},\ f(0) \leq f(1)\}$

A) (i) and (iii) B) (ii) and (iv) C) (i) only D) (ii) only

50. The equation of the plane which passes through (1,2,3) and parallel to the plane $4x + 5y - 3z = 7$ is

A) $3x + 4y - 3z = 7$ B) $4x + 5y - 3z = 5$

C) $5x - 4y + z = 3$ D) $4x + 5y - 3z + 7 = 0$

51. For what value of k is the function f(x)=$\begin{cases} \frac{1-\cos 2x}{2x^2} & , x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at x = 0?

A) 0 B) $\frac{1}{2}$ C) 1 D) 2

52. Find $\frac{dy}{dx}$ if y=$\tan^{-1}\sqrt{\frac{1+sinx}{1-sinx}}$

A) $\frac{1}{2(1+x^2)}$ B) $\frac{1}{2}$ C) $\frac{\pi}{4}+\frac{x}{2}$ D) $\frac{2}{1+x^2}$

53. If the radius of a circle is increasing at the rate of 5.5cm/s then how fast is the area of the circle increasing when the radius of the circle is 6cm?

A) 12□ cm^2/s B) 36π cm^2/s C) 60□ cm^2/s D) 66□ cm^2/s

54. The value of the definite integral $\int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\cos\left(\frac{1}{x}\right)}{x^2} dx$

A) -1 B) 0 C) 1 D) $\frac{\pi}{2}$

55. The number of different symmetric square matrices of order n with each element being either 0 or 1 is

A) 2^n B) 2^{n^2} C) $2^{\frac{n^2+n}{2}}$ D) $2^{\frac{n^2-n}{2}}$

56. $\lim\limits_{n\to}\left(\frac{1}{n^2+1}+\frac{2}{n^2+2}+\cdots+\frac{n}{n^2+n}\right)$ is

A) 0 B) $\frac{1}{2}$ C) 1 D) ∞

57. Let $\sum_{n=1} x_n$ be a series of real numbers. Which of the following is true?

A) If $\sum_{n=1} x_n$ is convergent then $\sum_{n=1} x_n$ is absolutely convergent

B) If $\sum_{n=1} x_n$ is divergent, then $\{x_n\}$ does not converge to 0

C) If $x_n \to 0$ then $\sum_{n=1} x_n$ is convergent

D) If $\sum_{n=1} x_n$ is convergent then $x_n^2 \to 0$ as $n \to$

58. The number of discontinuities of a monotone function is
A) finite B) infinite
C) countable D) uncountable

59. The value of $\sqrt{i} + \sqrt{-i}$ is
A) 0 B) 1 C) i D) $\sqrt{2}$

60. The function $f(z) = \frac{e^z+1}{e^z-1}$ has
A) a removable singularity at $z = 0$
B) a simple pole at $z = 0$ with residue 1
C) a simple pole at $z = 0$ with residue 2
D) an essential singularity at $z = 0$

61. The bilinear transformation which maps the points $z = 1, -i, -1$ into the points $w = i, 0, -i$ is
A) $\frac{i(1-z)}{1+z}$ B) $\frac{i(1+z)}{1-z}$ C) $\frac{z-i}{1+iz}$ D) $\frac{z+i}{z-i}$

62. The value of the integral $\int_c \frac{e^{-z}}{z+1}\, dz$ where c is the circle $|z| = {}^1/_2$ is
A) $2\pi i$ B) $2\pi i e$ C) 0 D) $4\pi i$

63. Let F be a field of order 256. Then
A) F has a subfield of order 8 B) F has a subfield of order 16
C) F has a subfield of order 32 D) F has a subfield of order 64

64. Which of the following is not true?

A) Every cyclic group is abelian
B) Every group of odd order is cyclic
C) The order of a cyclic group and that of its generating element are same
D) Every subgroup of a cyclic group is cyclic

65. The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 5 & 6 & 3 & 2 & 8 & 7 \end{pmatrix}$ in S_8 is

A) 4 B)6
C) 8 D) 16

66. The order of the element (1,2) in $\mathbb{Z}_5 \times \mathbb{Z}_{10}$ is
A) 5 B) 10
C) 15 D) 20

67. The splitting field of the set of polynomials $\{x^2-2 , x^2-3\}$ over $\mathbb{Q}$ is
A) $\mathbb{Q}(\sqrt{2})$ B) $\mathbb{Q}(\sqrt{3})$ C) $\mathbb{Q}(\sqrt[3]{2})$ D) $\mathbb{Q}(\sqrt{2},\sqrt{3})$

68. The gcd of 3+4i and -4+3i in the integral domain $(\mathbb{Z}[i] , + , .)$ is
A) 3+4i B) - 4+3i
C) Both A and B D) neither A nor B

69. Which of the following is not true?
A) If A is a m×n matrix and B is an n × p matrix, then rank(AB) ≤ min{rank(A),rank(B)}
B) If A is a m×n matrix and B is a non singular matrix of order n, then rank(AB)= rank A
C) If A is a m×n matrix and B is a n×p matrix, then rank(AB)≤ rank(A)
D) If A is a m×n matrix and B is a n×p matrix, then rank(AB)=min{rank(A),rank(B)}

70. Let W be the solution space of the system of homogeneous equations $2x+2y+z=0$, $3x+3y-2z=0$, $x+y-3z=0$. Then dim W is
A) 0 B) 1
C) 2 D) 3

71. Which of the following is not a linear transformation?

A) T:$\mathbb{R}^3 \to \mathbb{R}^3$ defined by T(x,y,z) =(x+y , x+z+2 , y+z)

B) T:$\mathbb{R}^2 \to \mathbb{R}^2$ defined by T(x,y)=(x.0)

C) T:$\mathbb{R}^2 \to \mathbb{R}^2$ defined by T(x,y)=(y,x)

D) T:$\mathbb{R}^3 \to \mathbb{R}^2$ defined by T(x,y,z)=(y,z,x)

72. The solution of the linear congruence $4x \equiv 3(mod\ 7)$ is

A) 2(mod7) B) 4(mod 7) C) 6(mod 7) D) 8(mod 7)

73. The integrating factor of the differential equation $(xy^2 - e^{x^{\frac{1}{3}}})dx - x^2ydy = 0$ is

A) $\frac{-4}{x}$ B) x^4 C) $\frac{x}{4}$ D) $\frac{1}{x^4}$

74. The wronskian of the differential equation $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$ is

A) 2 B) cos2x C) sin2x D) $\frac{1}{2}$

75. If $J_n(x)$ is the Bessel's function of order $n, n \in \mathbb{Z}$. Then

A) $J_{-n}(x) = -J_n(x)$

B) $J_{-n}(x) = J_n(-x)$

C) $J_n(x)$and$J_{-n}(x)$ are independent

D) $J_{-n}(x) = (-1)^n J_n(x)$

76. The generating function for the Legendre polynomial $P_n(x)$ is

A) $(1 + 2xz + z^2)^{\frac{1}{2}}$ B) $(1 - 2xz + z^2)^{\frac{1}{2}}$

C) $(1 - 2xz + z^2)^{\frac{-1}{2}}$ D) $(1 + 2xz + z^2)^{\frac{-1}{2}}$

77. The order and degree of the partial differential equation $\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial z}\right)^3$ are

A) 2, 3 B) 3, 2 C) 2, 1 D) 3, 1

78. Which of the following is not true?

A) The product of two T_1 spaces is a T_1 space

B) The product of two completely regular spaces is completely regular

C) The product of two first countable spaces is first countable

D) The product of two second countable space is second countable

79. Which of the following is not a Banach space?

A) K^n B) p C) c_{00} D) $L^p(E)$

80. If $\{x_1,x_2,x_3\}$ is an orthogonal set of an inner product space X with $||x_i||=2$, i=1,2,3, then $||x_1+x_2+x_3||^2$ is

A) $2\sqrt{3}$ B) 6 C) 12 D) 36