1. Let f(x) be a function on the set of reals defined by $f(x) = 2 + x - [x - 2]$ where [x] denotes the greatest integer less than or equal to x. Then the range of f is

A) [0, 2] B) [2, 3] C) [2, 4] D) [4, 5)

2. The set of values of x for which $2x^2 - 7x + 3$ is negative is

A) [1/2, 3] B) (1/2, 3)
C) (–3, –1/2) D) [–3, –1/2]

3. Which of the following is a countable set ?

A) $\{(x, y) \in \mathbb{R}^2 : 0 \le x \le 1, y = 0\}$
B) $\{(x, y) \in \mathbb{R}^2 : 0 \le x \le 1, 0 \le y \le 1\}$
C) $\{(x, y) \in \mathbb{R}^2 : x, y \text{ are rationals}\}$
D) $\{(x, y) \in \mathbb{R}^2 : x, y \text{ are irrationals}\}$

4. If the equation $x - \sqrt{3}\,y + 2 = 0$ reduces to its normal form $x \cos \alpha + y \sin \alpha = p$ then which of the following is true ?

A) $\alpha = 2\pi/3, p = 2$ B) $\alpha = 2\pi/3, p = 1$
C) $\alpha = \pi/6, p = 1$ D) $\alpha = 2\pi/3, p = -1$

5. The length of the y – intercept on the positive y–axis made by the circle for which (–4, 3) and (12, –1) are extremities of a diameter is

A) $1 + \sqrt{52}$ B) $\sqrt{52} - 1$
C) $4 + \sqrt{67}$ D) $4 - \sqrt{67}$

6. The equation of the parabola with focus (–1, –1) and directrix 2y = 3 is

A) $4x^2 + 8x + 20y - 1 = 0$ B) $3y^2 + 2x + 8y - 7 = 0$
C) $4x^2 + 2x + 4y - 1 = 0$ D) $4y^2 - 8x - 14y + 7 = 0$

7. The distance between the planes $2x + 3y - 6z + 12 = 0$ and $4x + 6y - 12z + 3 = 0$

A) 9 B) 15 C) 3/2 D) 9/7

8. The angle between the line joining (3, 2, –2) and (4, 1, –4) and the line joining (4, –3, 3) and (6, –2, 2) is

A) $\pi/6$ B) $\pi/4$
C) $\pi/3$ D) $\pi/2$

9. The value of $\lim_{x \to 1} \frac{x^x - 1}{x \log x}$ is

A) 0 B) 1 C) 2 D) ∞

10. If $y = \sin(e^{-x} \log x)$ then $\frac{dy}{dx} =$

A) $\cos(e^{-x} \log x)$

B) $\cos(e^{-x} \log x) \left(\frac{1}{x} - e^{-x}\right)$

C) $\cos\left(-e^{-x} + \frac{1}{x}\right)$

D) $\cos(e^{-x} \log x) \left(\frac{1 - x\log x}{xe^x}\right)$

11. $\int_0^1 x(1-x)^{20}\, dx =$

A) 1/380 B) 1/420 C) 1/400 D) 1/462

12. The area between one arch of the curve $y = \cos 4x$ and the x – axis is

A) 4/3 B) 3/4 C) 1/2 D) 1/4

13. A fair die is tossed twice. The probability that 3 turns up at least once is

A) 1/36 B) 11/36 C) 1/6 D) 1/3

14. Let $a_n = \sqrt{\frac{n^2 + 5n + 1}{n^2 + 3n + 1}}$. Then $\lim_{n \to \infty} a_n =$

A) 0 B) 1 C) 3 D) 5

15. $\lim_{x \to 0} x\sin(1/x) =$

A) 0 B) 1 C) $1/\pi$ D) π

16. Let $f_n(x) = \begin{cases} x^n : 0 \le x \le (1/2) \\ (1-x)^n : (1/2) \le x \le 1 \end{cases}$ and let $f(x) = \lim_{n \to \infty} f_n(x)$ for $x \in [0, 1]$.

Then which of the following is true ?

A) $f(x) = 0$ for all x

B) $f(0) = 1$

C) $f(1/2) = 1/2$

D) $f(1) = 1$

17. Let $f(x, y) = \begin{cases} \dfrac{xy - y}{x^2 - y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$

Then $\lim_{(x, y) \to (1, 1)} f(x, y)$ along $y = 1$ is

A) 0 B) 1 C) 1/2 D) –1/2

18. Let $f(x) = x^2$ and $\alpha(x) = x + 1$. Then $\int_0^1 f \, d\alpha =$

A) 1/3
B) 1 + (1/3)
C) 2 + (1/3)
D) 3

19. Let $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{otherwise} \end{cases}$

Let m denote the Lebesgue measure. Then $\int_0^1 f \, dm =$

A) 0 B) 1 C) 2 D) ∞

20. Let μ denote the outer measure on $\mathbb{R}$ defined as follows

$\mu(E) = \begin{cases} 0 & \text{if } E \text{ is countable} \\ 1 & \text{otherwise} \end{cases}$

Then which of the following is a μ – measurable set ?

A) Closed interval [0, 1]
B) Open interval (0, 1)
C) The set of all positive reals
D) The set of all nonzero reals

21. The real part of $\left(\dfrac{1 + i\sqrt{3}}{2}\right)^3$ is

A) 0 B) 1 C) –1 D) 1/2

22. If (r, θ) is the polar representation of the complex number $\dfrac{1 + i\sqrt{2}}{2}$ then r =

A) 1 B) 1/2 C) 1/4 D) $\sqrt{3}/2$

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23. Let e_1, e_2, e_3, e_4, e_5 be the fifth roots of unity. Then $|e_1 + e_2 + e_3 + e_4 + e_5| =$

A) 0 B) 1 C) 4 D) 5

24. Which of the following is not an analytic function in the complex plane ? Here $z = x + iy$.

A) $f(z) = xy + 2i$
B) $f(z) = 1 + x + iy$
C) $f(z) = x + i(y + 1)$
D) $f(z) = x^2 - y^2 + 2ixy$

25. Find $n(\gamma ; 0)$ where γ is the curve given by $\gamma(t) = e^{4\pi it} : 0 \leq t \leq 1$.

A) 0 B) 1 C) 2 D) 3

26. Let γ be the circle $|z| = 2$. Then $(1/2\pi i)\int_{\gamma} \frac{\sin z}{z-(\pi/2)} =$

A) 0 B) 1 C) $\pi/2$ D) π

27. Which of the following is true about the function $f(z) = \sin(1/z)$?

A) $\lim_{z \to 0} f(z) = 0$

B) $|f(z)| > 1$ for $|z| < 1$

C) there exists z with $|z| < 1$ such that $|f(z)| > 2$

D) there exists real z with $|z| > 1$ such that $|f(z)| > 1$

28. Which of the following pairs of groups are isomorphic ?

A) $\mathbb{Z}_{10} \oplus \mathbb{Z}_{10}$ and $\mathbb{Z}_{100}$
B) $\mathbb{Z}_{10} \oplus \mathbb{Z}_5$ and $\mathbb{Z}_{50}$
C) $\mathbb{Z}_{10} \oplus \mathbb{Z}_6$ and $\mathbb{Z}_{60}$
D) $\mathbb{Z}_{10} \oplus \mathbb{Z}_7$ and $\mathbb{Z}_{70}$

29. Let α be a permutation of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ defined by $\alpha(n) = 9 - n$. Then $\alpha =$

A) (8 7 6 5 4 3 2 1)
B) (1 8 7) (3 6 5) (2 4)
C) (1 8) (2 7) (3 4 5 6)
D) (1 8) (2 7) (3 6) (4 5)

30. Let H be the subgroup of the symmetric group S_4 generated by (1 2) (3 4). Then which of the following is a member of the coset H(1 2 3) ?

A) (1 3 2) B) (1 3 4) C) (1 2 4) D) (1 4 3 2)

31. The order of the commutator subgroup of the group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ of quaternion units is

A) 1 B) 2 C) 4 D) 8

32. The number of mutually non isomorphic abelian groups of order 8 is

A) 1 B) 2 C) 3 D) 4

33. The number of homomorphisms from the additive group $(\mathbb{Q}, +)$ of rationals to the additive group $(\mathbb{Z}, +)$ of integers is

A) 1 B) 2 C) 4 D) infinite

34. Which of the following is a zero divisor in the ring $\mathbb{Z}_{25}$ of integers mod 25 ?

A) 3 B) 9 C) 10 D) 12

35. Which of the following is not an irreducible polynomial in $\mathbb{Z}_2[x]$?

A) $x^4 + x^2 + 1$ B) $x^4 + x^3 + 1$

C) $x^3 + x^2 + 1$ D) $x^3 + x + 1$

36. Let $\mathbb{R}[x]$ be the ring of polynomials over the reals and I be the ideal generated by $x^2 + 1$. Then $I + (x^2 + x + 1) =$

A) $I + x$ B) $I + x^2$ C) $I + (x + 1)$ D) $I + (x^2 + 1)$

37. Let $\phi : \mathbb{Z}[x] \to \mathbb{R}$ be the ring homomorphism given by $f(x) \mapsto f(1 + \sqrt{2})$. Then $\ker\phi =$

A) $\langle x^2 - 2x - 1 \rangle$ B) $\langle x^2 - 2 \rangle$

C) $\langle x^2 - x - 2 \rangle$ D) $\langle x^2 - 2x - 2 \rangle$

38. The order of the group of units in the ring $\mathbb{Z}_{25}$ of integers mod 25 is

A) 24 B) 20 C) 16 D) 12

39. Let $\alpha = \sqrt{2}$ and $\beta = \sqrt[3]{2}$. Then the degree $[\mathbb{Q}(\alpha, \beta) : \mathbb{Q}] =$

A) 3 B) 4 C) 5 D) 6

40. Let K be a field of order 3^8 and F be a subfield of K. Then which of the following is a possible order of F ?

A) 3^3 B) 3^4 C) 3^5 D) 3^6

41. The number of automorphisms of the field $Q(\sqrt[3]{2})$ is

A) 1 B) 2 C) 3 D) 6

42. Let A be a 3 × 3 matrix which is nilpotent. Then which of the following is not true of A.

A) The rank of A is less than 3
B) A is invertible
C) 0 is an eigen value of A
D) det A = 0

43. Let $A = (a_{ij})$ be a 10 × 10 matrix where $a_{ij} = \begin{cases} 1 \text{ if } i < j \\ 0 \text{ otherwise} \end{cases}$. Then det A =

A) 0 B) 1 C) –1 D) 10

44. The rank of the matrix $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is

A) 1 B) 2 C) 3 D) 4

45. Consider the system of equations

$2x + 3y + 4z = 1$
$3x + 2y + z = 2$
$x + y + z = 3$

Then which of the following is true ?

A) The system has a unique solution
B) The system has exactly two solutions
C) The system has infinitely many solutions
D) The system has no solution

46. Let W be the subspace of $\mathbb{R}^3$ spanned by (1, 2, 1) and (0, 1, 1). Then which of the following is in W ?

A) (1, 2, 3)
B) (2, 3, 4)
C) (2, 3, 1)
D) (1, 1, 1)

47. If {(1, 1, 1), (1, 2, 1), (x, 1, 0)} is a linearly dependent set in $\mathbb{R}^3$ then x =

A) 0 B) 1 C) 2 D) 3

48. Let W be the subspace of $\mathbb{R}^2$ spanned by (1, 2). Then which of the following lines represents W ?

A) $x + 2y = 0$
B) $2x + y = 0$
C) $x - 2y = 0$
D) $y - 2x = 0$

49. Which of the following is not a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$?

A) $f(x, y) = (2x + y, 0)$
B) $f(x, y) = (x, x + y)$
C) $f(x, y) = (x + 1, y + 1)$
D) $f(x, y) = (x, x)$

50. Let $f : \mathbb{R}^4 \to \mathbb{R}^4$ be defined by $f(x, y, z, t) = (x - y, x - z, x - t, 0)$. Then dimension of null space of f is

A) 0
B) 1
C) 2
D) 3

51. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator which is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

Then which of the following is also a matrix representing T ?

A) $\begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$
B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C) $\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$
D) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

52. Which of the following is a linear functional on $\mathbb{R}^3$ which annihilates the subspace $\{(x, y, z) : z = 0\}$?

A) $f(x, y, z) = x - y + z$
B) $f(x, y, z) = x - z$
C) $f(x, y, z) = y - z$
D) $f(x, y, z) = z$

53. Which of the following is an eigen value of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$?

A) 0
B) 1
C) 2
D) 3

54. Which of the following is a non diagonalizable matrix ?

A) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
B) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$
C) $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$
D) $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

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55. Let (a, b) denote the GCD of two numbers a and b. Then which of the following is not true ? Here a, b, c are positive integers.

A) ((a, b), c) = (a, (b, c))

B) (ac, bc) = (a, b) c

C) If (a, b) = (a, c) = 1 then (a, bc) = 1

D) If (a, b) = 1 then (a + b, a – b) = 1

56. Let ϕ denote the Euler totient function. Then $\phi(187) =$

A) 160

B) 186

C) 28

D) 26

57. If x = a + b satisfies the congruence relation $4x \equiv 3 \pmod{15}$ then which of the following is true ?

A) If a = 4 then b is a multiple of 12

B) If a = 4 then b is a multiple of 15

C) If a = 12 then b is a multiple of 15

D) If a = 10 then b is a multiple of 15

58. If a and b are two solutions of the system

$x \equiv 3 \pmod 5$

$x \equiv 5 \pmod 7$

$x \equiv 7 \pmod{11}$

then which of the following is necessarily true ?

A) $a \equiv b \pmod{105}$

B) $a \equiv -b \pmod{105}$

C) $a \equiv b \pmod{385}$

D) $a \equiv -b \pmod{385}$

59. The differential equation which represents the family of curves $y^2 = -x^2 + cx$ is

A) $2yy'x = x^2 - y^2$

B) $2yy'x = y^2 - x^2$

C) $2yy'x = x^2 + y^2$

D) $2yy'x = -(x^2 + y^2)$

60. The Wronskian of the two functions $y_1 = e^{-x}$ and $y_2 = xe^{-x}$ is

A) e^{-x}

B) e^{-2x}

C) $2e^{-x}$

D) xe^{-2x}

61. Let $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ....$ and $f'(x)$ be the power series obtained by term differentiation of the power series of f(x). Then the radius of convergence of $f'(x)$ is

A) 0

B) 1

C) 1/2

D) ∞

62. $\int \frac{J_4(x)}{x^3} dx$ where $J_p(x)$ is the Bessel function of order p is

A) $-\frac{J_4(x)}{x^4} + c$

B) $-\frac{J_3(x)}{x^4} + c$

C) $-\frac{J_3(x)}{x^3} + c$

D) $-\frac{J_4(x)}{x^3} + c$

63. The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2} = 0$ where z is a function of x and y is of the form

A) $z = f(x) + yg(x)$

B) $z = f(y) + xg(y)$

C) $z = f(x) + g(x)$

D) $z = f(y) + g(y)$

64. The solution of the differential equation $x\,dy - y\,dx - 2x^2 z\,dz = 0$ is

A) $x = y\,(z^2 + c)$

B) $y = x\,(z^2 + c)$

C) $x = yz^2 + c$

D) $y = xz^2 + c$

65. Which of the following is a hyperbolic equation for all values of x ?

A) $u_{xx} + \sin^2(x)u_{yy} + u_y = 0$

B) $u_x + (2N/x)u_y = -(1/a^2)u_{yy}$ where N and a are constants

C) $u_{xx} - xu_{yy} = 0$

D) $(n - 1)\,(u_{xx} - y^{2n}u_{yy}) = ny^{2n-1}u_y$ where $n \in \mathbb{N}$ and $n \neq 1$

66. If u(x, t) satisfies the one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = (1/9)\frac{\partial^2 u}{\partial t^2}$ where $-\infty < x < \infty$; $t > 0$ and the initial conditions $u(x, 0) = 2x - 3$, $u_t(x, 0) = 0$ then $u(3, 2) =$

A) -4

B) 4

C) -3

D) 3

67. If u(x, t) satisfies the equation $u_{xx} + u_{yy} = 0$ in the rectangle $1 \leq x \leq 3$, $2 \leq y \leq 4$ and the boundary conditions $u(x, 2) = 0$, $u(x, 4) = 0$, $u(1, y) = 0$, $u(y, 3) = 4y - 3$ then the minimum value of u(x, y) in the rectangle is

A) 0

B) 2

C) -3

D) 5

68. Which of the following property of metric is not satisfied by the function $d : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by $d(x, y) = (x_1 - y_1)^2 + (x_2 - y_2)^2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$?

A) $d(x, y) \geq 0$ for all $x, y \in \mathbb{R}^2$

B) $d(x, y) = d(y, x)$ for all $x, y \in \mathbb{R}^2$

C) $d(x, y) = 0$ implies $x = y$

D) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in \mathbb{R}^2$

69. Let d be the discrete metric on $\mathbb{R}$. Which of the following sequence (x_n) is convergent in this space ?

A) $x_n = 1 + \frac{1}{n}$

B) $x_n = \begin{cases} 1 & \text{if } n \leq 10 \\ 2 & \text{otherwise} \end{cases}$

C) $x_n = \begin{cases} 0 & \text{if } n < 10 \\ 1/n & \text{otherwise} \end{cases}$

D) $x_n = \frac{1}{n} + \frac{1}{n+1}$

70. Let ρ be the sup metric on the space C[0, 1] of continuous real valued functions on [0, 1]. Let $f(x) = \begin{cases} x : 0 \leq x \leq 1/2 \\ 1 - x : 1/2 \leq x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 0 : 0 \leq x \leq 1/2 \\ 1 - 2x : 1/2 \leq x \leq 1 \end{cases}$.

Then $\rho(f, g) =$

A) 0 B) 1 C) 1/2 D) 3/2

71. Let τ be the topology on the reals $\mathbb{R}$ for which $\{(-\infty, a) : a > 0\} \cup \{(b, \infty) : b > 0\}$ is a subbase. Which of the following is not an open set in this topology ?

A) (0, 1) B) (–1, 0) C) (1, 2) D) $(-\infty, 1)$

72. Let τ_1 be the cofinite topology on $\mathbb{R}$ and τ_2 be the usual topology on $\mathbb{R}$. Then which of the following is a continuous function from $(\mathbb{R}, \tau_1)$ to $(\mathbb{R}, \tau_2)$?

A) f(x) = 1

B) f(x) = x

C) $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

D) $f(x) = \begin{cases} 1 + x & \text{if } x \geq 1 \\ 2 & \text{otherwise} \end{cases}$

73. Which of the following pairs of spaces are homeomorphic ? Here $\mathbb{R}$ is the real line.

A) $\mathbb{R}$ and $\mathbb{R} \times \mathbb{R}$

B) $\mathbb{R}$ and the open interval (0, 1)

C) the closed interval [0, 1] and the unit circle $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$

D) the closed interval [0, 1] and the unit disk $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq 1\}$

74. Let $X_p = \mathbb{R}^2$ be the normed linear space with norm $\| \|_p : 1 \leq p \leq \infty$. Then for which value of p the point $(3/4, -1/2)$ lies in the closed unit ball in X_p ?

A) 1 and 2

B) 2 and ∞

C) 1 and ∞

D) ∞ only

75. Let X be the normed linear space c_{00} with norm $\| \|_\infty$ and T be a linear operator on X which is continuous at the point (0, 0, ...). Then which of the following is not true ?

A) T is continuous on X

B) T is bounded

C) T(B) is bounded where B is the closed unit ball in X

D) For some convergent sequence (x_n) in X the sequence $(T(x_n))$ is not convergent

76. Let $X = \mathbb{R}^2$ be the normed linear space with norm $\| \|_2$. Let A be a bounded linear operator on X given by $A(x, y) = \left(\frac{1}{\sqrt{2}}(x + y), \frac{1}{\sqrt{2}}(y - x)\right)$. Then $\| A \| =$

A) 1

B) 2

C) $\sqrt{2}$

D) $2\sqrt{2}$

77. Let $X = \mathbb{R}^3$ be the normed linear space with norm $\| \ \|_2$ and $Y = \mathbb{R}^2$ be the normed linear space with norm $\| \ \|_2$. If $F : X \to Y$ is defined by $F(x_1, x_2, x_3) = (x_1, x_3)$ then which of the following is not true ?

A) F is continuous

B) F maps open sets onto open sets

C) If $\|x\| = 1$ then $\|F(x)\| = 1$

D) F has closed graph

78. Let X = C[–1, 1] be the space of all real valued continuous functions on [–1, 1] with inner product defined by $\langle x, y \rangle = \int_{-1}^{1} x(t)y(t)\,dt$. If x(t) = sin t then the set of all elements in X orthogonal to x is

A) the set of all odd functions

B) the set of all even functions

C) the set $\{\cos nt : n \in \mathbb{N}\}$

D) the set $\{\sin nt : n \in \mathbb{N}\}$

79. Let $H = l^2$ be the real Hilbert space and $f \in H'$ be defined by

$f(x) = x(1) + \frac{x(2)}{2} + \frac{x(3)}{3} + \dots$. Then $\|f\| =$

A) $\pi/\sqrt{6}$

B) $\pi/6$

C) $\pi^2/6$

D) $\pi^2/\sqrt{6}$

80. Let $H = L^2[-\pi, \pi]$ be the complex Hilbert space and $u_n(t) = e^{int}/\sqrt{2\pi}$ for each integer n and $t \in [-\pi, \pi]$. Then which of the following is not true ?

A) $x = \Sigma_n \langle x, u_n \rangle u_n$ for all $x \in H$

B) $\|x\| = \left(\Sigma_n |\langle x, u_n \rangle|^2 \right)^{1/2}$ for all $x \in H$

C) $\overline{\text{span}\{u_n\}} = H$

D) $\{u_n : \langle x, u_n \rangle \neq 0\}$ is finite for each $x \in H$