1. The number of one-to-one functions from the set {1, 2, 3, 4} to itself such that all the functions map 1 to 4 and 2 to 3 is

   A) 1 B) 2 C) 3 D) 4

2. Which of the following is an uncountable set ? Here $\mathbb{R}$ is the set of reals and $\mathbb{Q}$ is the set of rationals ?

   A) $\{x \in \mathbb{R} : \text{integral part of } x \text{ is } 1\}$

   B) $\{x \in \mathbb{R} : x^2 \text{ is rational}\}$

   C) $\{x \in \mathbb{R} : x \geq 0 \text{ and } \sqrt{x} \text{ is rational}\}$

   D) $\{x \in \mathbb{R} : x \geq 0 \text{ and } x + \sqrt{x} \text{ is an integer}\}$

3. Which of the following is a one to one function from the set $\mathbb{Z}$ of integers to $\mathbb{Z}$?

   A) $f(x) = x^2 - x$ B) $f(x) = x^2 + x$ C) $f(x) = x^3 + x$ D) $f(x) = x^3 - x$

4. Which of the following is a period of the function $f(x) = \sin\left(\frac{4x+1}{2\pi}\right)$?

   A) π B) 2π C) π^2 D) $2\pi^2$

5. Let θ be the angle between the lines $2x - y + 1 = 0$ and $3x - y + 2 = 0$. Then $\tan \theta =$

   A) 1/5 B) 2/3 C) 3/7 D) 1/7

6. At which among the following points the tangent to the parabola $y^2 = 4x$ has slope 1.

   A) (2, 1) B) (1, 2) C) $(2, 2\sqrt{2})$ D) (4, 4)

7. The direction cosines of the line joining (1, 2, 1) and (2, 1, –1) are

   A) $\frac{1}{6}, \frac{-1}{6}, \frac{-2}{6}$ B) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$ C) $\frac{1}{3}, \frac{-1}{3}, \frac{-2}{3}$ D) $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

8. Which of the following is a plane perpendicular to the plane $2x - 3y + z = 1$?

   A) $3x - 2y - z = 1$ B) $3x + 2y + 2z = 1$

   C) $2x + 3y - z = 1$ D) $2x + y - z = 1$

9. The value of $\lim_{x \to 0} x \sin\left(1 - \frac{1}{x}\right)$ is

A) 0
B) 1
C) –1
D) sin1

10. $\lim_{x \to 0} \frac{e^{2x} - e^{x}}{x} =$

A) 0
B) 1
C) e
D) $e - 1$

11. The function f(x) = sinx + 2cosx is

A) increasing in the interval $[0, \pi/4]$
B) increasing in the interval $[0, \pi/2]$
C) decreasing in the interval $[0, \pi/2]$
D) decreasing in the interval $[\pi/4, \pi/2]$

12. There are two coins. One has Head marked on both sides. The other is a fair coin. One coin is chosen at random and tossed. What is the probability that Head appears ?

A) 1
B) 1/2
C) 1/4
D) 3/4

13. If A and B are two events with probabilities $p(A) = \cdot 7$ and $p(B) = \cdot 8$ then which of the following is a consequence ?

A) $p(A \cup B) \le \cdot 8$
B) $p(A \cap B) \ge \cdot 5$
C) $p(A|B) = 1$
D) $p(B|A) = 1$

14. $\int_0^{\pi} (e^x \cos x - e^x \sin x)dx =$

A) 0
B) e^{π}
C) $1 - e^{\pi}$
D) $-1 - e^{\pi}$

15. Let $x_n = \frac{1}{n^2+1}$ and $y_n = (-1)^n$. Then which of the following is true about the sequences (x_n) and (y_n) ?

A) (x_n) and (y_n) are convergent
B) $(x_n y_n)$ is convergent
C) $(x_n + y_n)$ is convergent
D) $(x_n - y_n)$ is convergent

16. Let $f_n(x) = x^n \sin(n\pi x)$ and let $f(x) = \lim_{n \to \infty} f_n(x)$ for $x \in [0, 1]$. Then $f(1/4) =$

A) 0
B) 1
C) 1/2
D) 1/4

17. Let $f(x, y) = \begin{cases} \frac{x^2 + y}{x + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$. Then the directional derivative of f at (0, 0) along (2, 1) is

A) 0
B) 1
C) 1/2
D) 1/4

18. Let [x] denote the greatest integer $\leq x$. Then $\int_0^3 x^2 \, d[x] =$

A) 0
B) 1
C) 2
D) 3

19. Let E_1, E_2 be Lebesgue measurable sets and m be the Lebesgue measure such that $m(E_1) = 1$ and $m(E_2) = 4$. If $E_1 \cap E_2$ is singleton then $m(E_1 \cup E_2) =$

A) 2
B) 3
C) 4
D) 5

20. Let $f = \chi_E + 2\chi_F + 2\chi_G$ where χ denotes the characteristic function. If $m(E \cap F \cap G) = 1$ then $\int_{E \cap F \cap G} f \, dm =$

A) 3
B) 4
C) 5
D) 6

21. The imaginary part of $(1 + i\sqrt{3})^6$ is

A) 0
B) 1
C) 8
D) 27

22. Let e_1, e_2 be tenth roots of unity. Then $|e_1 + e_2|^2 + |e_1 - e_2|^2 =$

A) 2 B) 4 C) 6 D) 8

23. Which of the following transformation of the plane is a rotation ?

A) $T(z) = \frac{z + iz}{\sqrt{2}}$ B) $T(z) = \frac{z + iz}{z + 2}$ C) $T(z) = \frac{z + 2i}{2}$ D) $T(z) = \frac{z + 1}{z - 1}$

24. The radius of convergence of the series $\sum n^2 z^n$ is

A) 1 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{4}$

25. Let γ be the circle given by $\gamma(t) = 1 + e^{2\pi i t} : 0 \leq t \leq 1$. Then $\int_\gamma \frac{z \cos \pi z}{z - 1} dz =$

A) 0 B) 1 C) $2\pi i$ D) $-2\pi i$

26. The residue of $\frac{e^z}{(z - 1)^3}$ at z = 1 is

A) e B) e^2 C) $\frac{e}{2!}$ D) $\frac{e}{3!}$

27. Let γ be the curve given by $\gamma(t) = \exp(4\pi i t) : 0 \leq t \leq 1$. Then $\int_\gamma \frac{z^2 + 2}{z} dz =$

A) 0 B) $2\pi i$ C) $4\pi i$ D) $8\pi i$

28. Let S_4 be the symmetric group on 4 symbols. Then the order of the subgroup generated by {(1, 2), (2, 3)} is

A) 2 B) 3 C) 4 D) 6

29. Which of the following is a cyclic group ?

A) $\mathbb{Z}_4 \oplus \mathbb{Z}_{10}$ B) $\mathbb{Z}_5 \oplus \mathbb{Z}_{10}$ C) $\mathbb{Z}_6 \oplus \mathbb{Z}_{10}$ D) $\mathbb{Z}_7 \oplus \mathbb{Z}_{10}$

30. Let $f : \mathbb{Z}_{10} \to \mathbb{Z}_{12}$ be a nonzero homomorphism of groups. Then f(1) =

A) 1 B) 2 C) 3 D) 6

31. The commutator subgroup of the group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ of quaternion units is

A) Q_8 B) {1} C) {1, –1} D) {1, –1, i, –i}

32. Let G be a group of order 45 and H, K be subgroups of order 9 in G. Then which of the following is true ?

A) H = K B) $|H \cap K| = 1$ C) $|H \cap K| = 3$ D) $|H \cap K| = 5$

33. Let $\mathbb{Z}_{10}$ be the ring of integers mod 10 and $f : \mathbb{Z}_{10} \to \mathbb{Z}_{10}$ be a ring homomorphism. Then which of the following is a possible choice of f(1) ?

A) 2 B) 3 C) 4 D) 5

34. Which of the following is not an irreducible polynomial in $\mathbb{Q}[x]$ where $\mathbb{Q}$ is the field of rational numbers ?

A) $x^5 + 3x^4 + 6x + 6$ B) $x^5 + 12x^4 - 4x + 6$

C) $x^5 + x^2 - 16x - 4$ D) $x^5 + 12x^2 + 6x + 6$

35. Let $p(x) = x^2 - 5x \in \mathbb{Z}_6[x]$. Then the number of zeros of p(x) is $\mathbb{Z}_6$ is

A) 2 B) 3 C) 4 D) 6

36. Let $\mathbb{Z}_2(\alpha)$ be an extension of the field $\mathbb{Z}_2$ where α is a zero of $x^3 + x + 1 \in \mathbb{Z}_2[x]$. Then $\alpha^2 + \alpha + 1 =$

A) α^2 B) α^3 C) α^4 D) α^5

37. The order of the automorphism group $Aut(\mathbb{Q}(\sqrt{2}, \sqrt{3}))$ is

A) 1 B) 2 C) 4 D) 6

38. Let α be the real cube root of 2 and ω be a non real cube root of 1. Which of the following is not a splitting field over $\mathbb{Q}$?

A) $\mathbb{Q}(\alpha)$ B) $\mathbb{Q}(\alpha, \omega)$ C) $\mathbb{Q}(\alpha, \omega^2)$ D) $\mathbb{Q}(\alpha^2, \omega)$

39. The minimal polynomial of $1+\sqrt[3]{2}$ over the rationals is

A) $x^3 - 6x^2 + 3x - 6$
B) $x^3 - 3x^2 + 3x - 3$
C) $x^3 - 3x^2 + 6x + 3$
D) $x^3 + 6x^2 - 6x + 3$

40. Let $K = \mathbb{Q}(\sqrt{2}, i)$ and $G = \text{Aut}(K/\mathbb{Q})$. Let $H = \{\alpha \in G : \alpha(i) = i\}$. Then which of the following is an element in the fixed field of H ?

A) $\sqrt{2} + i$
B) $\sqrt{2} - i$
C) $1 + i$
D) $1 + \sqrt{2}$

41. Which of the following is a nilpotent matrix ?

A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
B) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
C) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
D) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

42. Which of the following is a row reduced echelon matrix obtained by transforming the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$?

A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
C) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
D) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

43. Which of the following statements is true about the solutions of the system of equations ?

$x + y + z = 1$
$2x + y + z = 2$
$3x + 2y + 2z = 3$

A) has a unique solution
B) has exactly two solutions
C) has infinitely many solutions
D) has no solution

44. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 1 & 0 & 4 \\ 0 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is

A) 1
B) 2
C) 3
D) 4

45. Which of the following is in the span of {(1, 3, 2), (2, 3, 1)} in $\mathbb{R}^3$?

A) (1, 2, 3)
B) (2, 3, 4)
C) (3, 2, 5)
D) (2, 5, 3)

46. Which of the following is true about the rows and columns of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 1 \\ 2 & 0 & 2 & -2 & 1 \\ 3 & 0 & 3 & -3 & 2 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix} ?$$

A) rows are independent
B) columns are independent
C) any two rows are independent
D) any two columns are independent

47. Let V be the space of all 3×3 matrices over $\mathbb{R}$ with all diagonal entries zero. Then dimension of V is

A) 3
B) 6
C) 8
D) 9

48. Consider the following subsets of $\mathbb{R}^2$.
$W_1 = \{(x, y) : 2x + 3y = 0\}$, $W_2 = \{(x, y) : x + y = 0\}$, $W_3 = \{(x, y) : x + y = 1\}$.
Then which of the following is true ?

A) W_1 and W_3 are subspaces of $\mathbb{R}^2$
B) W_2 and W_3 are subspaces of $\mathbb{R}^2$
C) $W_2 = (1, -1) + W_1$
D) $W_3 = (1, 0) + W_2$

49. Which of the following is a linear transformation from $\mathbb{R}^3 \to \mathbb{R}^2$?

A) T(x, y, z) = (x + y, xy)
B) T(x, y, z) = (x + 2y, 3y + z)
C) T(x, y, z) = (xy, yz)
D) T(x, y, z) = (x + z, xy + z)

50. The rank of the linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$ given by T(x, y, z, w) = (x + y, x + 2y, x + 3y, 0) is

A) 1
B) 2
C) 3
D) 4

51. Let T(x, y) = (x + y, 2y) be a linear operator on $\mathbb{R}^2$. Then which of the following is a matrix of T relative to some basis of $\mathbb{R}^2$?

A) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$
B) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
C) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$
D) $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

52. Let V, W be finite dimensional vector spaces and $T : V \to W$ be an invertible linear transformation. Then which of the following is not necessarily true ?

A) dim V = dim W
B) range T = W
C) For each $\beta \in W$ there is an $\alpha \in V$ such that $T(\alpha) = \beta$
D) There exists $\alpha \neq 0$ such that $T(\alpha) = 0$

53. The minimal polynomial of the matrix $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ is

A) $(x - 1)(x - 2)$
B) $(x - 1)^2 (x - 2)$
C) $(x - 1)(x - 2)^2$
D) $(x - 1)^2 (x - 2)^2$

54. Which of the following is a non-diagonalizable matrix ?

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
B) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$
C) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
D) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

55. Let $a = p_1^{a_1} p_2^{a_2} \dots p_k^{a_k}$ and $b = p_1^{b_1} p_2^{b_2} \dots p_k^{b_k}$ where $p_1, p_2, \dots, p_k$ are primes and $a_1 > b_1$ and $a_i \le b_i$ for all $i \ge 2$. Then gcd(a, b) =

A) $p_1^{a_1-b_1} p_2^{a_2-b_2} \dots p_k^{a_k-b_k}$
B) $p_1^{a_1-b_1} p_2^{b_2-a_2} \dots p_k^{b_k-a_k}$
C) $p_1^{b_1} p_2^{a_2} \dots p_k^{a_k}$
D) $p_1^{a_1} p_2^{b_2} \dots p_k^{b_k}$

56. Let x, y be relatively prime integers. The gcd of 2x + y and 5x + 3y is

A) 1
B) 2
C) 3
D) 5

57. Let ϕ denote Euler totient function. Then which of the following is not necessarily true ?

A) $\phi(p) = p - 1$ whenever p is a prime
B) $\phi(mn) = \phi(n)\ \phi(n)$ for all positive integers m and n
C) $\phi(p^r q^s) = \phi(p^r)\ \phi(q^s)$ where p, q are primes and $p \ne q$
D) $\phi(n)$ is either 1 or an even number for all n

58. Which of the following is not necessarily true about congruences (mod n) ?

A) if $a \equiv b$ (mod n) then $a^2 \equiv b^2$ mod (n) for all a, b
B) if $a^2 \equiv b^2$ (mod n) then $a \equiv b$ mod (n) for all a, b
C) if $a \equiv b$ (mod n) then $ac \equiv bc$ mod (n) for all a, b, c
D) if $ac \equiv bc$ (mod n) for all $c \ne 0$ then $a \equiv b$ mod (n) for all a, b

59. Which of the following congruence equations has a solution ?

A) $2x \equiv 3 \pmod 4$
B) $3x \equiv 4 \pmod 5$
C) $4x \equiv 5 \pmod{10}$
D) $5x \equiv 2 \pmod{10}$

60. Which of the following is a multiple of 23 ?

A) $15^{12} - 1$
B) $15^{15} - 1$
C) $15^{20} - 1$
D) $15^{22} - 1$

61. The general solution of the differential equation $x'' - x' - 6x = 0$ is

A) $C_1e^{3t} + C_2e^{-2t}$
B) $C_1e^{-3t} + C_2e^{2t}$
C) $C_1e^{2t} + C_2e^{-4t}$
D) $C_1e^{4t} + C_2e^{-2t}$

62. A particular solution of the differential equation $x'' + x = 0$ is

A) $\sin t + t \cos t$ B) $\sin t + \cos t$

C) $\sin t - t \cos t$ D) $t \sin t + \cos t$

63. Which of the following functions satisfies the Laplace equation in two variables ?

A) $U(x, y) = x^2 - y^2$ B) $U(x, y) = x^2 + y^2$

C) $U(x, y) = x^2 + xy$ D) $U(x, y) = y^2 + xy$

64. The differential equation representing the parabolas $y^2 = 4ax$ is

A) $y'' + y' = 0$ B) $yy'' + (y')^2 = 0$

C) $yy'' - y' = 0$ D) $y'' + yy' = 0$

65. Which of the following is a metric on $\mathbb{R}^2$? Here $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

A) $d(x, y) = |x_1 + y_1| + |x_2 + y_2|$

B) $d(x, y) = (x_1 - y_1)^2 + (x_2 - y_2)^2$

C) $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$

D) $d(x, y) = |x_1 - y_1|\ |x_2 - y_2|$

66. Let X be the metric space of all real valued functions on [0, 1] with metric given by

$d(f, g) = \max\{|f(x) - g(x)| : x \in [0, 1]\}$

Then which of the following (f_n) is a Cauchy sequence in X ?

A) $f_n(x) = x^n$ B) $f_n(x) = (-1)^n x^n$

C) $f_n(x) = n + x$ D) $f_n(x) = nx$

67. Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{X, \phi, \{1, 2, 3\}, \{2, 3\}\}$ be a topology on X. Then the closure of $\{1, 4\}$ is

A) $\{1, 4\}$ B) $\{1, 4, 5\}$

C) $\{1, 2, 4\}$ D) $\{1, 3, 4\}$

68. Let τ be a topology on $\mathbb{R}$ for which $\Sigma = \{(a, \infty) : a > 0\} \cup \{(-\infty, b) : b < 1\}$ is a subbase. Then which of the following is an open set in this space ?

A) (0, 1) B) (1, 2) C) (– 1, 1) D) (– 1, 0)

69. Let $X = \mathbb{R}$ and Y denote the real line. Let $f : X \to Y$ be defined by

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

and τ be the weak topology on X generated by f. Then which of the following is an open set in X ?

A) $(0, 1)$
B) $(-\infty, 1)$
C) $(-\infty, 0]$
D) $[0, \infty)$

70. Which of the following pairs of sub-spaces of the real line are not homeomorphic ?

A) $[0, 1]$ and $(0, 1)$
B) $(0, 1)$ and $(0, 2)$
C) $(0, 1)$ and $(-\infty, 0)$
D) $(0, 1)$ and $(0, \infty)$

71. Let X be a normed linear space. Then which of the following is a norm on $X \times X$? Here $(x, y) \in X \times X$

A) $f(x, y) = \| x \|$
B) $f(x, y) = \| x \| + \| y \|$
C) $f(x, y) = \| x \|^2 + \| y \|$
D) $f(x, y) = \| x \| \ \| y \|$

72. Let N_1, N_2, N_3 be norms on $\mathbb{R}^2$ given as follows. $N_1(x, y) = |x| + |y|$, $N_2(x, y) = \sqrt{x^2 + y^2}$ and $N_3(x, y) = \max\{|x|, |y|\}$. Let U, V, W denote the unit open balls in $\mathbb{R}^2$ relative to N_1, N_2 and N_3 respectively. Then which of the following is true ?

A) $U \subseteq V \subseteq W$
B) $U \subseteq W \subseteq V$
C) $V \subseteq U \subseteq W$
D) $V \subseteq W \subseteq U$

73. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (x + y, x + y)$. Then $\| f \| =$

A) 1
B) 2
C) $\sqrt{2}$
D) $\frac{1}{2}$

74. Which of the following is a linear functional of norm 2 on $\mathbb{R}^2$ with usual norm ?

A) $f(x, y) = x + y$
B) $f(x, y) = \sqrt{2}\,(x + y)$
C) $f(x, y) = x + \sqrt{2y}$
D) $f(x, y) = \sqrt{2}x + y$

75. Let $W = \{(x, y, z) : x + y = z\}$ be a sub-space of the Hilbert space $\mathbb{R}^3$ with usual inner product. Then which of the following is in the orthogonal complement $W^{\perp}$?

A) (1, 0, – 1)
B) (1, 1, – 1)
C) (0, 1, 1)
D) (0, 1, – 1)

76. Let U, W be sub-spaces of a Hilbert space H such that $U \neq W$ and $U^{\perp} = W^{\perp}$. Let P be the orthogonal projection on U, and Q be the orthogonal projection on W and R be a projection on W which is not orthogonal. Then which of the following is not necessarily true ? Here composition is defined as (x) PQ = ((x)P) Q.

A) PQ = Q
B) QP = P
C) PR = R
D) QR = Q

77. Let f be a bounded linear operator on the Hilbert space $\mathbb{C}^2$ given by $f(x, y) = (x - iy, x - y)$. Then the adjoint f* is given by f*(x, y) =

A) (x + y, ix – y)
B) (x + iy, x – y)
C) (x – y, x – iy)
D) (x – y, x + iy)

78. Let $\{e_1, e_2, e_3, e_4\}$ be an orthonormal set in a Hilbert space. Then which of the following is an orthogonal set ?

A) $\{e_1 + e_2, e_1 - e_2, e_3, e_4\}$
B) $\{e_1 + e_2, e_2 - e_3, e_3 + e_4, e_4\}$
C) $\{e_1 + e_2, e_1 - e_3, e_3, e_4)$
D) $\{e_1 + e_2, e_3 - e_4, e_3, e_4\}$

79. Let $\{e_1, e_2, ...\}$ be an orthonormal basis of a Hilbert space H and $x \in H$. Then which of the following is not necessarily true ?

A) $x = \Sigma \langle x, e_n \rangle e_n$
B) $||x|| = \Sigma |\langle x, e_n \rangle|$
C) $\Sigma \langle x, e_n \rangle$ is convergent
D) if $\langle x, e_n \rangle e_n = 0$ for all n then $x = 0$

80. Let $\mathbb{R}^3$ be the Hilbert space with usual inner product. Let $W = \{(x, y, 0) : x, y \in \mathbb{R}\}$. Then the best approximation to (1, 2, 1) from W is

A) (1, 2, 0)
B) (1, 1, 0)
C) (2, 2, 0)
D) (2, 1, 0)