A

16121 120 MINUTES

1. If $z = \cos\theta + i\sin\theta$ then $\dfrac{z^{2n}+1}{z^{2n}-1}$ is

A) $i\tan n\theta$ B) $-i\tan n\theta$ C) $i\cot n\theta$ D) $-i\cot n\theta$

2. Let X be a set with 11 elements. The number of subsets of X containing more than 5 elements is

A) 2^4 B) 2^5 C) 2^6 D) 2^{10}

3. Let a, b, c be real numbers with $a \neq 0$. Let α be a root of $a^2x^2 + bx + c = 0$ and β be a root of $a^2x^2 - bx - c = 0$ where $0 < \alpha < \beta$. Then one root γ of $a^2x^2 + 2bx + 2c = 0$ satisfies the relation

A) $\gamma = \dfrac{\beta - \alpha}{2}$ B) $\gamma = \dfrac{\alpha + \beta}{2}$ C) $\alpha < \beta < \gamma$ D) $\alpha < \gamma < \beta$

4. Which of the following is a period of the function $f(x) = \sin\left(\dfrac{2x+3}{6\pi}\right)$

A) 6π B) $6\pi^2$ C) 2π D) $2\pi^2$

5. The domain of the function $f(x) = \dfrac{2}{9 - x^2} + \log(x^3 - x)$ is

A) $(-3, \infty)$ B) $(-3, 0) \cup (3, \infty)$
C) $(-1, 0) \cup (1, 3) \cup (3, \infty)$ D) $(-3, 0) \cup (1, 3) \cup (3, \infty)$

6. The function $f(x) = \log_2 x$ maps the interval (4, 16) in to

A) (2, 4) B) (1, 4)
C) (1, 2) D) (0, 2)

7. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ then $A^3 =$

A) I B) A (A–I)
C) $3A^2 - 2A$ D) 2A (A–I)

8. The system of equations $2x + 3y = 5$; $4x + ay = 0$ is inconsistent for

A) $a = 3$ B) $a = 6$
C) All values of $a \neq 3$ D) All values of $a \neq 6$

9. The value of $\lim_{x\to 0}\frac{(1+x)^{1/2}-(1-x)^{1/2}}{x}$ is

A) 0 B) 1 C) $\frac{1}{2}$ D) ∞

10. The maximum value of $y = x(\log x)^2$ in the interval (0,1) is

A) $4e^2$ B) $4e^{-2}$ C) $2e^2$ D) $2e^{-2}$

11. Let $x = \log t$ and $y = t^3 - 2$. Then $\frac{d^2y}{dx^2}$ at $t = 1$ is

A) 2 B) 3 C) 6 D) 9

12. $\int x^{2x}(1+\log x)\,dx$ is

A) $2x^{2x} + c$ B) $\frac{1}{2}x^{2x} + c$

C) $x^{2x} + x + c$ D) $x^{2x} + \log x + c$

13. $\int_{-1}^{1}\frac{e^x+1}{e^x-1}dx =$

A) 0 B) 1 C) e^2 D) e^{-2}

14. The area bounded by one arch of the curve $y = \sin 4x$ is

A) 1 B) $\frac{1}{2}$ C) $\pi/4$ D) $\pi/2$

15. The number of common tangents to the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ is

A) 1 B) 2 C) 3 D) 4

16. If a plane meets the co ordinate axes at the points A, B, C such that the triangle ABC has centroid (3, 4, 5). Then the equation of the plane is

A) $3x + 4y + 5z = 1$ B) $3x + 4y + 5z = 12$

C) $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=1$ D) $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=3$

17. The eccentricity of the rectangular hyperbola is

A) $\sqrt{2}$ B) $\sqrt{3}$

C) $\sqrt{2}/2$ D) $\sqrt{3}/2$

18. The centre of the sphere passing through the four points (0, 0,0), (a, 0, 0), (0, b, 0) and (0, 0, c) is

A) (a, b, c) B) (a/2, b/2, c/2)

C) (–a, –b, –c) D) (–a/2, –b/2, –c/2)

19. The angle between the line

$$\frac{x-1}{2}=\frac{y-3}{3}=\frac{z-4}{1} \text{ and the plane } x-2y+4z+7=0 \text{ is}$$

A) 0 B) $\pi/2$ C) $\pi/3$ D) $\pi/4$

20. The portion of a straight line intercepted between the axes is divided by the point $(4, -3)$ in the ratio $4 : 5$. Then the equation of the line is

A) $15x - 16y = 1$ B) $15x - 16y = 27$
C) $15x - 16y = 36$ D) $15x - 16y = 108$

21. The equation $1 - x^2 = x$ with $0 < x < 1$ has

A) Exactly one solution B) Exactly two solutions
C) More than two solutions D) No solution

22. For an irrational number x and a rational number y which of the following is true?

A) $x + y$ can be rational B) xy can be rational
C) $\frac{x}{y}$ can be rational D) x^y can be rational

23. Let (α_n) be the sequence given by $\alpha_n = \left(1+\frac{1}{n}\right)^n$. Then which of the following is true about the limit, $\lim \alpha_n = \alpha$,

A) $0 < \alpha < 1$ B) $1 < \alpha < 2$
C) $2 < \alpha < 3$ D) $3 < \alpha < 4$

24. Let $X = \{x \in Q : \geq 3\}$ and $Y = \{x \in Q : x^2 \leq 3\}$. Then which of the following is true about $X \cap Y$

A) ϕ B) is singleton
C) has exactly two elements D) is infinite

25. Let $f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$, then

A) f is continuous at 0 and 1
B) f is continuous at 1 and differentiable at 0
C) f is continuous and differentiable at 1
D) f is continuous at 0 and differentiable at 0

26. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{otherwise} \end{cases}$.

Then $\lim_{(x,y) \to (0,0) \text{ along } x=2y} f(x, y)$ is

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{2}{5}$

27. Let f be a continuous real valued function on [0, 1] such that $f(x)$ is rational for all x and $f(0) = 0$. Then $f(1) =$

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$

28. Let $f(x) = \begin{cases} 1 \text{ if } 0 \le x \le 1 \\ x \text{ if } 1 \le x \le 3 \end{cases}$, then $\int_0^3 f(x)\,dx =$

A) 4 B) $4\frac{1}{2}$ C) 5 D) $5\frac{1}{2}$

29. Let $f(x) = \begin{cases} 0 \text{ if } x \text{ is rational} \\ 1 \text{ otherwise.} \end{cases}$

Then Lebesgue integral of f on [0, 1] is

A) 0 B) 1 C) 2 D) $\frac{1}{2}$

30. Let $*(A) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ \text{otherwise } 0 \end{cases}$

Then which of the following is not a $*-$ measurable set.

A) {1} B) set of all rationals
C) set of all irrationals D) set of all positive irrationals

31. Which of the following is not an analytic function on the open unit disk?

A) $f(z) = \sin z$ B) $f(z) = \frac{z}{2+z}$
C) $f(z) = |z|$ D) $f(z) = e^{1+z}$

32. Let $\gamma(t) = 2e^{it}$ for $0 \le t \le 2\pi$. Then the value of the integral $\frac{1}{2\pi i}\int_\gamma \frac{(3z+1)dz}{z(z-1)}$ is

A) 0 B) 1 C) 2 D) 3

33. Let $\sum_{n=0}^{\infty} \alpha_n z^n$ be the power series expansion of $(1-z)^{-3}$ about the origin. Then for each n, α_n is equal to

A) $n(n+1)$ B) $(n+1)(n+2)$
C) $\frac{(n+1)(n+2)}{2}$ D) n^2

34. The radius of convergence of all the series $\sum_{n=0}^{\infty} \frac{(4+3i)^n}{5^{2n}} z^n$ is

A) 5 B) $\frac{1}{5}$ C) $\sqrt{5}$ D) $\frac{1}{\sqrt{5}}$

35. Let $f(z)$ be an entire function such that $|f(z)| \to 0$ as $|z| \to \infty$. If $f(1) = i$ then $f(2) =$

A) i B) 1 C) 2 D) 2i

36. Let $\gamma(t) = 1 + e^{it}$ for $0 \le t \le 2\pi$. Then $\int_\gamma e^z \sin z \, dz =$

A) 0 B) 1 C) 2π D) $2\pi i$

37. Let $f = u + iv$ be an entire function with $f(0) = 0$. Let $u(x, y) = x^2+2x - y^2$ for all $x + iy \in C$. Then $v(x, y) =$

A) $2xy$ B) $2x(y+1)$
C) $x(y+1)$ D) $2(x + 1)y$

38. The Mobius transformation $\frac{2z-1}{2-z}$ maps the disk $U = \{z : |z| < 1\}$ onto

A) $\frac{1}{2}U$ B) U C) 2U D) C

39. Which of the following Mobius transformation maps the disk $\{z : |z| < 1\}$ onto the upper half plane

A) $\frac{z+1}{z-1}$ B) $i\frac{1+z}{1-z}$ C) $\frac{i+z}{i-z}$ D) $\frac{i+z}{1-z}$

40. Let $f(z) = 1 + \sum_{n=1}^{\infty} \alpha_n z^n$ be analytic in the disk $D = \{z : |z| < 1\}$. If $|f|$ is bounded by 1 in D then which of the following statements is not true

A) $\alpha_n = 0$ for $n = 1, 2, \ldots$ B) $f(\frac{1}{2}) = 1$
C) $f'(0) = 1$ D) $f''(0) = 0$

41. Let A and B be nonempty subsets of a set X such that $(A \cup B) - (A \cap B) \subseteq A$. Then which of the following is true

A) $A \subseteq B$ B) $B \subseteq A$
C) $A - B = A$ D) $B - A = B$

42. Which of the following is a congruence on the group $(\mathbf{Z}, +)$ of integers.

A) $\{(x, y) : x + y = 0\}$ B) $\{(x, y) : x - y = 1\}$
C) $\{(x, y) : x - y \text{ is even}\}$ D) $\{(x, y) : x - y \text{ is odd}\}$

43. The number of automorphisms of the cyclic group of $\mathbf{Z}_6$ is

A) 1 B) 2 C) 3 D) 6

44. Let $\mathbf{R}^*$be the multiplicative group all non zero reals and H be the subgroup of $\mathbf{R}^*$ generated by the set $\{2, 3\}$. Then which of the following is true

A) $H = \mathbf{Q}^*$, the multiplicative group of all rationals
B) $\mathbf{R}^* / H$ is finite
C) $3 + \sqrt{2} \in H$
D) $\frac{1}{2} - \frac{1}{3} \in H$

45. Choose the correct statement from the following
A) Every infinite group is abelian
B) Every group of order 11 is cyclic
C) Every group of order 8 is abelian
D) Every abelian group of order 28 is cyclic

46. Let S_3 be the symmetric group on three symbols. Then the commutator subgroup of S_3 is of order
A) 1 B) 2 C) 3 D) 6

47. Let I = 2**Z** and J = 5**Z** be ideals of the ring **Z** of integers. Then I + J =
A) I B) J C) **Z** D) 7**Z**

48. Let I be an ideal of **Q**[x] generated by $\{1 + x, 1 - x^2\}$. Then which of the following is a generator of I
A) x B) 1 + x C) $1 - x^2$ D) $1 + x - x^2$

49. Which of the following is an irreducible polynomial in **Q**[x]
A) $x^5 + 3x^3 - 2x - 2$ B) $x^5 + 3x^3 - 3x - 6$
C) $x^5 + 4x^3 + x^2 + 4$ D) $x^5 - 4x^4 - 2x + 5$

50. Let σ be a non identity automorphism of the field C of complex numbers. Then which of the following can not be true for σ
A) $\sigma\left(\sqrt{2}\right) = \sqrt{3}$ B) $\sigma(3) = 3$
C) $\sigma(e) = \pi$ D) $\sigma(1 + i) = 1 - i$

51. The degree of the irreducible polynomial of $\sqrt{2} + \sqrt{3}$ over **Q** is
A) 1 B) 2 C) 3 D) 4

52. The degree of the splitting field of $x^3 - 2$ over **Q** is

A) 1 B) 2 C) 4 D) 6

53. Let F be a field of 8 elements and $\alpha \in F$ be such that $\alpha^3 + \alpha + 1 = 0$. Then $1 + \alpha^2 =$
A) α B) α^2 C) α^4 D) α^6

54. Let A, B be 2 X 2 matrices of order 2 in the group of all invertible 2 X 2 matrices over **R**. Then which of the following is necessarily true:
A) A = B B) A = B or – B
C) AB = BA D) det A = det B or – det B

55. Let W = {(x, y) : 2x + y = 0} be a subspace of $\mathbf{R}^2$. Then which of the following is in W + (2, 1).
A) (4, 0) B) (0, 3) C) (4, – 3) D) (–1, 3)

56. Which of the following is in the span of {(1, 2, 1), (1, 3, 1)} in $\mathbf{R}^3$
A) (2, 3, 4) B) (2, 5, 1) C) (3, 3, 3) D) (3, 5, 4)

57. Which of the following is an eigen vector of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

58. Which of the following is orthogonal to (1, 1, 2) in the space $\mathbf{R}^3$ with usual inner product

A) (1, 1, –1) B) (1, 1, –2) C) (1, –1, 2) D) (1, 2, –1)

59. Which of the following matrix is a conjugate of $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

A) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

60. Consider the system of equations

$$x + 2y - z = 2$$
$$x - y + 2z = 3$$
$$2x + y + z = 4$$

Then which of the following is true about the system

A) has a unique solution
B) has exactly two solutions
C) has infinitely many solutions
D) has no solution

61. The rank of the linear transformation T: $R^4 \to R^4$ given by T (x, y, z, w) = (x – y, x – 2y, x – 3y, x – 4y) is

A) 1 B) 2 C) 3 D) 4

62. Let X = {1, 2, 3, 4, 5} and T = {X, Φ, {1, 2, 3}} be a topology on X. Then the interior of {3, 4, 5} is

A) {1} B) {3, 4} C) (3, 4, 5} D) Φ

63. Let X = R with usual topology and Y = **R** with discrete topology. Then which of the following is a continuous function from X to Y

A) f (x) = x for all $x \in X$ B) f (x) = 0 for all $x \in X$
C) f (x) = 1 + x for all $x \in X$ D) f (x) = 1 + x^2 for all $x \in X$

64. Let X = **R** and Y be the real line. Let f: X → Y be defined by

$$f(x) = \begin{cases} 1 \text{ if } x \in Q \\ 0 \text{ otherwise} \end{cases}$$

And τ be the weak topology on X generated by f. Then which of the following is an open set in X

A) Q
B) the set of all positive rationals $\mathbf{Q}^+$
C) the set of all positive reals $\mathbf{R}^+$
D) the open interval (0, 1)

65. Let τ_1, τ_2, τ be topologies on R such that $\tau_1 \subseteq \tau_2$. Let X = (**R**, τ_1), Y = (R, τ_2) and Z = (R, τ). If f : X → Z is continuous then which of the following is necessarily true

A) f : X → Y is continuous
B) f : Y → Z is continuous
C) f : Z → X is continuous
D) f : Z → Y is continuous

66. Let $\{(\alpha, \infty) : \alpha \in \mathbf{R}\}$ be a subbase for a topology on the reals. Then the closure of (0, 1) in this topology is

A) [0, 1]
B) (0, 1]
C) $[0, \infty)$
D) $(-\infty, 1]$

67. Which of the following is a connected subset of the real line?

A) $\{x : x^2 + y^2 = 1\}$
B) $\{x : x^2 < 1\}$
C) $\{x : x^2 \geq 1\}$
D) $\{x : 1 + x^2 > 1\}$

68. Let **R** be the real line and **R** x **R** be the product space. Which of the following is an open set in **R** x **R**

A) $\{(x, y) : x \geq 0\}$
B) $\{(x, y) : y \leq 0\}$
C) $\{(x, y) : |x| < 1\}$
D) $\{(x, y) : |x + y| = 1\}$

69. Which of the following pairs of topological spaces are not homeomorphic?

A) [0, 1] and **R**
B) (0, 1) and (0, 2)
C) (0, 1) and **R**
D) (0, 1) and $(0, \infty)$

70. Let d denote the metric on $\mathbf{R}^2$ defined as follows. For z = (x, y) and w = (a, b),

$$d(z, w) = \frac{|x - a| + |y - b|}{3}$$

Then which of the following is a point in the open unit ball centered at the origin in this metric space

A) (0, 2)
B) (0, 3)
C) (2, 2)
D) (3, 2)

71. Which of the following is a norm on $\mathbf{R}^2$?

A) $f(x, y) = |x| + |y|^2$
B) $f(x, y) = |x| - |y|$
C) $f(x, y) = 2|x| + 2|y|$
D) $f(x, y) = |x|^2 + |y|$

72. Let X be the space of all bounded sequences of real numbers with sup norm. Let $x = (x_n)$ where $x_n = \sin(\pi / n)$. Then $\|x\| =$

A) 1 B) 2 C) $\frac{1}{\sqrt{2}}$ D) $\frac{\sqrt{3}}{2}$

73. Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be defined by $f(x, y) = (2y, x)$. Then $\|f\| =$

A) 1 B) 2 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$

74. Let H be a Hilbert space and $\{e, f, g\}$ be an orthonormal set in H. Let $x = 2e + f + g$, $y = e + 2f - g$ and $z = e - f + 2g$. Then which of the following is true

A) $\|x + y\| = \|x + z\|$ B) $\|x + y\| = \|y + z\|$

C) $\|x + z\| = \|x - y\|$ D) $\|y + z\| = \|y - z\|$

75. Let W be the line $x - y = 0$ in the inner product space $\mathbf{R}^2$ with usual inner product. Then the orthogonal projection of the point (1, 2) on W is

A) (1, 1) B) (2, 1) C) $(\frac{1}{2}, \frac{1}{2})$ D) $(\frac{3}{2}, \frac{3}{2})$

76. Let A, B be subsets of an inner product space X such that $0 \notin A$, $0 \notin B$ and $A^{\perp} \oplus B^{\perp} = X$. Then which is necessarily true

A) $A = B$ B) $A \cap B = \phi$ C) $A \cup B = X$ D) $A = B^{\perp}$

77. Let $\{e_1, e_2, \ldots\}$ be an orthonormal set in a Hilbert space H and let $x = \sum a_i e_i \in H$. Then which of the following is not necessarily true

A) $\langle x, e_i \rangle = a_i$ for all i B) $(a_i) \in l^2$

C) $\sum |a_i| = \|x\|$ D) $\sum |a_i|^2 = \|x\|^2$

78. Let $\mathbf{R}^2$ be the inner product space with usual inner product. Let f be a functional on $\mathbf{R}^2$ defined by $f(x) = \langle x, \alpha \rangle$ where $\alpha = (1, 1)$. Then $\|f\| =$

A) 1 B) 2 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$

79. Let S be the right shift operator on l^2 defined by $S : (x_1, x_2, \ldots) \mapsto (0, x_1, x_2, \ldots)$. Which of the following is not true about S?

A) $\|S\| = 1$ B) 1 is an eigen value of S

C) S is one to one D) $\|S(x)\| = \|x\|$ for all x

80. Which of the following is the adjoint of the operator $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x, y) = (x + y, y)$

A) $(x, y) \mapsto (x, x + y)$ B) $(x, y) \mapsto (x + y, + y)$

C) $(x, y) \mapsto (x + y, x + y)$ D) $(x, y) \mapsto (y, x + y)$

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