**14221** **120 MINUTES**

1. If $1, \omega, \omega^2$, -----, $\omega^9$ are the $10^{th}$ roots of unity, then $(1 + \omega)(1 + \omega^2)$ ----- $(1 + \omega^9)$ is
A) 1 B) – 1 C) 10 D) 0

2. If $\left(\frac{1+i}{1-i}\right)^{50} = a + ib$, then
A) a = – 1, b = 0 B) a = 0, b = – 1
C) a = 1, b = 0 D) a = 0, b= 1

3. Find the integral values for which the quadratic equation $(x – a)(x – 10) = – 1$ has integral roots
A) 12 B) 8
C) 12 or 8 D) 12 and 8

4. The system of linear equations $3x + 2y + z = 3$, $2x + y + z = 0$, $6x + 2y + 4z = 6$ is
A) Inconsistent
B) Consistent and has a unique solution
C) Consistent and has infinite number of solutions
D) Consistent and has three solutions

5. If $\alpha$ and $\beta$ are the roots of the equation $2x^2 – ax + b = 0$, then the equation whose roots are $2\alpha$ and $2\beta$ is
A) $x^2 – 2ax + 4b = 0$ B) $2x^2 – ax + 2b = 0$
C) $x^2 – ax + b = 0$ D) $x^2 – ax + 2b = 0$

6. If $n = mC_2$, then the value of $nC_2$ is
A) $(m + 1) C_4$ B) $3((m – 1) C_4)$
C) $(m + 2) C_4$ D) $3((m + 1) C_4)$

7. If A = {1, 2, 3, 4}, then the number of mappings from A into A whose range set contains two or more elements is
A) 256 B) 252
C) 16 D) 240

8. The domain of $y = \sin^{-1}\left(\frac{x - 3}{2}\right) – \log_{10}(4 – x)$ is
A) $(-\infty, 4)$ B) $[1, 4)$
C) $(-\infty, 3)$ D) $[3, 4]$

9. The inverse of the function $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ is given by
A) $\frac{1}{2}\log_e(2x - 1)$ B) $\frac{1}{2}\log_e(2 - x)$
C) $\frac{1}{2}\log_e\left(\frac{x+1}{x-1}\right)$ D) $\frac{1}{2}\log_e\left(\frac{x-1}{x+1}\right)$

10. If A is the matrix $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ then $A^{50}$ is

A) $-I$ B) $I$ C) $A$ D) $-A$

11. The points (1, 2) and (5, 2) are two vertices of a rectangle. If the other two vertices lie on the straight line $y = 3x + c$, then the value of c is

A) $-5$ B) 2 C) 3 D) $-7$

12. The number of common tangents to the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ is

A) 1 B) 2 C) 3 D) 0

13. The area bounded by the curves $x = 0$, $y = 0$ and $2x + 5y = 1$ in square units is

A) $\frac{1}{20}$ B) $\frac{1}{10}$ C) 10 D) 5

14. If $2x + y - 2z = 3$ and $3x + 4y + 5z = 3$ are two planes $P_1$ and $P_2$ respectively, then the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

A) Is parallel to $P_1$ but not perpendicular to $P_2$
B) Is perpendicular to $P_2$ but not parallel to $P_1$
C) Is parallel to $P_1$ and is perpendicular to $P_2$
D) Is perpendicular to $P_1$ and is parallel to $P_2$

15. The centre and radius of the sphere $x^2 + y^2 + z^2 - 2x + 6y + 4z - 35 = 0$ are

A) $(1, -3, -2)$ and 7 B) $(1, 3, -2)$ and 5
C) $(-1, 3, 2)$ and 7 D) $(1, -3, 2)$ and 7

16. If the line $y = 2x + c$ is a tangent to the parabola $y^2 = 4ax$, then

A) $a + c = 2$ B) $ac = 2$ C) $c = 2a$ D) $a = 2c$

17. $\lim_{x \to 0} \left(\frac{1+7x^2}{1+5x^2}\right)^{1/x^2}$ is

A) $\frac{7}{5}$ B) $e^2$ C) 1 D) 0

18. If $xe^{xy} = y + \sin^2 x$, then $\frac{dy}{dx}$ at (0, 0) is equal to

A) 0 B) 1 C) $-1$ D) e

19. The area included between the curves $y = e^{2x}$ and $y = e^{-2x}$ and the lines $x = 0$ and $x = 1$ is

A) $e^2 + e^{-2}$ B) $\frac{(e + e^{-1})^2}{2}$ C) $\frac{(e - e^{-1})^2}{2}$ D) $\frac{e^2 + e^{-2}}{2}$

20. The value of the integral $\int_1^4 (|x-3| + |1 - x|)dx$ is

A) 7 B) 8 C) 12 D) 4

21. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ is

A) 1 B) 0 C) e D) $\infty$

22. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^{n!}}{n!}$ is

A) 1 B) $\infty$ C) $\frac{1}{2}$ D) 2

23. Let the function f be defined on the set of real numbers **R**by

$$f(x) = \begin{cases} x^3, \text{for } 0 \le x \le 1 \\ x^2, \text{for } -1 \le x < 0 \\ 1, \text{otherwise} \end{cases}$$

Then the set of points at which f is not differentiable is

A) {–1, 0, 1} B) {1, –1} C) {0, 1} D) {–1, 0}

24. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ is

A) Absolutely convergent
B) Convergent, but not absolutely convergent
C) Divergent and diverges to infinity
D) Oscillating

25. Let f be defined on **R**by

$$f(x) = \begin{cases} 0, \text{if } x \text{ is rational} \\ x, \text{otherwise} \end{cases}$$

Then the Lebesgue integral $\int_0^1 fd$ has the value

A) 2 B) ½ C) 0 D) 1

26 Let f and $\alpha$ be defined on [0, 1] by $f(x) = x^2$ and $\alpha(x) = x^3$. Then the Riemann – Stieltjes integral $\int_0^1 fd\alpha$ has the value

A) ½ B) 3/5 C) 1/3 D) 2/5

27. Which of the following is not a property of the Lebesgue measure?
A) Countable sub additivity B) Countable additivity
C) Monotonicity D) Non – negativity

28. Let E be a subset of **R** with Lebesgue outer measure zero ie.m*(E) = 0. Then which of the following statements is not necessarily true?
A) E is measurable
B) E is countable
C) If A is any subset of **R**, then $m^*(A \cup E) = m^*(A)$
D) Every subset of E is measurable

29. The residue of $\frac{\sin z}{z^2}$ at z = 0, is
A) 0 B) 1 C) ½ D) 1/3

30. The singularity of the function $f(z) = -1/z + \sin 1/z$ at z = 0, is
A) A simple pole B) A pole of order 2
C) An essential singularity D) A removable singularity

31. Suppose f has an isolated singularity at z = 0. If z = 0 is a simple pole of f, then $\lim_{z \to 0} z f(z)$
A) Is finite and non-zero B) Is zero
C) Is infinity D) Does not exist

32. The value of the integral
$\int_\gamma \frac{dz}{z^2 - 4z + 3}$ where $\gamma(t) = 2e^{it}, 0 \le t \le 2\pi$ is
A) $2\pi i$ B) $-\pi i$ C) $\pi i$ D) 0

33. Let $\gamma(t) = 3e^{it}$, $0 \le t \le 4\pi$. Then $\int_\gamma \frac{dz}{z-2}$ has the value
A) 0 B) $2\pi i$ C) $-2\pi i$ D) $4\pi i$

34. Let $\gamma$ be the positively oriented rectangular path with vertices 0, 1, 1+i, i. Then $\int_\gamma z^2\, dz$ is equal to
A) ¼ B) 0 C) 1 D) 1/3

35. Which of the following Mobius transformation maps the open unit disk {z:|z| < 1} onto itself
A) $T_1(z) = \frac{2z - 1}{2 - z}$ B) $T_2(z) = \frac{z - 2}{z - 3}$
C) $T_3(z) = \frac{3z - 2}{1 - z}$ D) $T_4(z) = \frac{z + 1}{z - 1}$

36. Let f be a non constant analytic function on a region G. Then from the following statements pick the one which is incorrect?
A) f is infinitely differentiable on G
B) f(G) is open
C) f attains its maximum on G
D) The set of zeros of f in G has no limit point in G.

37. The coefficient of $\frac{1}{z}$ in the Laurent series expansion of $f(z) = \frac{1}{z(z-1)}$ in the region $0 < |z| < 1$ is
A) 0  B) 1
C) −1  D) 2

38. There exists no analytic branch of the logarithm in the region
A) $\{z : \text{Re } z > 0\}$  B) $\{z : \text{Im } z > 0\}$
C) $\{z : |z-1| < 1\}$  D) $\{z : |z| > 1\}$

39. Which of the following regions is connected, but not simply connected?
A) $\{z : |z| < 1\}$  B) $\{z : |z| > 1\}$
C) $\{z : |z-1| + |z+1| < 2\}$  D) $\{z : \text{Re } z > 0\}$

40. Let f be an analytic function on a region G. Let U = Re f and V = Imf. Then which of the following functions is not necessarily harmonic in G?
A) U  B) $U^2$
C) $U^2 - V^2$  D) UV

41. Let Z be the ring of integers. Which of the following values of m and n gives mZ + nZ = Z.
A) m = 2, n = 6  B) m = 3, n = 6
C) m = 4, n = 6  D) m = 5, n = 6

42. Let R[x] be the ring of polynomials over the reals. Let f(x) = 2x + 3 and g(x) = $2x^2 + 3$. Let I be the ideal generated by f(x) and J be the ideal generated by g(x). Then which of the following is true?
A) $I \subseteq J$  B) $J \subseteq I$
C) I + J = R[x]  D) $I \cap J = \phi$

43. Let R = $Z_{12}$ be the ring of integers mod 12. Let A be the ideal generated by 4 and B be the ideal generated by 6. Then which of the following holds?
A) R/A is a field  B) R/B is a field
C) R/(A+B) is a field  D) R/(A∩B) is a field

44. Which of the following is an irreducible polynomial in $Z_2[x]$?
A) $x^3 + x^2 + 1$  B) $x^3 + x^2 + x + 1$
C) $x^4 + x^2 + x + 1$  D) $x^4 + x^2 + 1$

45. If a, b are divisors of zero in a ring R then which of the following is not necessarily a divisor of zero?

A) a + b B) ab C) $a^2$ D) aba

46. Which of the following pairs of fields are isomorphic?

A) $Q(i), Q(\sqrt{2})$ B) $Q(\sqrt{2}), Q(\sqrt{3})$

C) $Q(\sqrt{2}+i), Q(\sqrt{2}-i)$ D) $Q(\sqrt{2}+i), Q(\sqrt{3}+i)$

47. Let f(x) and g(x) be polynomials of degree 4 over a field F. Then which of the following is true always?

A) deg f(x) + deg. g(x) is 8

B) deg f(x) + deg. g(x) is 4

C) deg f(x) + deg. g(x) is 3

D) deg f(x) + deg. g(x) is less than or equal to 4

48. Which of the following is not an algebraic extension?

A) $Q(\sqrt{\pi})$ over $Q(\pi)$ B) $Q(\pi)$ over Q

C) $Q(\sqrt{5})$ over Q D) $Q(\sqrt{2}+1)$ over Q

49. Let E be the splitting field of $x^3 - 2$ over Q. Then [E : Q] =

A) 2 B) 3

C) 4 D) 6

50. Let $\{0, 1, \alpha, 1+\alpha\}$ be a field of four elements. Then $1+\alpha^2 =$

A) $\alpha$ B) $1+\alpha$

C) 0 D) 1

51. Let $\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$ be a linearly independent set in a vector space. Then which of the following is also a linearly independent set?

A) $\{\upsilon_1, \upsilon_1+\upsilon_2, \upsilon_2+\upsilon_3+\upsilon_4, \upsilon_1+\upsilon_3+\upsilon_4\}$

B) $\{\upsilon_1, \upsilon_1+\upsilon_2, \upsilon_2+\upsilon_3+\upsilon_4, \upsilon_1+\upsilon_2+\upsilon_4\}$

C) $\{\upsilon_1+\upsilon_2, \upsilon_2+\upsilon_3, \upsilon_3+\upsilon_4, \upsilon_1+\upsilon_4\}$

D) $\{\upsilon_1+\upsilon_2, \upsilon_3+\upsilon_4, \upsilon_1+\upsilon_3+\upsilon_4, \upsilon_2+\upsilon_3+\upsilon_4\}$

52. Consider the system of equations

$2x + 3y + z + \omega = 0$

$3x - 2y - 2z + 2\omega = 0$

$4x - 7y - 5z + 3\omega = 0$. Then the dimension of the space of solutions is

A) 0 B) 1

C) 2 D) 3

53. Which of the following triples of points lie on a straight line?

A) (0, 1), (1, 2), (–1, 3) B) (0, 2), (1, 0), (–1, 4)

C) (1, 1), (2, 1), (3, 2) D) (1, –1), (–1, 2), (0, 1)

54. Let $T : R^2 \to R^2$ be defined by $T(x,y) = (2x + y, x + 2y)$. Then which of the following is a matrix of T with respect to some basis of $R^2$?

A) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}$

55. Which of the following is a diagonalizable matrix?

A) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

56. Let $S_4$ be the group of all permutations on four symbols. Let $\alpha = (1\ 2\ 3)$ and $\beta = (1\ 2)$. Then the order of the subgroup generated by $\alpha$ and $\beta$ is

A) 2 B) 3 C) 4 D) 6

57. The number of homomorphisms from the group $Z_{10}$ to the group $Z_{12}$ is

A) 1 B) 2 C) 3 D) 4

58. Let G be the group of quaternion units. Then the order of the centre of G is

A) 1 B) 2 C) 4 D) 8

59. Which of the following is a generator of the group $Z_5 \times Z_{12}$?

A) (2, 6) B) (2, 3) C) (3, 4) D) (3, 5)

60. The number of mutually non-isomorphic groups of order 49 is

A) 1 B) 2 C) 3 D) 4

61. Let d denote the sup metric on the set of bounded real valued functions on $[0, \pi]$. Let $f(x) = \sin x$ and $g(x) = \cos x$. Then $d(f,g) =$

A) 0 B) 1 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$

62. Let A and B be open balls of radius 1 and centre (0, 0) and (1, 1) respectively in R x R with usual metric. Then which of the following is a point in $A \cap B$?

A) $\left(\frac{1}{3}, \frac{1}{3}\right)$ B) $\left(\frac{1}{4}, \frac{1}{4}\right)$ C) $\left(\frac{3}{4}, \frac{3}{4}\right)$ D) $\left(\frac{1}{6}, \frac{1}{6}\right)$

63. Let $\tau$ be the topology on the set R of reals consisting of R, $\phi$ and all intervals of the form (a, b) where $a < 0$ and $b > 0$. Then the closure of [0, 1] in this topology is

A) [0,1] B) $(-\infty, 1]$ C) $[0, \infty)$ D) R

64. Which of the following is a compact subset of the real line?

A) $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, ---\right\}$ B) (0, 1) U (1, 2)

C) $[0, \infty)$ D) $(0, \infty)$

65. Let Q be the subspace of rationals of the real line R. Let $f : R \to Q$ be a continuous function. Then which of the following holds?

A) $f(1) > f(2)$ B) $f(1) > f(0)$

C) $f(1) < f(0)$ D) $f(1) = f(0)$

66. Which of the following pairs of spaces ishomeomorphic. Here R is the real line and the subsets given are subspaces of R; N is the set of naturals and Z is the set of integers?

A) [0, 1) and (0, 1) B) R and R x R

C) R \ {0, 1} and R \ {0} D) R \ N and R \ Z

67. Let X = {1, 2, 3, 4, 5} and $\tau$ = {X, ϕ, {1, 2, 3}, {4, 5}}. Let R be the real line. Which of the following is a continuous function $f : X \to R$?

A) $f(1) = f(2) = 0,\ f(3) = f(4) = f(5) = 1$

B) $f(1) = f(2) = f(3) = 0,\ f(4) = f(5) = 2$

C) $f(1) = f(2) = 0,\ f(3) = f(4) = 1,\ f(5) = 2$

D) $f(1) = 0,\ f(2) = 1,\ f(3) = f(4) = f(5) = 2$

68. Which of the following is not a continuous image of the real line?

A) The open interval (0, 1)

B) The closed interval [0, 1]

C) The subspace $\{x: 0 < |x| < 1\}$

D) The subspace $\{x: x > 0\}$

69. Let X = {0, 1} be the discrete space and f: $R \to X$ be defined by

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Let $\tau$ be the weak topology on R induced by f. Then which of the following is an open set in (R, $\tau$)?

A) (0, 1) B) [0, 1]

C) $(-\infty, 0]$ D) $[0, \infty)$

70. Which of the following sequence $(x_n)$ is a cauchy sequence in the metric space R with discrete metric?

A) $x_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$ B) $x_n = \begin{cases} 0 & \text{if } n < 10 \\ 1 & \text{if } n \ge 10 \end{cases}$

C) $x_n = n$ for all n D) $x_n = \frac{1}{n}$ for all n

71. Let V be the normed linear space $R^3$ with Euclidean norm and W= span {(1,0,0), (1,1,1)}. If $\overline{W}$ and $W^0$ denote respectively the closure and interior of W, then which one of the following is not true?

A) $\overline{W} \cup W^0 = W$
B) $\overline{W} \cap W^0 = \phi$
C) $\overline{W} \cup W^0 = V$
D) $\overline{W} + W^0 = W$

72. For $x = (x(1),x(2)) \varepsilon R^2$, let $\|x\|_p = \left[ |x(1)|^p + |x(2)|^p \right]^{1/p}$. If $E = \{x : \|x\|_p \le 1\}$ is not convex, then the possible value of p from the following is

A) $p = 1$
B) $p = 2$
C) $p = \infty$
D) $p = \frac{1}{2}$

73. Let X be the normed linear space $\ell^2$ and $T : X \to X$ be defined by

$T : (x(1), x(2), x(3), ---) = \left( x(1), \frac{x(2)}{2}, \frac{x(3)}{3}, --- \right)$ for $x = (x(1), x(2), ---)$ in X.

Then which one of the following is not correct?

A) T is continuous at the origin (zero element)
B) T is uniformly continuous
C) T is unbounded
D) One of the above is not true

74. Let M be a closed subspace of a normed linear space X. Then X is a Banach space if

A) $X/M$ is a Banach space
B) M is a separable Banach space
C) M and X are separable spaces
D) M and $X/M$ are Banach spaces

75. Let X be the normed linear space $\{x \varepsilon \ell^\infty$ : x(j) converges in K$\}$ and M = span {(1, 0, 0, ---), (0, 1, 0, ---), (0, 0, 1, 0, ---)}. Let F: $X \to X/M$ be the quotient map. If $C_{00} = \{x \varepsilon \ell^\infty : x(j) = 0$ for all but finitely many j$\}$, then

A) $F(C_{00})$ is linearly homeomorphic to $X/M$
B) $F(C_{00})$ is open in $X/M$
C) $F(C_{00})$ is closed in $X/M$
D) F(M) is a nonzero subspace of $X/M$

76. Let H be the complex Hilbert space $L^2([0, 2\pi])$. If $\int_0^{2\pi} x(t) e^{int} dt = 0$ for x ε H and n = 0, ±1, ±2, ---, then

A) $x = 0$
B) $x(t) = 1(t) = 1$ for t ε [0, 2π]
C) $x(t) = \sin t$
D) $x(t) = e^{-int}$

77. Let H be the real Hilbert space $R^2$, $e_1 = (1, 0)$ and $e_2 = (0, 1)$. Let A $\varepsilon BL(H)$ be represented by the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ with respect to the basis $\{e_1, e_2\}$. If B = $\{A(e_1), A(e_2)\}$ and B' = $\{A^*(e_1), A^*(e_2)\}$ then

A) B is a basis for H while B' is not a basis for H
B) B' is an orthonormal basis for H while B is not an orthonormal basis for H
C) Both B and B' are orthonormal basis for H
D) Neither B nor B' is an orthonormal basis for H

78. If $|<x, y>| = \| x \| \, \| y \|$ for two vectors x, y in an inner product space X, then

A) x and y are linearly independent
B) x and y are linearly dependent
C) $\| x \| = \| y \|$ always
D) $\| x + y \| = \| x - y \|$

79. For x, y ina Hilbert space H, if $\| x \| = 2$, $\| x - y \| = 3$ and $\| y \| = 5$, then $\| x + y \|$ is

A) 5 B) 4
C) 7 D) 6

80. If x and y are two vectors in a Hilbert space H such that $x \perp y$, then which one of the following is not true?

A) $\| x + y \|^2 = \| x \|^2 + \| y \|^2$ B) $\| x - y \|^2 = \| x \|^2 + \| y \|^2$
C) Span $\{x\} \perp$ Span $\{y\}$ D) Span $\{x\}^{\perp} \perp$ Span $\{y\}^{\perp}$

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