Mathematical Sciences
Paper II

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Note : This Paper contains **Fifty (50)** multiple choice questions. Each question carries **Two (2)** marks. Attempt *All* questions.

1. The segment (a, b) of the x-axis of $\mathbf{R}^2$ is :

   (A) Neither open nor closed

   (B) Open

   (C) Closed

   (D) Both open and closed

2. An open set in $\mathbf{R}$ is :

   (A) an interval

   (B) finite union of open intervals

   (C) countable union of disjoint open intervals in $\mathbf{R}$

   (D) intersection of open intervals

3. Consider the two statements for $z \in \mathbb{C}$

   (a) e^z is a one-one function

   (b) $\sin z$ is bounded entire function

   then :

   (A) both (a) and (b) are false

   (B) both (a) and (b) are true

   (C) only (a) is true

   (D) only (b) is true

4. Let $f(z) = e^z$, where $z = x + iy$, then :

   (A) $e^z > 0$ when y is an even multiple of π, and $e^z < 0$ when y is an odd multiple of π

   (B) $e^z > 0$ if y is an odd multiple of π

   (C) $e^z < 0$ if y is an even multiple of π

   (D) $e^z > 0$ if y is a multiple of π

5. $f(z) = \tan z,\ z \in \mathbb{C}$:
   (A) assumes all complex numbers
   (B) assumes none of the complex numbers
   (C) assumes all complex numbers except i
   (D) assumes all complex numbers except i and $-i$

6. Let V(F) be a vector space. Consider the following statements :
   (I) If α, β, γ are linearly independent vectors in V(F), then $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.
   (II) The set of vectors $A \subseteq V(F)$ which contains the zero vector is always linearly independent.

   Then :
   (A) Both (I) and (II) are true
   (B) Both (I) and (II) are false
   (C) only (I) is true
   (D) only (II) is true

7. Which of the following functions $T : \mathbf{R}^2 \to \mathbf{R}^2$ is *not* a linear transformation ?
   (A) $T(a, b) = (a + b, a - b)$
   (B) $T(a, b) = (a + b, b)$
   (C) $T(a, b) = (1 + a, b)$
   (D) $T(a, b) = (b, a)$

8. Let V be a vector space over a field F and $T : V \to V$ be a linear transformation such that T has 0 as a characteristic root, then :
   (A) T is diagonalizable over F
   (B) Multiplicity of each characteristic root of T is 1
   (C) T is invertible
   (D) T is nilpotent

9. Let $f : U \to V$ be a linear map and $\{u_1, u_2, \ldots\ldots, u_n\}$ be the set of linearly independent vectors in U. Then the set $\{f(u_1), f(u_2), \ldots, f(u_n)\}$ is linearly independent iff :

(A) f is one-one and onto

(B) f is one-one

(C) f is onto

(D) U = V

10. Let $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{pmatrix}$ be a matrix of transformation. Then the rank of A is equal to :

(A) 2

(B) 3

(C) 4

(D) 1

11. If P(E) = 1 and P(F) = 0.3, which of the following is *not* correct ?

(A) $P(E \cup F) = 1$ and $P(E \cap F) = 0.3$

(B) $P(E^C \cup F) = 0.3$ and $P(E \cap F) = 0.3$

(C) $P(E \cup F^C) = 1$ and $P(E \cap F^C) = 0.4$

(D) $P(E \cup F) = 1$ and $P(E^C \cap F) = 0$

12. Let E and F be two independent and mutually exclusive events. Which of the following statements is *not* correct ?

(A) Either P(E) = 1 or P(F) = 1

(B) Either P(E) = 0 or P(F) = 0

(C) $P(E) = P(E \cup F) - P(F)$

(D) $P(E|F) = P(E)$

13. Let X and Y be two independent Poisson r.v.s with parameters λ and θ respectively. Which of the following statements is *not* correct ?

(A) $P[X + Y = 10] = e^{-(\lambda+\theta)}\dfrac{(\lambda + \theta)^{10}}{10!}$

(B) $P[X - Y = 10] = e^{-(\lambda-\theta)}\dfrac{(\lambda - \theta)^{10}}{10!}$

(C) $P[X \leq 10 \mid Y \leq 20] = \sum_{k=0}^{10} e^{-\lambda}\dfrac{\lambda^k}{k!}$

(D) $P[X = 10 \mid X + Y = 20] = \dfrac{\binom{20}{10}(\lambda\theta)^{10}}{(\theta + \lambda)^{20}}$

14. Let X be a normal random variable with mean 1 and variance 1. Define the events E = {–2 < X < 0}, F = {–1 < X < 1} and G = {0 < X < 2} Which of the following statements is *correct* ?

(A) P(E) = P(F) = P(G)

(B) P(F) < P(E) < P(G)

(C) P(E) < P(F) < P(G)

(D) P(G) < P(E) < P(F)

15. The optimality of a basic feasible solution of a transportation problem is checked by :

(A) Graphical method

(B) Stepping stone method

(C) Cutting plane method

(D) Branch and bound method

16. The optimal solution of the linear programming problem :

Maximize $Z = x_1 + 2x_2$

Such that $x_1 + x_2 \leq 1$

$x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

(A) is unbounded

(B) is infeasible

(C) is $x_1 = 0; x_2 = 1$

(D) is $x_1 = 1; x_2 = 0$

17. The series $\sum_{n=0}^{\infty} a_n z^n$ represents an entire function iff :

(A) $\overline{\lim_{n \to \infty}} \, |a_n|^{1/n} = 0$

(B) $\lim_{n \to \infty} |a_n|^{1/n} = 0$

(C) $\lim_{n \to \infty} |a_n|^{1/n}$ is a finite positive real number

(D) $\overline{\lim_{n \to \infty}} \, |a_n|^{1/n} = \infty$

Or

17. Let X be a normal random variable with mean 0 and variance 2. Then :

(A) $E[\cos 2X] = e^{-4}$

(B) $E[\cos 2X] = 1$

(C) $E[\cos 2X] = 0$

(D) $E[\cos 2X] = e^{-2}$

18. $f(x) = e^{-1/x^2}$, $x \neq 0$, $x \in \mathbf{R}$

$= 0$, $x = 0$

Then :

(A) f is only a C^1 function

(B) f is real analytic at 0

(C) f is nowhere real analytic

(D) f is a C^{∞} function

Or

18. Let X be a random variable with mean 3.6 and variance 6. Which of the following is *not* correct ?

(A) X can not follow a Binomial distribution

(B) X can not follow a Poisson distribution

(C) X can not follow an exponential distribution

(D) X can not follow a normal distribution

19. If $A_n = \left[\frac{1}{n}, 1\right]$, then $\bigcup_{n=1}^{\infty} A_n$ is :

(A) (0, 1)

(B) (0, 1]

(C) [0, 1]

(D) [0, 1)

Or

19. Let X follows a Poisson distribution with parameter λ. The estimator T is defined as T = 1 if X = 0 and T = 0, otherwise. Which of the following statements is *false* ?

(A) T is unbiased for $e^{-\lambda}$

(B) Variance of T attains Crammer-Rao lower bound

(C) T is not uniformly minimum variance unbiased estimator of $e^{-\lambda}$

(D) Variance of T is greater than Crammer-Rao lower bound

20. Let $a_n = 1 + \frac{1}{2n}$, n is even

$= 1 - \frac{1}{3n}$, n is odd

Then :

(A) $\overline{\lim_{n \to \infty}} a_n > \underline{\lim}_{n \to \infty} a_n$

(B) $\overline{\lim_{n \to \infty}} a_n = \underline{\lim}_{n \to \infty} a_n = -1$

(C) $\overline{\lim_{n \to \infty}} a_n$ and $\underline{\lim}_{n \to \infty} a_n$ both do not exist

(D) $\overline{\lim}\, a_n = \underline{\lim}\, a_n = 1$

Or

20. Let $X_1, X_2, \ldots.. X_n$ be a random sample from one parameter exponential family. Let $T = \sum_{i=1}^{n} X_i$.

Test the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. Then the uniformly most powerful unbiased test is given by :

(A) $\phi(X) = 1$ if $C_1 < T < C_2$
$= 0$ otherwise

(B) $\phi(X) = 1$ if $T > C_1$
$= 0$ if $T \leq C_2$

(C) $\phi(X) = 1$ if $T < C_1$
$= 0$ if $T \geq C_2$

(D) $\phi(X) = 1$ if $T < C_1$ or $T > C_2$
$= 0$ otherwise

21. If a and b are real numbers, then $\sup \{a, b\} =$

(A) $\frac{a + b - |a - b|}{2}$

(B) $\frac{a + b + |a - b|}{2}$

(C) $\frac{a - b + |a + b|}{2}$

(D) $\frac{a - b - |a - b|}{2}$

Or

21. Let $X_1, X_2, \ldots\ldots X_n$ be iid geometric r.v.s. with parameter p. Then the distribution of $X_{(1)} = \underset{i}{\text{Min}}\, X_i$ is :

(A) Binomial

(B) Geometric

(C) Poisson

(D) Hypergeometric

22. Consider the sequence

$$a_n = 1 + \frac{1}{n} \text{ if } n \text{ is even.}$$
$$= 1 - \frac{1}{n} \text{ if } n \text{ is odd.}$$

Then :

(A) $\overline{\lim_{n \to \infty}} = \underline{\lim_{n \to \infty}}\, a_n = 1$

(B) $\overline{\lim_{n \to \infty}}\, a_n < \underline{\lim_{n \to \infty}}\, a_n$

(C) $\overline{\lim_{n \to \infty}}\, a_n > \underline{\lim_{n \to \infty}}\, a_n$

(D) $\overline{\lim_{n \to \infty}}\, a_n = \underline{\lim_{n \to \infty}}\, a_n = 2$

Or

22. The r.v. X has Bernoulli distribution defined by $P[X = 1] = 1 - P[X = 0] = \theta$, where $0 < \theta < 1$. The mle of θ based on X is :

(A) $1 - X$

(B) $\frac{1 - X}{3}$

(C) $\frac{3 - X}{3}$

(D) X

23. If $I_n = \left\{x \in \mathbf{R} \mid 0 < x < \frac{1}{n}\right\}$, then $\bigcap_{n=1}^{\infty} I_n$ is :

(A) $\{0\}$

(B) ϕ

(C) $\{x \in \mathbf{R} \mid -1 < x < 1\}$

(D) $\mathbf{R}$

Or

23. Which of the following distributions is *not* an exact sampling distribution ?

(A) χ^2-distribution

(B) Beta-distribution

(C) t-distribution

(D) F-distribution

24. Let $f(z) = \frac{\sin z}{z^3}, z \neq 0$. Then at $z = 0$, f has :

(A) a removable singularity

(B) a pole of order 1

(C) a pole of order 3

(D) a pole of order 2

Or

24. Let $X_1, X_2, ... X_n$ be iid random sample from the following probability mass function

$$P[X = x] = \frac{1}{N}; \ x = 1, 2, \ldots N$$

Then $P[X_{(n)} \leq Z]$, where $X_{(n)} = \underset{i}{\text{Max}}\ X_i$ is :

(A) $\frac{Z^N}{N^n}$

(B) $\frac{Z^N}{n^N}$

(C) $\frac{Z}{N}$

(D) $\frac{Z^n}{N^n}$

25. $\int_{|z|=2} \frac{\cos z}{z^3} dz =$

(A) πi

(B) $-\pi i$

(C) $2\pi i$

(D) $-2\pi i$

Or

25. Let y_1, y_2 be two independent observations having expectations $E(y_1) = \theta_1 + \theta_2$, $E(y_2) = \theta_1 - \theta_2$, $V(y_1) = V(y_2) = \sigma^2$. $\hat{\theta}_1$ and $\hat{\theta}_2$ are least squares estimators of θ_1 and θ_2 respectively. Which of the following statements is *false* ?

(A) $\hat{\theta}_1 = (y_1 + y_2)/2$

(B) $V(\hat{\theta}_1) = V(\hat{\theta}_2) = \sigma^2$

(C) $\hat{\theta}_2 = (y_1 - y_2)/2$

(D) $\text{Cov}(\hat{\theta}_1, \hat{\theta}_2) = 0$

26. If $f(z) = \frac{1}{e^z - 1}$, $z \neq 2n\pi i$, $n = 0, 1, 2, \ldots$, then :

(A) f has removable singularities at $z = 2n\pi i$, $n = 0, 1, \ldots$

(B) f has essential singularities at $z = 2n\pi i$, $n = 0, 1, \ldots$

(C) f has double poles at the omitted points at which the residue is zero

(D) f has poles at $z = 2n\pi i$, $n = 0, 1, 2, \ldots$

Or

26. In a two-way classification model with equal observations per cell, $E(y_{ijk}) = \mu + \alpha_i + \beta_j + \gamma_{ij}$ and $V(y_{ijk}) = \sigma^2$; $i = 1 \ldots\ldots a$; $j = 1, \ldots\ldots b$; $k = 1 \ldots c$. Which parametric function is estimable ?

(A) $\sum_i c_i \alpha_i$ where $\Sigma c_i = 0$

(B) $\sum_j d_j \beta_j$ where $\Sigma d_j = 0$

(C) $\mu + \alpha_i + \beta_j$

(D) $\sum_i \sum_j f_{ij} \gamma_{ij}$ where $\sum_i f_{ij} = \sum_j f_{ij} = 0$

27. Let N be a deleted neighbourhood of z_0. Then *correct* statement from the following is :

(A) z_0 is a removable singularity of f if f is analytic and unbounded in N

(B) z_0 is a removable singularity of f if f is analytic and bounded in N

(C) z_0 is a removable singularity of f if $\lim_{z \to z_0} (z - z_0) f(z) \neq 0$

(D) z_0 is a removable singularity of f if $\lim_{z \to z_0} (z - z_0) f(z)$ does not exist

Or

27. In one-way ANOVA with n observations and 8 treatments, the experimenter while calculating the F ratio forgot to divide the sum of squares by the corresponding degrees of freedom. However the statistician said there is no mistake, because :

(A) $n = 8$

(B) $n = 15$

(C) $n = 16$

(D) $n = 14$

28. The function $f(z) = |z|^2$, $z \in \mathbb{C}$ is :

(A) continuous and differentiable everywhere

(B) everywhere continuous but nowhere differentiable

(C) continuous everywhere but differentiable only at the origin

(D) neither continuous nor differentiable anywhere

Or

28. In a RBD with 5 blocks and 4 treatments, number of plots in each block is :

(A) 20

(B) 5

(C) 4

(D) 9

29. $f(z) = e^{\bar{z}}$, $z \in \mathbb{C}$ is :

(A) analytic in $\mathbb{C}$

(B) analytic only in the upper half plane

(C) analytic in the first quadrant

(D) analytic nowhere in $\mathbb{C}$

Or

29. Let r denote the minimum number of times any treatment is replicated in completely randomised design with v treatments and n plots. Which of the following statements is always *true* ?

(A) $r = v$

(B) $r \geq$ integer part of (n/v)

(C) $r =$ integer part of $(n/v) + 1$

(D) $r \leq$ integer part of (n/v)

30. If $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, then $(\bar{z})^4$ is :

(A) $\frac{1 - \sqrt{3}i}{2}$

(B) $\frac{1 + \sqrt{3}i}{2}$

(C) $\frac{-1 + \sqrt{3}i}{2}$

(D) $\frac{-1 - \sqrt{3}i}{2}$

Or

30. In a RBD with b blocks and v treatments which of the following statements always holds ?

(A) $b > v$

(B) $b < v$

(C) $b = v$

(D) b and v are arbitrary positive integers

31. Which of the following statements is *true* ?

(A) Every group is isomorphic to a permutation group

(B) A group G is isomorphic to a permutation group iff it is finite

(C) A group G is commutative iff it is cyclic

(D) A group G is non-abelian iff order of G is not prime

Or

31. When desired information is available for all objects in the population, we have what is called a :

(A) Census

(B) Population

(C) Sample

(D) Stem-and-leaf display

32. Let G be a group of order 15. Then :
    - (A) $d \mid o(G)$ implies G has a unique subgroup of order d
    - (B) G has only one normal subgroup
    - (C) G has two subgroups of order 3 and one subgroup of order 5
    - (D) (e) is the only proper subgroup of G

Or

32. Which of the following statements is *false* ?
    - (A) All companies listed on the New York Stock Exchange is an example of a sample
    - (B) In enumerative studies, interest is focussed on a finite, identifiable, unchanging collection of individuals or objects that make up a population
    - (C) A sampling frame is a listing of the individuals or objects to be sampled
    - (D) All possible yields (in grams) from a certain chemical reaction carried out in a laboratory is an example of conceptual or hypothetical population

33. The ring of 2×2 matrices over Z :
    - (A) is commutative
    - (B) is an integral domain
    - (C) has non-zero zero-divisors
    - (D) is a field

Or

33. Which of the following statements involves descriptive statistics as opposed to inferential statistics ?
    - (A) The FAA samples 1000 traffic controllers in order to estimate the percent retiring due to job stress related illness
    - (B) Based on sample of 250 professional tennis players, a tennis magazine reported that 25% of the parents of all professional tennis players did not play tennis
    - (C) The Alcohol, Tobacco and Firearms Department reported that Houston had 1,791 registered gun dealers in 1997
    - (D) Based on survey of 500 magazine readers, the magazine reports that 40% of its readers prefer double column articles

34. Let F be a field with a finite number of elements. Then :

(A) the order of F may be 4

(B) the order of F may be 10

(C) the order of F is prime

(D) the characteristic of F is zero

Or

34. You asked ten of your classmates about their weight. On the basis of this information, you stated that the average weight of all students in your college is 150 pounds. This is an example of :

(A) inferential statistics

(B) sample

(C) population

(D) descriptive statistics

35. Let M be a proper ideal of a Boolean ring R. Consider the following statements :

(I) $\frac{R}{M}$ is a Boolean ring

(II) $\frac{R}{M} \cong \frac{Z}{(2)}$ iff M is a maximal ideal.

Then :

(A) Both (I) and (II) hold

(B) Only (I) holds

(C) Only (II) holds

(D) Both (I) and (II) are false

Or

35. The measure most unaffected by outliers is the :

(A) median

(B) trimmed mean

(C) mean

(D) range

36. Let R be an integral domain and R[x] be the polynomial ring over R. Then :

(A) R[x] is an Euclidean domain

(B) If every ideal of R is principal then so is true for R[x]

(C) R[x] is an integral domain

(D) R[x] is a field

Or

36. The average score for a class of 25 students was 75. If the 15 female students in a class averaged 70, then the male students in the class averaged :

(A) 75.0

(B) 85.0

(C) 82.5

(D) 77.5

37. Consider the following two statements :

(I) If A is a triangular matrix and no entry on the main diagonal is zero, then A is singular.

(II) If A is triangular matrix and an entry on the main diagonal is zero, then A is invertible.

Then :

(A) only (I) is true

(B) only (II) is true

(C) both (I) and (II) are true

(D) both (I) and (II) are false

Or

37. At a certain car service station with service facility for only one car and no waiting space, cars arrive for service in Poisson manner at the rate of one per hour and service time is also exponential with an average one hour, with independence of service times and interarrival times, the probability that an incoming car finds the service facility available is :

(A) $\frac{1}{2}$

(B) e^{-1}

(C) $\frac{3}{4}$

(D) None of the above

38. If the transformation T : $\mathbf{R}^n \to \mathbf{R}$ is defined by T(X) = X'AX, where A is any $n \times n$ symmetric matrix, then T(X + Y) is equal to :

(A) T(X) + 2X'AY + T(Y)

(B) T(X) + T(Y)

(C) T(X) + X'AY + T(Y)

(D) T(X) + Y'AX + T(Y)

Or

38. The staff replacement policy in replacement model :
    - (A) arises due to resignation, retirement, or death of a staff member from time to time
    - (B) is like replacement policy for items whose values deteriorate gradually
    - (C) can be easily formulated because people retire at known time
    - (D) does not yield the optimum replacement interval

39. Let A = $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ be the matrix, then the corresponding quadratic form is :
    - (A) $x^2 - y^2$
    - (B) $x^2 + xy + y^2$
    - (C) $x^2 + y^2$
    - (D) $x^2 + 2xy + y^2$

Or

39. Which of the following is *not* an assumption underlying the fundamental problem of EOQ in Inventory Model ?
    - (A) Demand is known and uniform
    - (B) Holding cost per unit per time period is constant
    - (C) Lead time is not zero
    - (D) Stock-outs (Shortages) are not permitted

40. Let T be the linear transformation on $\mathbf{R}^2$ represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, then the characteristic values of T are :
    - (A) 1, –1
    - (B) 1, 0
    - (C) 1, i
    - (D) i, $-i$

Or

40. A mixed strategy game in Game theory can be solved by :
    - (A) Matrix method
    - (B) Graphical method
    - (C) Algebraic method
    - (D) All of the above

41. Let $p, q, r \in \mathbf{Z}$. If the vectors $\begin{pmatrix} p \\ q \\ r \end{pmatrix}, \begin{pmatrix} q \\ r \\ p \end{pmatrix}, \begin{pmatrix} r \\ p \\ q \end{pmatrix}$ are mutually orthogonal in Euclidean space $\mathbf{R}^3$ then :

(A) $p^2 + q^2 + r^2 = 0$

(B) $p^2 + q^2 + r^2 \neq 0$

(C) $pq + qr + rp \neq 0$

(D) $pq + qr + rp = 0$

Or

41. Which of the following is *not* one of the principles of sampling ?

(A) Sampling units must be identifiable and clearly defined

(B) Sampling units are not interchangeable

(C) Sampling units must be independent of each other

(D) Sampling units must be representative and include males and females

42. Let $\alpha_1 = (1, 2), \alpha_2 = (3, 4) \in \mathbf{R}^2$ and $\beta_1 = (3, 2, 1), \beta_2 = (6, 5, 4) \in \mathbf{R}^3$. Let T be a linear transformation from $\mathbf{R}^2$ into $\mathbf{R}^3$ such that $T\alpha_i = \beta_i$ $\forall\ i = 1, 2$. Then :

(A) $T(x, y) = \left(\frac{3}{2}x,\ x + \frac{1}{2}y,\ 2x + \frac{1}{2}y\right)$

(B) $T(x, y) = \left(x + \frac{3}{2}y,\ x - \frac{1}{2}y,\ 2x + y\right)$

(C) $T(x, y) = \left(\frac{3}{2}y,\ x + \frac{1}{2}y,\ 2x - \frac{1}{2}y\right)$

(D) Such a linear transformation does not exist

Or

42. In a multi-stage sampling :

(A) the same number of people are studied more than once

(B) different respondents are studied more than once

(C) a sequence of samples are drawn from already selected samples but only the last sample is studied

(D) a sequence of samples are drawn from already selected samples and each one of them is studied

43. Consider the following two statements :

(I) If A is a nilpotent matrix then I + A is non-singular

(II) A is triangular matrix with all diagonal elements are zero, then A is nilpotent.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

Or

43. In a study of attitudes to University policies, a researcher initially chose 150 first-year students, 130 second-year students and 100 third-year students. Then the researcher selected 25 male and 25 female students from each year group, who were finally interviewed. The sampling procedure used in this study was :

(A) Probability sampling

(B) Stratified sampling

(C) Multi-stage sampling

(D) Multi-phase sampling

44. The complete solution of the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \sin x + \sin y$$

is :

(A) $z = a(x + y) + (\sin x - \sin y) + b$

(B) $z = a(x - y) - (\cos x + \cos y) + b$

(C) $z = a(x + y) - (\sin x - \cos y) + b$

(D) $z = a(x - y) - (\cos x - \sin y) + b$

Or

44. The time required to assemble an electronic component is normally distributed with a mean of 12 minutes and a standard deviation of 1.5 minutes. Find the probability that the time required to assemble nine components is greater than 117 minutes :

(A) 0.0013

(B) 0.0228

(C) 0.2486

(D) 0.2514

45. Let ϕ_1, ϕ_2 be two linearly independent solutions of the constant coefficient equation $y'' + a_1 y' + a_2 y = 0$. Then the Wronskian of ϕ_1, ϕ_2 is constant if and only if :

(A) $a_1 = 0$

(B) $a_2 = 0$

(C) $a_1 < 0$

(D) $a_1 > 0$

Or

45. The central limit theorem tells us that the sampling distribution of the mean is approximately normal. Which of the following conditions is necessary for the theorem to be valid ?

(A) The sample size has to be large

(B) The population variance has to be small

(C) We must be sampling from a normal population

(D) The population has to be symmetric

46. The characteristic curve of the two parameter family of surfaces $(x - a)^2 + (y - b)^2 + z^2 = 1$ is :

(A) a circle

(B) a great circle

(C) helix

(D) ellipse

Or

46. A canonical correlation cannot be negative because :

(A) we take only positive eigen-values

(B) it is generalization of the multiple correlation

(C) we take only positive square root

(D) we reject a negative value

47. Let ϕ_1, ϕ_2 be two differentiable functions on an interval I then

(I) $W(\phi_1, \phi_2)(x) = 0 \ \forall x \in I \Rightarrow \phi_1$, ϕ_2 are linearly dependent

(II) $W(\phi_1, \phi_2)(x) \neq 0 \ \forall x \in I \Rightarrow \phi_1$, ϕ_2 are linearly independent

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

Or

47. Wishart distribution is a multivariate generalization of :

(A) normal distribution

(B) beta distribution

(C) matrix beta distribution

(D) chi-square distribution

48. The initial value problem $y' = f(x, y)$, $y(x_0) = y_0$, where (x, y) belongs to a domain D, has unique solution :

(A) if $f(x, y)$ is continuous in D

(B) if $f(x, y)$ is continuous and bounded on D

(C) if $f(x, y)$ is continuous and satisfies Lipschitz condition for all points on D

(D) for all functions $f(x, y)$ on D

Or

48. Which of the following statements are true for the following data values 9, 7, 8, 6, 9, 10 and 14 ?

(A) The mean is smaller than the median

(B) The mean and the 10% trimmed mean are equal

(C) The mean and median are equal

(D) The mean is larger than the median

49. If the number of constants are to be eliminated from the given relation is just equal to the number of independent variables, then the partial differential equation obtained by eliminating these constants :

(A) is always a linear equation of first order

(B) can be a non-linear equation of first order

(C) is a linear equation of second order

(D) is a non-linear equation of second order

Or

49. Let X_1, X_2, ... be a sequence of independent and identically distributed r.v.s. with the common p.d.f. given by

$$f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Consider the statements :

(I) $\sum_{i=1}^{n} X_i/n \xrightarrow{P} \frac{4}{5}$ as $n \to \infty$

(II) $\sum_{i=1}^{n} X_i^2/n \xrightarrow{P} \frac{2}{3}$ as $n \to \infty$

(III) $\frac{1}{n}\sqrt{\frac{3}{2}} \sum_{i=1}^{n} \left(X_i - \frac{4}{5}\right) \xrightarrow{d}$ N(0, 1) as $n \to \infty$

Then :

(A) All statements are true

(B) Only statements I and II are true

(C) Only statements I and III are true

(D) Only statement I is true

50. The solution of a homogeneous initial value problem with constant coefficient is $y = 3xe^{2x} + 6 \cos 4x$. Then the least possible order of the differential equation is :

(A) 4

(B) 5

(C) 6

(D) 3

Or

50. The random variables X and Y assume only 4 values each, and cov (X, Y) = 0. Then :

(A) X and Y need not be independent

(B) X and Y are independent

(C) X and Y are indepedent only if four values are 0 and 1 for both X and Y

(D) The line is parallel to Y-axis or X-axis

ROUGH WORK

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