

Test Booklet Code & Serial No.

प्रश्नपत्रिका कोड व क्रमांक  
**Paper-II**

**A**

# MATHEMATICAL SCIENCE

**Signature and Name of Invigilator**

Seat No.

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1. (Signature) .....

(In figures as in Admit Card)

(Name) .....

Seat No. ....

(In words)

2. (Signature) .....

(Name) .....

OMR Sheet No.

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(To be filled by the Candidate)

**SEP - 30221**

**Time Allowed : 2 Hours]**

**[Maximum Marks : 200**

**Number of Pages in this Booklet : 48**

**Number of Questions in this Booklet : 190**

### Instructions for the Candidates

- Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
- This paper consists of **190** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
- At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
  - To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
  - Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
  - After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
- Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.  
**Example :** where (C) is the correct response.  

(A)	(B)	(C)	(D)
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- Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only**. If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
- Read instructions given inside carefully.
- Rough Work is to be done at the end of this booklet.
- If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
- You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
- Use only Blue/Black Ball point pen.
- Use of any calculator or log table, etc., is prohibited.
- There is no negative marking for incorrect answers.

### विद्यार्थ्यांसाठी महत्वाच्या सूचना

- परीक्षार्थींनी आपला आसन क्रमांक व पुस्तकील कर्त्या कोप्यात लिहावा. तसेच आपणास दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
- सदर प्रश्नपत्रिकेत 190 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास दोन गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील सर्व प्रश्न खेडविलेले आणकार्य आहे.
- परीक्षा सुरु झाल्यावर विद्यार्थ्यांला उत्तरपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून घ्याव्यात.
  - प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
  - पहिल्या पृष्ठाने व मुद्र केलेल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताना पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदीप प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकांस परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच केवळी पाहून घ्याव्यात नाही याची कुनचा विद्यार्थ्यांनी नोंद घ्यावी.
  - बरीलप्रमाणे सर्व पडताना पहाविल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा वेळ लिहावा.
- प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उतारे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळ/निळा करावा.  
**उदा. :** जर (C) हे योग्य उत्तर असेल तर.  

(A)	(B)	(C)	(D)
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- या प्रश्नपत्रिकेतील प्रश्नांची उतारे ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत. इतर ठिकाणी लिहिलेले उतारे तपासली जाणार नाहीत.
- आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
- प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोन्या पानावरच कपचे काम करावे.
- जर आवक ओ.एम.आर. वर व मुद्र केलेल्या लिखाणाचा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटल अशी कोणतीही खूप केलेली आडखून आल्यास अथवा असभ्य भाषेचे वापर किंवा इतर गैरमर्यादा अवलंब केल्यास विद्यार्थ्यांला परीक्षास आपण उरविण्यात येईल.
- परीक्षा संपल्यानंतर विद्यार्थ्यांने मुळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे पात करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय इत आपल्याबरोबर नेण्यास विद्यार्थ्यांचा परवानगी आहे.
- फक्त निळा किंवा काळ्या बॉल पेनचाच वापर करावा.
- कॅल्क्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
- चुकीच्या उत्तरासाठी गुण कापत केली जाणार नाही.

**SEP - 30221/II—A**

## Mathematical Science Paper II

**Time Allowed : 120 Minutes]**

**[Maximum Marks : 200**

**Note :** This Paper contains **One Hundred Ninety (190)** multiple choice questions in **THREE (3)** sections, each question carrying **TWO (2)** marks. Attempt **all** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

**Section I : Q. Nos. 1 to 10,**

**Section II : Q. Nos. 11 to 100,**

**Section III : Q. Nos. 101 to 190.**

SECTION I	
<p>1. Let <math>(x_n)_{n \in \mathbf{N}}</math> be a convergent sequence of positive real valued numbers. Which of the following statements is <i>true</i> ?</p> <p>(A) Limit of the sequence is positive</p> <p>(B) Limit of the sequence may be negative</p> <p>(C) There is a subsequence of the sequence which has positive limit</p> <p>(D) The limit of every subsequence of the sequence is non-negative</p>	<p>2. Let <math>f : \mathbf{R} \rightarrow \mathbf{Q}</math> be a non-constant increasing function. Then :</p> <p>(A) <math>f</math> is constant on some interval <math>[a, b]</math></p> <p>(B) <math>f</math> is one-one</p> <p>(C) <math>f</math> is onto</p> <p>(D) <math>f</math> is not constant on any interval</p> <p>3. Which of the following is <i>true</i> for a complex number <math>z = x + iy</math> ?</p> <p>(A) <math> x  +  y  =  z </math></p> <p>(B) <math> x  +  y  = \sqrt{2}  z </math></p> <p>(C) <math> x  +  y  \geq  z </math></p> <p>(D) <math> x  +  y  \leq  z </math></p>

4. The function  $f : \mathbf{C} \rightarrow \mathbf{C}$  defined by  $f(z) = \cos z$  is :
- (A) one-one  
 (B) onto  
 (C) bijective  
 (D) bounded
5. Let  $\mathbf{R}_n[x]$  denote the vector space of all polynomials over  $\mathbf{R}$  having degree atmost  $n$ . Then the dimension of  $\mathbf{R}_n[x]$  is :
- (A)  $n^2$   
 (B)  $n + 1$   
 (C)  $n$   
 (D)  $\infty$
6. Let  $V$  and  $V'$  be finite dimensional vector spaces over  $K$  and let  $\{\sqrt{1}, \dots, \sqrt{n}\}$  be a basis of  $V$ . If  $T \in L(V, V')$  and  $T$  is injective, then which of the following statements is FALSE ?
- (A)  $T\sqrt{1}, \dots, T\sqrt{n}$  are linearly independent  
 (B)  $\dim V \leq \dim V'$   
 (C)  $\dim V = \dim V'$   
 (D)  $\text{Ker } T = \{0\}$
7. Suppose  $P(x, y)$  denotes the joint probability mass function of two random variables  $X$  and  $Y$  where :  
 $P(0, 0) = 0.4$ ,  $P(0, 1) = 0.2$ ,  
 $P(1, 0) = 0.1$ ,  $P(1, 1) = 0.3$ .
- What is the conditional probability of  $X = 0$ , given that  $Y = 1$  ?
- (A)  $\frac{1}{5}$   
 (B)  $\frac{3}{5}$   
 (C)  $\frac{2}{5}$   
 (D) 0
8. If 3 balls are randomly drawn from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and other two black ?
- (A)  $\frac{3}{11}$   
 (B)  $\frac{4}{11}$   
 (C)  $\frac{8}{11}$   
 (D)  $\frac{2}{11}$

SECTION II

9. Which of the following sets are convex sets ?
- (i)  $\{(x, y) \mid y = x\}$
- (ii)  $\{(x, y) \mid y \leq |x|\}$
- (iii)  $\{(x, y) \mid y \geq x^2\}$
- (A) (i) only
- (B) (i) and (ii) only
- (C) (i) and (iii) only
- (D) (ii) and (iii) only
10. Which of the following statement(s) is/are true for an LP problem ?
- (I) The set of all feasible solutions to an LP problem is a convex set.
- (II) Every hyperplane is a convex set.
- (A) Both are true
- (B) Only (I) is true
- (C) Only (II) is true
- (D) Neither (I) nor (II) is true

11. Consider the set X of all sequences  $(x_n)$  of natural numbers.
- Let  $Y = \{(x_n) \in X : x_n = 0 \text{ for infinitely many } n\}$ ,
- $Z = \{(x_n) \in X : x_n = 0 \text{ for infinitely many } n\}$ ,
- $W = \{(x_n) \in X : x_n \neq 0 \text{ for finitely many } n\}$ .
- Then which of the following sets is countable ?
- (A) Y
- (B) Z
- (C) W
- (D) Complement of Z in X
12. Which of the following real numbers  $x$ , the inequality  $(1+x)^n \geq 1+nx$  holds for all  $n \in \mathbf{N}$  ?
- (A)  $x \in \mathbf{R}$
- (B)  $x \in [-4, 4]$
- (C)  $x \in (-1, \infty)$
- (D)  $x \in (-\infty, -4] \cup [0, \infty)$
13. How many solutions are there for the equation  $\cos x - e^{-x} = 0$  ?
- (A) No solution
- (B) Finitely many solutions
- (C) Countably infinite solutions
- (D) Uncountably many solutions

14. Which of the following statements holds for a function  $f : [0, 1] \rightarrow \mathbf{R}$  ?
- (A) If  $f$  is of bounded variation, then  $f$  is Riemann integrable
- (B) If  $f$  is of bounded variation, then it is uniformly continuous
- (C) If  $f$  is bounded, then it is of bounded variation
- (D) If  $f$  is uniformly continuous, then it is of bounded variation
15. Consider the following statements :
- (a) Every non-empty open subset of  $\mathbf{R}$  is uncountable.
- (b) Every connected subset of  $\mathbf{R}$  containing at least two elements is uncountable.
- Then :
- (A) Only (a) is true
- (B) Only (b) is true
- (C) Both (a) and (b) are true
- (D) Both (a) and (b) are false
16. Which of the following Mobius transformations maps the unit disc onto the right half plane ?
- (A)  $f(z) = \frac{z-i}{z+i}$
- (B)  $f(z) = \frac{z-1}{z+1}$
- (C)  $f(z) = \frac{1+z}{1-z}$
- (D)  $f(z) = i\left(\frac{1+z}{1-z}\right)$
17. Let  $S$  be the set of zeros of a non-zero entire function. Then the set  $S$  is :
- (A) non-empty open
- (B) infinite and bounded
- (C) closed
- (D) uncountable
18. If  $f = u + iv$  is analytic on  $\mathbf{C}$ , then which of the following is FALSE ?
- (A)  $u, v$  are harmonic
- (B)  $u^2, v^2$  are harmonic
- (C)  $u + v$  is harmonic
- (D)  $uv$  is harmonic

19. Let  $f$  be analytic on  $\mathbf{C} - \{0\}$  and let  $v_1, v_2 : [0, 1] \rightarrow \mathbf{C} - \{0\}$  be defined by  $v_1(t) = e^{2\pi it}$  and  $v_2(t) = 10 e^{2\pi it}$ .

Suppose  $I_1 = \int_{v_1} f(z) dz$  and

$$I_2 = \int_{v_2} f(z) dz.$$

Then :

(A)  $I_1 = I_2$

(B)  $I_1 = 10I_2$

(C)  $I_1 = \frac{1}{10} I_2$

(D) There is no relation between  $I_1$  and  $I_2$

20. Consider the function  $f(z) = \frac{\cot z}{z}$ .

Then :

(A)  $f$  has a simple pole at  $z = 0$

(B)  $f$  has a removable singularity at  $z = 0$

(C)  $f$  has uncountably many singularities

(D) the residue of  $f$  at  $z = 0$  is 1

21. Let  $G$  be a finite group and  $H, H'$  be two subgroups of  $G$ . Then :

(A)  $H = H' \Leftrightarrow \# H = \# H'$

(B)  $H = H' \Leftrightarrow [G : H] = [G : H']$

(C)  $H = H' \Leftrightarrow aH = bH'$  for some

$$a, b \in G$$

(D)  $H = H' \Leftrightarrow$  both  $H$  and  $H'$  are normal in  $G$

22. Let  $I$  be the ideal in the polynomial ring  $\mathbf{Z}_4[X]$  generated by the polynomial  $1 + 2X$ . Then :

(A)  $I$  is a prime ideal

(B)  $I$  is a maximal ideal

(C)  $I$  is a proper ideal which is not a prime ideal

(D)  $I$  is a unit ideal

23. In the group  $S_3$ , there are exactly :

- (A) 2 elements satisfying  $x^3 = e$  and  
3 elements satisfying  $x^2 = e$
- (B) 3 elements satisfying  $x^3 = e$  and  
4 elements satisfying  $x^2 = e$
- (C) 2 elements satisfying  $x^3 = e$  and  
4 elements satisfying  $x^2 = e$
- (D) 3 elements satisfying  $x^3 = e$  and  
3 elements satisfying  $x^2 = e$

24. Let  $G$  be a cyclic group of order 20.

Then the number of elements of  $G$  of order 10 is :

- (A) 10
- (B) 4
- (C) 6
- (D) 2

25. Which of the following is an integral domain but not a field ?

- (A)  $\mathbf{Z}_{10}$
- (B)  $\mathbf{Z}_{31}$
- (C)  $\mathbf{R}[x]$
- (D)  $\mathbf{C}(x)$

26.  $A^{35}$  if  $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  is :

- (A)  $\begin{pmatrix} 1 & 70 \\ 0 & 1 \end{pmatrix}$
- (B)  $\begin{pmatrix} 1 & 35 \\ 0 & 1 \end{pmatrix}$
- (C)  $\begin{pmatrix} 2 & 35 \\ 0 & 2 \end{pmatrix}$
- (D)  $\begin{pmatrix} 35 & 35 \\ 0 & 35 \end{pmatrix}$

27. Let  $W$  be the vector space of real skew symmetric  $n \times n$  matrices. Then the dimension of  $W =$

- (A)  $n^2$
- (B)  $n^2 - n$
- (C)  $\frac{n^2 - n}{2}$
- (D)  $\frac{n^2 + n}{2}$



28. If  $A$  is a real  $3 \times 3$  matrix, which of the following statements can *not* be true about  $A$  ?

- (A) Minimum polynomial = characteristic polynomial
- (B)  $A$  has at least one complex eigen value
- (C) Minimum polynomial of  $A = x^2 - 1$
- (D) Minimum polynomial of  $A = x^2 + 1$

29. Matrix  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$  is :

- (A) Diagonalizable over  $\mathbf{R}$
- (B) Triangular over  $\mathbf{R}$  but not diagonalizable over  $\mathbf{R}$
- (C) Triangular over  $\mathbf{C}$  but not diagonalizable over  $\mathbf{C}$
- (D) Diagonalizable over  $\mathbf{C}$

30. Let  $W$  be the subspace of  $\mathbf{R}^4$  given by :

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \middle/ \begin{matrix} x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ x_1 - x_2 + x_3 = 0 \end{matrix} \right\}$$

Then  $W^\perp$  is generated by :

(A)  $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

(D)  $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

31. The partial differential equation obtained by eliminating arbitrary constants from

$$z = ae^{bx} \sin by$$

is :

- (A)  $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} = 0$
- (B)  $\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$
- (C)  $\frac{\partial^2 z}{\partial y^2} + x \frac{\partial^2 z}{\partial x^2} = 0$
- (D)  $\frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} = 0$

32. The solution of the linear partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial y^2} = 0$$

is :

- (A)  $z = \phi_1(x + y) + \phi_2(x - y)$
- (B)  $z = \phi_1(x + y) + x\phi_2(x - y)$
- (C)  $z = \phi_1(x + y) + x\phi_2(x + y)$
- (D)  $z = \phi_1(x + y) + x^2\phi_2(x + y)$

33. If  $\phi_1$  and  $\phi_2$  are two linearly independent solutions of  $y'' + ay = 0$ , where  $a$  is a constant, on an interval  $I$ . Then which of the following statements about the Wronskian  $W(\phi_1, \phi_2)(x)$  is true ?

- (A)  $W(\phi_1, \phi_2)(x)$  is a constant function
- (B)  $W(\phi_1, \phi_2)(x)$  is a monotonically increasing function
- (C)  $W(\phi_1, \phi_2)(x)$  is a monotonically decreasing function
- (D)  $W(\phi_1, \phi_2)(x) = 0$  for all  $x \in I$

34. A solution of the differential equation

$$xy' = 2y$$

passing through  $(1, 2)$ , also passes through :

- (A)  $(2, 1)$
- (B)  $(2, 4)$
- (C)  $(2, 8)$
- (D)  $(2, 16)$

35. The differential equation obtained by eliminating arbitrary constants from the expression  $y = ae^{bx}$  is :

(A)  $y \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$

(B)  $y \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$

(C)  $\frac{d^2y}{dx^2} + xy \left(\frac{dy}{dx}\right)^2 = 0$

(D)  $\frac{d^2y}{dx^2} - xy \frac{dy}{dx} = 0$

36. Assign the programmers U, V, W to the programmes I, II, III in such way that the total computer time is minimum. The estimated computer time required by the programmers to execute the programmes is given in the table.

		Programmers		
		U	V	W
Programmes	I	120	100	80
	II	80	90	110
	III	110	140	120

The optimal assignment is :

- (A) I → W, II → V, III → U
- (B) I → V, II → U, III → W
- (C) I → U, II → W, III → V
- (D) Optimal assignment cannot be done

37. A company management and the labour unions are negotiating a new three year settlement. Each of these has 4 strategies mentioned below :  
 (I) : Hard and aggressive bargaining.  
 (II) : Reasoning and logical approach.  
 (III) : Legalistic strategy.  
 (IV) : Conciliatory approach.

The cost to the company are given for every pair of strategy choice.

		Company Strategies			
		I	II	III	IV
Labour Union Strategies	I	20	15	12	35
	II	25	14	8	10
	III	40	2	10	5
	IV	-5	4	11	0

Then the value of the game is :

- (A) 12
- (B) -5
- (C) 0
- (D) 2

38. Consider the following LP problem.

Max  $Z = 6x_1 - 4x_2$

Subject to the constraints :

$2x_1 + 4x_2 \leq 4$

$4x_1 + 8x_2 \geq 16$

$x_1, x_2 \geq 0.$

Then L.P. has :

- (A) Infeasible solution
- (B) Unbounded solution
- (C) Basic feasible solution
- (D) Feasible solution

39. Consider the LP problem :

$$\text{Max } Z = x_1 - x_2 + 3x_3$$

Subject to the constraints :

$$x_1 + x_2 + x_3 \leq 10$$

$$2x_1 - x_2 - x_3 \leq 2$$

$$2x_1 - 2x_2 - 3x_3 \leq 6 \text{ and}$$

$$x_1, x_2, x_3 \geq 0.$$

Then the dual of the above LP problem is :

$$\text{Min } W : py_1 + qy_2 + ry_3$$

Subject to the constraints :

$$ay_1 + by_2 + cy_3 \geq 1$$

$$y_1 - y_2 - 2y_3 \geq -1$$

$$y_1 - y_2 - 3y_3 \geq 3 \text{ and}$$

$$y_1, y_2, y_3 \geq 0$$

where the values of  $p, q, r, a, b, c$  are :

(A)  $p = 10, q = 2, r = -6, a = -1,$   
 $b = 2, c = -2$

(B)  $p = 10, q = 2, r = 6, a = 1,$   
 $b = 2, c = 2$

(C)  $p = 10, q = -2, r = -6, a = -1,$   
 $b = -2, c = 2$

(D)  $p = -10, q = 2, r = 6, a = -1,$   
 $b = -2, c = 2$

40. The number of basic variables in an  $(m \times n)$  transportation table are :

(A)  $m + n - 1$

(B)  $m + n + 1$

(C)  $mn$

(D)  $m + n$

41. Consider the function

$$f : [0, 1] \times [0, 1] \rightarrow \mathbf{R}$$

defined as :

$$f(x, y) = 1 \text{ if either } x \text{ or } y \text{ is}$$

irrational

$$= 0 \text{ otherwise}$$

Then  $\int_{[0,1] \times [0,1]} f$  is :

(A) 0

(B) 1

(C) Undefined

(D) 2

42. Consider the function  $f : \mathbf{R}^2 \rightarrow \mathbf{R}$

given by  $f(x, y) = x^2 + y^2 - 4y$ .

Then in a neighbourhood of  $(0, 0)$  :

(A)  $x$  variable can be expressed as a function of  $y$

(B)  $y$  variable can be expressed as a function of  $x$

(C) none of  $x$  and  $y$  be expressed as a function of other variable

(D) both  $x$  and  $y$  can be expressed as a function of the other variable

43. Consider the function  $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  defined as  $f(x, y) = (x^2 - y^2, 2xy)$ . Then the image of the ball  $B(P, \frac{1}{2})$ , where  $P = (1, 0)$  is the centre and  $\frac{1}{2}$  is radius, is :
- (A) a closed set  
 (B) an open set  
 (C) neither open nor closed set  
 (D) an unbounded set
44. Let  $f$  be a non-constant entire function. Then which of the following is *impossible* ?
- (A)  $f(\mathbf{C}) = \mathbf{C}$   
 (B)  $f(\mathbf{C}) = \mathbf{C} - \{0\}$   
 (C)  $\overline{f(\mathbf{C})} = \mathbf{C}$   
 (D)  $f(\mathbf{C}) \subseteq \mathbf{R}$
45. Consider the following statements :
- (a)  $|e^{\sin z}|$  does not assume its maximum value in  $\mathbf{C}$ .  
 (b)  $|\sin(e^z)|$  does not assume its minimum value in  $\mathbf{C}$ .
- Then :
- (A) Only (a) is true  
 (B) Only (b) is true  
 (C) Both (a) and (b) are true  
 (D) Both (a) and (b) are false
46. Let  $f : \mathbf{C} \rightarrow \mathbf{C}$  be a non-constant analytic such that  $|f(z)| \leq |z|$  for  $|z| \geq 2019$ . Then which of the following is *false* ?
- (A)  $f$  maps a line onto a circle  
 (B)  $f$  is one-one  
 (C)  $f$  is onto  
 (D)  $f'(z) \neq 0$  for all  $z \in \mathbf{C}$
47. Let  $L|K$  be an algebraic field extension and  $\sigma : L \rightarrow L$  be a  $K$ -algebra homomorphism. Then :
- (A)  $\sigma$  is bijective  
 (B)  $\sigma$  is injective, but not surjective  
 (C)  $\sigma$  is surjective, but not injective  
 (D)  $\sigma$  is neither injective nor surjective
48. The number of linear factors of the polynomial  $X^{16} + 1$  in  $\mathbf{R}[X]$  is :
- (A) 0  
 (B) 1  
 (C) 2  
 (D) 4
49. The number of irreducible quadratic factors of the polynomial  $X^8 + 1$  in  $\mathbf{R}[X]$  is :
- (A) 4  
 (B) 3  
 (C) 2  
 (D) 1

50. Let  $X$  be a metric space and  $Y, Z \subset X$  such that  $Y \cap Z = \phi$ . Then :
- (A)  $d(Y, Z) > 0$   
 (B)  $d(Y, Z) > 0$  if  $Y$  and  $Z$  are closed  
 (C)  $d(Y, Z) > 0$  if  $Y$  or  $Z$  is compact  
 (D)  $d(Y, Z) > 0$  if one of  $Y$  or  $Z$  is compact and the other is closed in  $X$
51. Let  $f, g : \mathbf{R} \rightarrow \mathbf{R}$  be defined by
- $$f(x) = \sin \frac{1}{x} \quad \text{and} \quad g(x) = x \sin \frac{1}{x}.$$
- Then :
- (A)  $f$  and  $g$  are of bounded variation  
 (B)  $f$  is of bounded variation, but  $g$  is not of bounded variation  
 (C)  $g$  is of bounded variation, but  $f$  is not of bounded variation  
 (D) neither  $f$  nor  $g$  is of bounded variation
52. Let  $X, Y$  be metric spaces and  $f : X \rightarrow Y$  be a surjective continuous map. Then :
- (A)  $f$  maps open sets in  $X$  to open sets in  $Y$   
 (B)  $f$  maps closed sets in  $X$  to closed sets in  $Y$   
 (C)  $f$  maps dense subsets of  $X$  to dense subsets of  $Y$   
 (D)  $f$  maps nowhere dense subsets of  $X$  to nowhere dense subsets of  $Y$
53. Let  $G$  be a group of order  $n$ ,  $p$  a prime such that  $p | n$ . Then :
- (A) every Sylow  $p$ -subgroup of  $G$  is normal in  $G$   
 (B) every Sylow  $p$ -subgroup of  $G$  is abelian  
 (C) the number of Sylow  $p$ -subgroups of  $G$  is equal to the index of  $N(P)$  in  $G$ , where  $N(P)$  is the normaliser of any Sylow  $p$ -subgroup  $P$  of  $G$   
 (D) the number of Sylow  $p$ -subgroups of  $G$  is equal to the index of  $P$  in  $G$ , where  $P$  is any Sylow  $p$ -subgroup of  $G$
54. The number of elements of order 3 in the direct product of groups  $\mathbf{Z}_9 \times \mathbf{Z}_3$  is :
- (A) 6  
 (B) 8  
 (C) 9  
 (D) 18
55. The field  $\mathbf{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$  is not equal to :
- (A)  $\mathbf{Q}(\sqrt{30})$   
 (B)  $\mathbf{Q}(\sqrt{2} + \sqrt{3} + \sqrt{5})$   
 (C)  $\mathbf{Q}(\sqrt{2} + \sqrt{3}, \sqrt{5})$   
 (D)  $\mathbf{Q}(\sqrt{2} + \sqrt{3} - \sqrt{5})$

56. Let  $X$  and  $Y$  be Banach spaces with  $\|\cdot\|_1$  and  $\|\cdot\|_2$  respectively denoting their norms. Let  $T : X \rightarrow Y$  be a bijective bounded linear map. If  $\|x\|_3 = \|T_x\|_2$  and  $\|y\|_4 = \|x\|_1$ , where  $T_x = y$ , for all  $x \in X$  and  $y \in Y$ . Then :

- (A)  $\|\cdot\|_3$  and  $\|\cdot\|_4$  are norms and complete
- (B)  $\|\cdot\|_3$  is a complete norm and  $\|\cdot\|_4$  is not a norm
- (C)  $\|\cdot\|_4$  is a complete norm and  $\|\cdot\|_3$  is not a complete norm
- (D) neither  $\|\cdot\|_3$  is a complete norm nor  $\|\cdot\|_4$  is a complete norm

57. Let  $V_k = \{(a_n) \in l^2 : a_n = 0 \text{ for all } n > k\}$ . Then :

- (A)  $V_k$  is finite dimensional for all  $k$  and  $l^2$  is a direct sum of  $\{V_k \mid k \in \mathbf{N}\}$
- (B)  $V_k$  is finite dimensional for all  $k$  and  $l^2$  is a sum of  $\{V_k \mid k \in \mathbf{N}\}$  and is not a direct sum
- (C)  $V_k$  is infinite dimensional for all  $k$  and  $l^2$  is a sum of  $\{V_k \mid k \in \mathbf{N}\}$
- (D)  $V_k$  is finite dimensional for all  $k$  and  $l^2$  is not a sum of  $\{V_k \mid k \in \mathbf{N}\}$

58. Let  $X$  be the space of all continuous complex valued functions on  $[0, 1]$  with inner product

$$\langle f, g \rangle = \int_0^1 f(t) \overline{g(t)} dt .$$

Let  $T : X \rightarrow X$  be defined by  $T(f)(t) = t e^{it} f(t)$ . Then :

- (A)  $T$  is self-adjoint, but not unitary
- (B)  $T$  is unitary, but not self-adjoint
- (C)  $T$  is normal, but neither self-adjoint nor unitary
- (D)  $T$  is not normal

59. Let  $X_i$  denote the set  $X$  with topology  $\theta_i$  and  $Y_i$  denote the set  $Y$  with topology  $\tau_i$  for  $i = 1, 2$ . If  $f : X_1 \rightarrow Y_1$  is continuous, then  $f : X_2 \rightarrow Y_2$  is continuous provided :

- (A)  $\tau_1 \subset \tau_2$  and  $\theta_1 \subset \theta_2$
- (B)  $\tau_2 \subset \tau_1$  and  $\theta_1 \subset \theta_2$
- (C)  $\tau_1 \subset \tau_2$  and  $\theta_2 \subset \theta_1$
- (D)  $\tau_2 \subset \tau_1$  and  $\theta_2 \subset \theta_1$

60. Let  $X = \{(x, y) \in \mathbf{R}^2 : (x - n)^2 + y^2 = 1, n \in \mathbf{Z}\}$  be the subspace of the Euclidean space  $\mathbf{R}^2$ . Then :
- (A)  $X$  is compact and connected  
 (B)  $X$  is connected, but not compact  
 (C)  $X$  is compact, but not connected  
 (D)  $X$  is neither connected nor compact
61. Let  $X$  be a separable first countable space, then  $X \times X$  in the product topology is :
- (A) first countable and separable  
 (B) first countable, but may not always be separable  
 (C) separable and may not always be first countable  
 (D) neither always first countable nor always separable
62. The subgroup lattice of a finite cyclic group is :
- (A) finite direct product of chains  
 (B) modular but not distributive  
 (C) not modular  
 (D) not distributive
63. Let  $G$  be a simple graph having no isolated vertex and having no vertex induced subgraph with exactly two edges. Then :
- (A)  $G$  is planar  
 (B)  $G$  is self-complementary  
 (C)  $G$  is complete  
 (D)  $G$  is a tree
64. The coefficient of  $x^8$  in  $(x^2 + x^3 + x^4 + x^5)^5$  is :
- (A) 13  
 (B) 10  
 (C) 20  
 (D) 0
65. Let  $u_1$  and  $u_2$  be solutions of the equation :
- $$\Delta u = 0 \text{ in } D$$
- $$\frac{\partial u}{\partial n} = g(s) \text{ on } \partial D$$
- where  $D = \{(x, y) \in \mathbf{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$  is an ellipse with boundary  $\partial D = \{(x, y) \in \mathbf{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} = 1\}$ . If  $u_1(0, \frac{1}{2}) - u_2(0, \frac{1}{2}) = 1$ . Then :
- (A)  $u_1$  and  $u_2$  are dependent solutions  
 (B)  $u_1 \equiv u_2 + 1$  in  $D$   
 (C)  $u_1 - u_2$  changes sign in  $D$   
 (D)  $u_1 \equiv u_2$  in  $D$



66. The equation

$$xu_{xx} - yu_{yy} + \frac{1}{2}(u_x - u_y) = 0$$

is hyperbolic in the maximal domain :

- (A)  $\{(x, y) \in \mathbf{R}^2 / x > 0, y > 0\}$
- (B)  $\{(x, y) \in \mathbf{R}^2 / x < 0, y > 0\}$
- (C)  $\{(x, y) \in \mathbf{R}^2 / xy > 0\}$
- (D)  $\{(x, y) \in \mathbf{R}^2 / x < 0, y < 0\}$

67. The partial differential equation

$$(x^2 + z^2) \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} = z^3 x + y^2 \text{ is :}$$

- (A) linear
- (B) semi-linear
- (C) quasi-linear
- (D) non-linear

68. Every natural number is :

- (A) a sum of distinct powers of 2
- (B) a sum of distinct powers of 3
- (C) a sum of distinct factorials
- (D) a sum of distinct triangular numbers

69. Which of the following is a solution to the system of congruences :

$$\begin{aligned} x &\equiv 2 \pmod{5} \\ x &\equiv 3 \pmod{4} \\ x &\equiv 1 \pmod{3} \end{aligned}$$

- (A) 22
- (B) 27
- (C) 47
- (D) 67

70. Modulo the prime 101, the number  $(50!)^2$  is congruent to :

- (A) -1
- (B) 1
- (C) 0
- (D) 2

71. Which of the following statements is *correct* ?

- (A) A system of  $n$  particles can have infinite degrees of freedom
- (B) A system of  $n$  particles with  $k$  holonomic constraints has  $3n - k$  degrees of freedom
- (C) A system of  $n$  particles with  $k$  constraints has  $n - k$  degrees of freedom
- (D) A system of  $n$  particles has 6 degrees of freedom

72. The Lagrangian of a dynamical system is given by :

$$\begin{aligned} L(q_1, q_2, \dot{q}_1, \dot{q}_2) &= \frac{m}{2}(\dot{q}_1^2 + \dot{q}_2^2) \\ &\quad + a(q_1 \dot{q}_2 - q_2 \dot{q}_1) \end{aligned}$$

where  $m$  and  $a$  are constants. Then canonical momenta of the system are given by :

- (A)  $p_1 = m\dot{q}_1, p_2 = m\dot{q}_2$
- (B)  $p_1 = m\dot{q}_1 + aq_2, p_2 = m\dot{q}_2 - aq_1$
- (C)  $p_1 = a\dot{q}_2, p_2 = -a\dot{q}_1$
- (D)  $p_1 = m\dot{q}_1 - aq_2, p_2 = m\dot{q}_2 + aq_1$

73. If the Hamiltonian of the dynamical system is

$H(q_1, q_2, p_1, p_2) = p_1^2 + p_2^2 - k q_1^2$ ,  
then the Lagrangian is given by :

(A)  $L = \frac{\dot{q}_1^2}{2} + \frac{\dot{q}_2^2}{2} + k q_1^2$

(B)  $L = \dot{q}_1^2 + \dot{q}_2^2 - k q_1^2$

(C)  $L = \frac{\dot{q}_1^2}{4} + \frac{\dot{q}_2^2}{4} + k q_1^2$

(D)  $L = \frac{\dot{q}_1^2}{2} + \frac{\dot{q}_2^2}{2} - k q_1^2$

74. Which of the following statements is correct ?

(A) Finite rotation can be represented by a single vector

(B) Infinitesimal transformation can be represented by a single vector

(C) The matrix representing infinitesimal transformation is idempotent matrix

(D) Addition of finite rotations is commutative

75. Identify the velocity field  $(u, v)$  which satisfies conservation of mass for incompressible plane flow :

(A)  $u = 3x^2 + y^2, v = x^3 - 4xy^2$

(B)  $u = x^2, v = -y^2$

(C)  $u = 2xy - x^2, v = 2xy - y^2$

(D)  $u = x^2y, v = xy^2$

76.  $u = ax + by, v = cx + dy$  where  $a, b, c, d$  are non-zero constants are velocity components of a possible incompressible fluid motion only, if :

(A)  $a = d$

(B)  $a + d = 0$

(C)  $b = c$

(D)  $b + c = 0$

77. If the velocity potential of the fluid is given by :

$$\phi = x^2 - y^2 + \frac{xy^3}{3} - \frac{x^3y}{3} + 6,$$

then the velocity components of the fluid flow are :

(A)  $u = x^2y - 2x - \frac{y^3}{3},$

$$v = 2y - xy^2 + \frac{x^3}{3}$$

(B)  $u = 2y - xy^2 - \frac{x^3}{3},$

$$v = 2x + \frac{y^3}{3} - x^2y$$

(C)  $u = -2y + xy^2 - \frac{x^3}{3},$

$$v = -2x - \frac{y^3}{3} + x^2y$$

(D)  $u = 2x, v = -2y$

78. In two-dimensional fluid flow consider a source of strength  $m$  at  $z = f$ , where  $f$  is real, outside the circular cylinder of radius  $a$  whose centre is at the origin. The image of the source in a cylinder is :

- (A) a sink of strength  $m$  at  $z = \frac{a^2}{f}$  and a source of strength  $m$  at the origin
- (B) a source of strength  $m$  at  $z = \frac{a^2}{f}$  and a sink of strength  $m$  at the origin
- (C) a sink of strength  $m$  at  $z = \frac{a^2}{f}$  and a sink of strength  $m$  at the origin
- (D) a source of strength  $m$  at  $z = \frac{a^2}{f}$  and a source of strength  $m$  at the origin

79. Let  $Edu^2 + 2Fdudv + Gdv^2$  and  $Ldu^2 + 2Mdudv + Ndv^2$  be the first and second fundamental forms respectively of a surface patch in  $\mathbf{R}^3$ .

If  $X = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$  and  $Y = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$ , then :

- (A)  $X^{-1}Y$  is non-singular
- (B)  $X^{-1}Y$  may not be non-singular, but symmetric
- (C)  $X^{-1}Y$  may not be symmetric, but have real eigen values
- (D)  $X^{-1}Y$  is non-singular, but not symmetric
80. For any unit speed curve in  $\mathbf{R}^3$  with tangent  $\bar{t}$ , normal  $\bar{n}$ , binormal  $\bar{b}$ , curvature  $k$  and torsion  $\tau$ ,  $\tau\bar{t}' + k\bar{b}'$  is in :
- (A) the linear span of  $\bar{t}$  and  $\bar{b}$
- (B) the linear span of  $\bar{t}$  and  $\bar{n}$
- (C) the linear span of  $\bar{b}$  and  $\bar{n}$
- (D) none of the linear spans of any two vectors in  $\{\bar{t}, \bar{n}, \bar{b}\}$

81. The Gaussian curvature of the elliptic cylinder

$$\{(x, y, z) \in \mathbf{R}^3 : \frac{x^2}{2} + \frac{y^2}{3} = 1\}$$

is :

- (A) always  $> 0$
- (B) always  $< 0$
- (C) always  $= 0$
- (D)  $> 0$  at some points and  $< 0$  at some points

82. The extremal problem

$$J[y(x)] = \int_0^{\pi} (y'^2 - y^2) dx, \quad y(0) = 1, \quad y(\pi) = \lambda$$

has :

- (A) a unique extremal if  $\lambda = 1$
- (B) infinitely many extremals if  $\lambda = 1$
- (C) a unique extremal if  $\lambda = -1$
- (D) infinitely many extremals if  $\lambda = -1$

83. If the integrand  $f$  does not depend on  $y$ , the Euler Lagrange's equation of the functional

$$I[y(x)] = \int_{x_1}^{x_2} f(x, y', y'') dx$$

under the conditions that both  $y$  and  $y'$  are prescribed at the end points has the first integral as :

- (A)  $\frac{\partial f}{\partial y''} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'''} \right) = \text{constant}$
- (B)  $\frac{\partial f}{\partial y'} - \frac{d}{dx} \left( \frac{\partial f}{\partial y''} \right) = \text{constant}$
- (C)  $\frac{\partial f}{\partial y'} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = \text{constant}$
- (D)  $\frac{\partial f}{\partial y''} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = \text{constant}$

84. An extremum of the functional

$$\int_0^1 (y'^2 - 12xy) dx, \quad y(0) = 0, \quad y(1) = 1$$

can occur only along the curve :

- (A)  $y(x) = x^3$
- (B)  $y(x) = -x^3 + 2x$
- (C)  $y(x) = x^3 - 2x$
- (D)  $y(x) = x^2 - x^3$

85. The curve connecting two given points A and B that do not lie on a vertical line such that a particle sliding down this curve under gravity from the point A reaches point B in shortest time is :

- (A) a straight line connecting A and B
- (B) a parabola passing through A and B
- (C) there is no curve that minimizes time
- (D) the cycloid of which one arc contains both the points A and B

86. Let  $\lambda_1 \neq \lambda_2$  be the characteristic numbers and  $\phi_1, \phi_2$  be the corresponding eigen functions for the homogeneous Fredholm integral equation with separable kernel :

$$u(t) = \lambda \int_0^1 k(t, s) u(s) ds.$$

Then which of the following is *not* true ?

- (A) Fredholm determinant  $D(\lambda) = 0$
- (B)  $\int_0^1 \phi_1(t) \phi_2(t) dt = 0$
- (C)  $\phi_1, \phi_2$  are linearly dependent
- (D)  $\phi_1, \phi_2$  are linearly independent

87. The initial value problem

$$y'' + y = 0, \quad y(0) = y'(0) = 0$$

is equivalent to the integral equation :

- (A)  $y(t) = \int_0^t (t-s) y(s) ds$
- (B)  $y(t) = \int_0^t t(t-s) y(s) ds$
- (C)  $y(t) = \int_0^t (s-t) y(s) ds$
- (D)  $y(t) = \int_0^t t(s-t) y(s) ds$

88. If  $K(t, s)$  is symmetric kernel of the Fredholm integral equation. Define the operator  $K$  by :

$$K\phi = \int_a^b K(t, s) \phi(s) ds$$

and consider the following statements :

- (I)  $K$  is self-adjoint operator.
- (II) Inner product  $\langle K\phi, \phi \rangle$  is always real.

Then :

- (A) Only (I) is true
- (B) Only (II) is true
- (C) Both (I) and (II) are true
- (D) Both (I) and (II) are false

89. The third divided difference of the function  $f(x) = 1/x$  with arguments  $p, q, r, s$  is :

- (A)  $\frac{1}{pq}$   
 (B)  $\frac{1}{pqr}$   
 (C)  $-\frac{1}{pqrs}$   
 (D)  $\frac{-s}{pqr}$

90. Using Newton's forward interpolation formula, the cubic polynomial which takes the following value

$x :$	0	1	2	3
$f(x) :$	1	2	1	10

is :

- (A)  $2x^3 - 7x^2 + 6x + 1$   
 (B)  $2x^3 + 7x^2 - 6x + 1$   
 (C)  $2x^3 + x^2 - 2x + 1$   
 (D)  $2x^3 + 2x^2 + 3x + 1$
91. For a given initial value problem

$$y' = y - x, \quad y(0) = 2$$

the value of  $y(0.1)$  by Runge-Kutta second order method is :

- (A) 2.2100  
 (B) 2.0050  
 (C) 2.2050  
 (D) 2.1900

92. If

$$f(t) = \begin{cases} e^{-xt}g(t), & t > 0 \\ 0 & , \quad t < 0. \end{cases}$$

Then which of the following is true ?

- (A)  $L\{f(t)\} = e^s F\{g(t)\}$   
 (B)  $F\{f(t)\} = sL\{g(t)\}$   
 (C)  $F\{f(t)\} = L\{g(t)\}$   
 (D)  $L\{f(t)\} = F\{g(t)\}$

93. If  $\tilde{f}(s)$  is the Fourier transform of  $f(t)$ , then Fourier transform of  $f(t - a)$  is :

- (A)  $e^{isa} \tilde{f}(s + a)$   
 (B)  $e^{sa} \tilde{f}(s + a)$   
 (C)  $e^{sa} \tilde{f}(s)$   
 (D)  $e^{isa} \tilde{f}(s)$

94. Solution of the Fredholm integral equation

$$\int_0^{\infty} \cos \lambda t u(t) dt = e^{-\lambda}$$

is :

- (A)  $\frac{2}{\pi} \frac{1}{(1+t^2)}$   
 (B)  $\frac{2}{\pi} \frac{1}{(1-t^2)}$   
 (C)  $\frac{2}{\pi} \frac{1}{(1+t)^2}$   
 (D)  $\frac{2}{\pi} \frac{1}{(1-t)^2}$

95. Consider the following auxiliary LP problem for two-phase simplex method.

$$\text{Max } Z^* = \sum_{i=1}^m (-1) A_i,$$

where  $A_i$ 's are artificial variables, subject to

$$\sum_{j=1}^n a_{ij} x_j + A_i = b_i, \quad i = 1, 2, \dots, m$$

and  $x_j, A_i \geq 0$ .

If  $\text{Max } Z^* = 0$  and at least one artificial variable is present in the basis with positive values then the original L.P. problem has :

- (A) no feasible solution  
 (B) a feasible solution  
 (C) degenerate feasible solution  
 (D) cannot say anything about the existence of solution
96. There exists a graph with vertex-connectivity  $a$ , edge-connectivity  $b$  and the minimum degree  $c$ , if :
- (A)  $a = 4, b = 2, c = 3$   
 (B)  $a = 4, b = 3, c = 4$   
 (C)  $a = 2, b = 3, c = 4$   
 (D)  $a = 2, b = 4, c = 3$

97. Which of the following statements is true for a graph  $G$  ?

- (A) Every graph  $G$  has a perfect matching  
 (B) Every regular graph  $G$  with even degree  $\geq 1$  has a 2-factor  
 (C) Every bipartite graph  $G$  having no pendant vertex has a 2-factor  
 (D) Line graph of a regular graph  $G$  has a 1-factor

98. Consider the signed measure  $\nu$  on  $[0, 2]$  defined by  $\nu(A) = \int_A f(x) dx$

where 
$$f(x) = \begin{cases} 1 & \text{if } x \in [0, 1) \\ -1 & \text{if } x \in [1, 2] \end{cases}$$

Then  $\nu\left(\frac{1}{2}, \frac{3}{2}\right)$  is :

- (A) equal to the Lebesgue measure of  $(0, 1)$   
 (B) 1  
 (C) 0  
 (D)  $-\frac{1}{2}$

99. Let  $(X, M, \mu)$  be a measurable space and  $E \in M$  be a measurable set. Let  $f$  be a non-negative measurable function with support contained in  $E$ . Define  $\lambda(A) = \int_{A \cap E} f d\mu$ .

Which of the following statements is *false* ?

- (A)  $\lambda$  is a measure on  $M$
- (B)  $\lambda$  is a measure on

$$M(E) = \{A \cap E / A \in M\}$$

- (C)  $\lambda$  is absolutely continuous with respect to  $\mu$  on  $M(E)$
- (D)  $\lambda$  is singular with respect to  $\mu$  on  $M(E)$

100. Let  $f: [0, 1] \rightarrow \mathbf{R}$  be a continuous function. Then  $f$  is absolutely continuous, if :

- (A)  $f$  is uniformly continuous
- (B)  $f$  is of bounded variation
- (C)  $f$  is Riemann integrable
- (D)  $f'$  exists and is continuous

**SECTION III**

101. In a class of 100 students, 40 students got 60 marks and 60 students got 40 marks out of 100. Hence the mean, median and mode of the class marks are respectively.

- (A) 48, 40 and 60
- (B) 48, 40 and 40
- (C) 24, 60 and 40
- (D) 24, 40 and 60

102. The coefficient of variation of a random variable  $X$  is 1.2. If  $Y = \frac{X}{4}$ , the coefficient of variation of  $Y$  is :

- (A) 0.3
- (B) 0.6
- (C) 2.4
- (D) 1.2

103. The random variable  $X$  follows normal distribution with mean 5 and standard deviation 6. Hence :

- (A)  $P[X \geq 5] = P[X \leq 6]$
- (B)  $P[X \leq 5] > P[X \leq 6]$
- (C)  $P[X \geq 5] < P[X \geq 6]$
- (D)  $P[X \geq 5] > P[X \geq 6]$



104. Suppose  $P(A/B) = 0.4$  and  $P(B) = 0.8$ . Which of the following is a possible value for  $P(A)$  ?
- (A) 0.3  
 (B) 0.5  
 (C) 0.6  
 (D) 0.1

105. Let  $X$  be a random variable with  $E[|X|^3] = 3$

Then :

- (A)  $P[|X| \leq 2] \leq \frac{3}{8}$   
 (B)  $P[|X| \leq 2] \leq \frac{3}{5}$   
 (C) One cannot say anything about  $P[|X| \leq 2]$  from the given information  
 (D)  $P[|X| \leq 2] \geq \frac{5}{8}$

106. Let the joint distribution of  $(X, Y)$  be given by :

		X			
		0	1	2	3
Y	0	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$
	1	$\frac{3}{20}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{5}{20}$

Then  $P[X \leq 1]$  is :

- (A)  $\frac{3}{20}$   
 (B)  $\frac{1}{20}$   
 (C)  $\frac{1}{2}$   
 (D)  $\frac{1}{5}$

107. Which of the following is a characteristic function of a random variable ?

- (A)  $\Phi(t) = \cos(t^2)$   
 (B)  $\Phi(t) = \cos\left(\frac{t}{2}\right)$   
 (C)  $\Phi(t) = \frac{2}{1+2t}$   
 (D)  $\Phi(t) = \frac{t}{1+t^2}$

108. Suppose  $E, F$  and  $G$  are 3 events such that  $P(E \cap F \cap G) = P(E).P(F).P(G)$  and  $P(E \cap F \cap G) = P(E).P(F \cap G)$ .

Consider the statements :

- (I)  $F$  and  $G$  are independent events  
 (II)  $E$  and  $F^C \cup G^C$  are independent events  
 (III)  $E^C$  and  $F \cap G$  are independent events

- (IV)  $E, F$  and  $G$  are mutually independent events

- (A) Only statements I, II and III are correct  
 (B) Only statements II, III and IV are correct  
 (C) Only statements II and III are correct  
 (D) Only statement I is correct

109. Suppose  $X_1, X_2, X_3$  are independent and identically distributed random variables each having exponential distribution with mean 1. Then the probability density function of sample range is given by,

- (A)  $h(r) = e^{-r}, r > 0$
- (B)  $h(r) = re^{-r}, r \in \mathfrak{R}$
- (C)  $h(r) = (1 - e^{-r})e^{-r}, r \in \mathfrak{R}$
- (D)  $h(r) = 2(1 - e^{-r})e^{-r}, r > 0$

110. The distribution with moment generating function  $\frac{1}{3 - 2e^t}$  is :

- (A) Exponential distribution
- (B) Cauchy distribution
- (C) Geometric distribution
- (D) Logarithmic series distribution

111. Which of the following statements about an exponential distribution with mean  $\lambda$  is true ?

- (A) Its variance is  $\lambda$
- (B) Its moment generating function is  $1/(1 - \lambda t)$  exists for all  $t \in \mathbb{R}$
- (C)  $E(X) = \int_0^\infty e^{-\lambda x} dx.$
- (D)  $\bar{F}(t+s) = \bar{F}(t)\bar{F}(s), \quad \forall t, s \in \mathbb{R}^+,$   
where  $\bar{F}(x)$  is the survival function of X.

112. Suppose  $X_1, X_2, X_3$  and  $X_4$  are independent and identically distributed tri-variate random vectors having normal distribution.

Then the distribution of  $a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4$ , where  $a_1, a_2, a_3, a_4$  are real numbers, is :

- (A) Univariate normal
- (B) Four variate normal
- (C) Trivariate normal
- (D) Not normal

113. Let  $X_1, X_2, \dots, X_n$  be independently and identically distributed random variables with  $N(\mu, 1)$  distribution. Assume that  $\mu \geq 0$ . Let  $\hat{\mu}$  denote MLE of  $\mu$ . Then which of the following statements is true ?

- (A)  $\hat{\mu} = \max\{\bar{X}_n, 0\}$
- (B)  $\hat{\mu}$  is unbiased for  $\mu$
- (C) The sample mean  $\bar{X}_n$  is sufficient for  $\mu$
- (D)  $\hat{\mu}$  may not exist

114. Let  $X$  be a single observation from  $\theta e^{-\theta x} I_{[0, \infty)}(x)$ ,  $\theta > 0$ . Then  $(X, 2X)$  is a confidence interval for  $\frac{1}{\theta}$  with confidence coefficient :

- (A)  $\frac{1}{2}$
- (B)  $e^{-1}$
- (C)  $e^{-\frac{1}{2}}$
- (D)  $e^{-\frac{1}{2}} - e^{-1}$

115. Let  $X_1, X_2, \dots, X_n$  be a random sample from uniform  $\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$  distribution. Consider the problem of testing  $H_0 : \theta = -\frac{1}{2}$  Vs  $H_1 : \theta = \frac{1}{2}$ .

Let  $X_{(1)} = \min \{X_1, X_2, \dots, X_n\}$ . Consider the test, reject  $H_0$  if  $X_{(1)} > 0$  and accept otherwise. Then the power ( $p$ ) and size ( $s$ ) of the test ( $p, s$ ) are given by :

- (A) (0, 0)
- (B) (0, 1)
- (C) (1, 0)
- (D) (1, 1)

116. Which of the following theorems is useful for obtaining a sufficient statistics ?

- (A) Neyman-Pearson theorem
- (B) Neyman-factorization theorem
- (C) Basu's theorem
- (D) Rao-Blackwell theorem

117. Let  $X_1, X_2, \dots, X_n$  be a random sample from the density  $f(x, \theta) = c(\theta) h(x) e^{\pi(\theta)d(x)}$ ,  $\theta \in \mathbb{H}$ .

$$\text{Let } T(\underline{x}) = \sum_{i=1}^n d(x_i).$$

Then which of the following statements is *not* true ?

- (A)  $T(\underline{X})$  is a complete sufficient statistics for  $\theta$
- (B)  $T(\underline{X})$  is uniformly minimum variance unbiased estimator of its expected value
- (C)  $T(\underline{X})$  is unbiased for  $\theta$
- (D) The probability distribution depends on  $\theta$

118. Suppose  $t$  test is applied to test the equality of means of two random variables  $X$  and  $Y$ , where  $X$  and  $Y$  are independent. Then :

- (A) Distribution of both  $X$  and  $Y$  must be normal with equal variances
- (B) Distribution of both  $X$  and  $Y$  must be normal
- (C) Distribution of  $X$  and  $Y$  may not be normal
- (D) Both  $X$  and  $Y$  must have the same variance

119.  $X$  is a normal  $(\mu, \sigma^2)$  random variable and a 95% confidence interval for  $\mu$  based on a random sample of size  $n$  on  $X$  is given by  $(15, 25]$ . Hence it can be interpreted that :

- (A) The true value of  $\mu$  is 20
- (B) The true value of  $\mu$  lies in  $(15, 25]$  with probability 0.95
- (C) The true value of  $\mu$  is between 15 and 25
- (D) The interval  $(15, 25]$  covers the true value of  $\mu$  with probability 0.95

120. Suppose  $r_{12.3}$  is a partial correlation coefficient between  $(X_1, X_2)$  and  $X_3$ . Then which of the following statements is *not* correct ?

- (A)  $r_{12.3}$  is a correlation coefficient between  $X_1 - \hat{X}_1$  and  $X_2 - \hat{X}_2$ , where  $\hat{X}_1$  is a line of best fit based on  $X_3$  and  $\hat{X}_2$  is a line of best fit based on  $X_3$ .

- (B)  $-1 \leq r_{12.3} \leq 1$
- (C)  $r_{12.3} \geq 0$  always

(D) 
$$r_{12.3} = \frac{r_{12} - r_{13} \cdot r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

121. A frequency data is classified in 10 classes and normal  $N(\mu, \sigma^2)$  distribution is fitted to data after estimating the parameters. To use  $\chi^2$  goodness of fit test, two classes are combined into one class. Hence the degrees of freedom associated with  $\chi^2$  test are :

- (A) 9
- (B) 8
- (C) 7
- (D) 6

122. Let  $(X_i, Y_i)$   $i = 1, \dots, 5$  be the bivariate pair of observations and  $Z_i = X_i - Y_i$   $i = 1, \dots, 5$ . For a given sample the number of  $Z_i > 0$  is 2. Hence the  $p$ -value to test the null hypothesis that  $H_0 : \text{median}(Z) = 0$  against  $H_1 : \text{median}(Z) < 0$  is :

- (A) 0.5
- (B) 0.25
- (C) 0.10
- (D) 0.05

123. Suppose that there are 1000 components in operation. The number of failed replacements at the end of each week are as follows :

W1	W2	W3	W4
50	82	128	199

If the cost of replacing an individual failed component is ₹ 125 and the cost of group replacement of all 1000 components is ₹ 300, what is the best interval between group replacements ?

- (A) One week
- (B) Two weeks
- (C) Three weeks
- (D) Four weeks

124. A newspaper vendor buys papers for ₹ 3 each and sells them for ₹ 5 each. The unsold papers have no value. The daily demand for the paper has the following probability distribution :

# of customers	Probabilities
200	0.2
250	0.3
300	0.3
350	0.1
400	0.1

If each day's demand is independent of the other days, the average profit per day is :

- (A) ₹ 75
- (B) ₹ 120
- (C) ₹ 93
- (D) ₹ 50

125. A canteen has a single cash counter.

A customer has to buy coupons from the cash counter before getting the

food items. Customers arrive at the counter according to a Poisson

process with rate of 2 per 5 minutes.

It takes 1.5 minutes on average to

buy coupons. What is the long-run

probability that the cash counter is

free ?

(A)  $1 - \frac{1.5}{2}$

(B)  $1 - \frac{1.5}{5}$

(C)  $1 - \frac{1.5}{2.5}$

(D) 0

126. In a survey of finite population, the following method is applied to obtain a sample of size  $n$ .

(i) In the 1st draw, the  $i$ th unit is selected with probability

$$P_i, > 0, i = 1 \dots\dots\dots N, \sum_{i=1}^N P_i = 1$$

and  $P_i \neq \frac{1}{N}$  for some  $i$ .

(ii) The remaining  $(n - 1)$  units are obtained by SRSWOR method from remaining  $N - 1$  units of the population.

Let  $\bar{Y}$  denote the sample mean, then :

- (A)  $\bar{Y}$  is an unbiased estimator of population mean
- (B) There exists an unbiased estimator of the population mean and it is different from the sample mean
- (C) This is a stratified sampling procedure
- (D) There does not exist an unbiased estimator of the population mean

127. Let the variances of the regression estimator and the sample mean, both computed from SRSWOR of the same sample size, be denoted by  $V_{\text{Reg}}^2$  and  $V_{\text{SRS}}^2$  respectively. Then :

- (A)  $V_{\text{Reg}}^2 \leq V_{\text{SRS}}^2$  for a large  $n$ .
- (B)  $V_{\text{Reg}}^2 = V_{\text{SRS}}^2$
- (C) Both the estimators of the population mean are unbiased
- (D)  $V_{\text{Reg}}^2 \leq V_{\text{SRS}}^2$  for any  $n$

128. To examine whether five different skin creams A, B, C, D and E have different effect on the human body,  $n$  randomly chosen persons were enrolled in an experiment. The five creams were applied on each of one randomly chosen fingers of the same person; and the procedure is repeated for each person. The resulting design is :

- (A) a completely randomized design with 5 treatments
- (B) a randomized block design with 5 blocks and  $n$  treatments
- (C) a randomized block design with  $n$  blocks and five treatments
- (D) a repeated measures design

129. For one-way classification model the F-test is valid under the assumption that :

- (A) The observations on response variable are uncorrelated with common mean and variance
- (B) The errors are uncorrelated with common mean and variance
- (C) The errors are normally distributed with common mean zero and common positive variance
- (D) The errors are independently normally distributed with common mean zero and common +ve variance

130. Under a  $2^3$  factorial design with factors A, B, C arranged in a single replicate of two blocks, the contents of block-I (in Yate's notation) are  $\{1, a, ab, b\}$ . Which of the following effects is confounded with blocks ?

- (A) AB
- (B) AC
- (C) ABC
- (D) C

131. Let  $\mathbf{F}_1$  and  $\mathbf{F}_2$  be two  $\sigma$ -fields of subsets of  $\Omega$ .

Then which of the following is *not* true :

- (A) Both  $\mathbf{F}_1 \cap \mathbf{F}_2$  and  $\mathbf{F}_1 \cup \mathbf{F}_2$  are  $\sigma$ -fields
- (B)  $\mathbf{F}_1 \cap \mathbf{F}_2$  is not a  $\sigma$ -field but  $\mathbf{F}_1 \cup \mathbf{F}_2$  is a  $\sigma$ -field
- (C) Neither  $\mathbf{F}_1 \cap \mathbf{F}_2$  nor  $\mathbf{F}_1 \cup \mathbf{F}_2$  are  $\sigma$ -fields
- (D)  $\mathbf{F}_1 \cap \mathbf{F}_2$  is a  $\sigma$ -field but  $\mathbf{F}_1 \cup \mathbf{F}_2$  is not a  $\sigma$ -field

132. Let  $\Omega = \{1, 2, 3, 4\}$ .

Consider the two  $\sigma$ -fields :

$$\mathbf{F}_1 = \{\phi, \Omega, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{2, 3, 4\}, \{1, 3, 4\}\}$$

$$\mathbf{F}_2 = \{\phi, \Omega, \{1, 2\}, \{3, 4\}\}$$

Let  $f$  denote a function from  $(\Omega, \mathbf{F}_1)$  to  $(\Omega, \mathbf{F}_2)$ .

Then :

- (A) Any  $f$  is measurable
- (B)  $f$  defined by  $f(k) = k, k = 1, 2, 3, 4$  is measurable
- (C)  $f$  defined by  $f(1) = f(3) = 1$  and  $f(2) = f(4) = 4$  is measurable
- (D)  $f$  defined by  $f(1) = f(2) = f(3) = 1$  and  $f(4) = 4$  is measurable

133. Let  $\mathbb{R}$  denote the real line,  $\mathbf{B}$  the Borel  $\sigma$ -field and  $m$  the Lebesgue measure on  $\mathbb{R}$  :

For  $A \in \mathbf{B}$ , define  $\mu(A) = \int_A \exp\left(-\frac{x^2}{2}\right) dm(x).$

Consider the statements :

- (I)  $\mu$  is a measure on  $\mathbf{B}$
- (II) The Radon-Nikodym derivative of  $\mu$  with respect to  $m$ ,

$$\frac{d\mu}{dm}(x) = \exp\left(-\frac{x^2}{2}\right) \text{ a.e. } m.$$

- (III)  $m \ll \mu$ , ( $m$  is absolutely continuous with respect to  $\mu$ ).

Which of the above statements are true ?

- (A) All the three
- (B) (I) and (II) only
- (C) (I) and (III) only
- (D) (I) only

134. Let  $\{X_n\}$  be a sequence of random variables and  $X$  a random variable such that :

$$\sum_{n=1}^{\infty} P[|X_n - X| > \varepsilon] < \infty \text{ for every } \varepsilon > 0.$$

Consider the statements :

- (I)  $P\left[\lim_{n \rightarrow \infty} X_n = X\right] = 1$
- (II)  $P\left[\lim_{n \rightarrow \infty} X_n^2 = X^2\right] = 1$
- (III)  $P\left[\lim_{n \rightarrow \infty} X_n = X\right]$  only if  $\{X_n\}$  is a sequence of independent random variables.

Which of the above are always correct ?

- (A) (I) only
- (B) (I) and (II) only
- (C) (III) only
- (D) None of the three



135. Let  $\{X_n\}$  be i.i.d. random variables with mean zero and finite variance  $\sigma^2 > 0$ .

Which of the following is *false* ?

(A)  $\frac{1}{n}(\exp(X_1) + \exp(X_2) + X_3 + \dots + X_n) \rightarrow 0$  in probability as  $n \rightarrow \infty$

(B)  $\frac{1}{n^{5/8}}(\exp(X_1) + X_2 + \dots + X_n) \rightarrow 0$  in probability as  $n \rightarrow \infty$

(C)  $\frac{1}{n^{3/8}}(X_1 + \dots + X_n)$  does not converge in probability as  $n \rightarrow \infty$

(D)  $\frac{1}{n^{3/8}}(X_1 + \dots + X_n)$  converges in distribution to a r.v. as  $n \rightarrow \infty$

136. Suppose the sequence of random variables  $\{X_n\}$  converges in distribution to X.

Then which of the following is *not* always true ?

(A)  $\lim_n E[X_n] = E[X]$

(B)  $E[|X|] \leq \lim_n \inf E[|X_n|]$

(C)  $\lim_n E\left[\frac{|X_n|}{1+|X_n|}\right] = E\left[\frac{|X|}{1+|X|}\right]$

(D)  $\lim_n \frac{X_n}{\sqrt{n}} = 0$  in probability

137. Suppose  $\{X_n\}$  and  $\{Y_n\}$  are sequences of random variables and X and Y are random variables such that as  $n \rightarrow \infty$ ,  $X_n$  converges in quadratic mean to X and  $Y_n$  converges in quadratic mean to Y.

Consider the following statements :

(I) The sequence  $(X_n + Y_n)$  converges in quadratic mean to  $(X + Y)$  as  $n \rightarrow \infty$

(II)  $\lim_{n \rightarrow \infty} P[|X_n Y_n - XY| > \epsilon] = 0,$   
 $\forall \epsilon > 0$

(III)  $\lim_{n \rightarrow \infty} E[e^{iX_n}] = E[e^{iX}]$

Which of the above are *true* ?

(A) All the three

(B) (I) and (II) only

(C) (I) and (III) only

(D) (II) and (III) only

138. Let  $\{X_n\}$  be a sequence of independent random variables.

$$\text{Let } E = \left\{ \omega \mid \sum_{k=1}^{\infty} X_k(\omega) \text{ converges} \right\}$$

Then which of the following is always *correct* ?

- (A)  $P(E) = 0$
- (B)  $P(E) = 1$
- (C)  $0 < P(E) < 1$
- (D)  $P(E)$  is either '0' or 1

139. Let  $\{Y_n\}$  be a sequence of independent random variables with mean zero and variance 1.

Let  $\mathbf{F}_n = \sigma(Y_1, \dots, Y_n)$  the  $\sigma$ -field generated by  $\{Y_1, Y_2, \dots, Y_n\}$

Consider the statements :

- (I) The sequence  $X_n = Y_1 + Y_2 + \dots + Y_n$  is a  $\mathbf{F}_n$  - martingale.
- (II) The sequence  $X_n = \prod_{k=1}^n Y_k$  is a  $\mathbf{F}_n$  - martingale.

(III) The sequence  $X_n = \left( \sum_{k=1}^n Y_k \right)^2 - n$

is a  $\mathbf{F}_n$  - martingale.

Which of the following is *correct* ?

- (A) (I) and (III) only
- (B) (I) and (II) only
- (C) (II) and (III) only
- (D) All the three

140. Suppose a distribution function  $F : \mathbf{R} \rightarrow [0, 1]$  is as follows :

$$F(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1/4, & \text{if } 0 \leq x < 1, \\ 1/2, & \text{if } 1 \leq x < 2, \\ 1/2 + (x - 2)/2, & \text{if } 2 \leq x < 3, \\ 1, & \text{if } x \geq 3. \end{cases}$$

Then  $F(x) = l_1 F_1(x) + l_2 F_2(x)$  where :

- (A)  $l_1 = 1/4, l_2 = 3/4$ ,  $F_1$  is a distribution function of a discrete random variable with support  $\{0, 1\}$  with respective probabilities  $1/4, 3/4$  and  $F_2$  is a distribution function of uniform  $U(2, 3)$  distribution.
- (B)  $l_1 = 1/2, l_2 = 1/2$ ,  $F_1$  is a distribution function of a discrete random variable with support  $\{0, 1\}$  with respective probabilities  $1/2, 1/2$  and  $F_2$  is a distribution function of uniform  $U(2, 3)$  distribution.
- (C)  $l_1 = 1/4, l_2 = 3/4$ ,  $F_1$  is a distribution function of a discrete random variable with support  $\{0, 1\}$  with respective probabilities  $1/2, 1/2$  and  $F_2$  is a distribution function of uniform  $U(2, 3)$  distribution.
- (D)  $l_1 = 1/4, l_2 = 3/4$ ,  $F_1$  is a distribution function of a discrete random variable with support  $\{1, 2\}$  with respective probabilities  $1/4, 3/4$  and  $F_2$  is a distribution function of uniform  $U(2, 3)$  distribution.

141. Which of the following statements is *not* correct ? The set of points of discontinuities of a distribution function may be :

- (A) empty
- (B) finite
- (C) uncountable
- (D) countable

142. Suppose X is a non-negative random variable whose mean exists. Then :

- (A)  $E(\log(X)) < \log(E(X))$
- (B)  $E(\log(X)) \geq \log(E(X))$
- (C)  $E(\log(X)) = \log(E(X))$
- (D) None of (A), (B), (C) is true

143. Suppose  $\phi(\cdot)$  is a characteristic function of a random variable with distribution function F. Then :

- (A)  $F(b) - F(a) =$

$$\lim_{u \rightarrow \infty} \frac{1}{2\pi} \int_{-u}^u \frac{[\exp(-iua) - \exp(-iub)] \phi(u) du}{iu}$$

- (B)  $F(b) - F(a) =$

$$\lim_{u \rightarrow \infty} \int_{-u}^u \frac{[\exp(-iua) - \exp(-iub)] \phi(u) du}{iu}$$

- (C)  $F(b) - F(a) =$

$$\lim_{u \rightarrow \infty} \frac{1}{2\pi} \int_{-u}^u \frac{[\exp(iua) - \exp(iub)] \phi(u) du}{iu}$$

- (D)  $F(b) - F(a) =$

$$\lim_{u \rightarrow \infty} \frac{1}{2\pi} \int_{-u}^u \frac{[\exp(-iub) - \exp(-iua)] \phi(u) du}{iu}$$

144. Suppose X has bivariate normal distribution with mean vector (3, 4)' and

$V(X_1) = 2$ . Further, it is known that  $X_1$  and  $X_2$  are positively correlated. Which of the following can be the conditional distribution of  $(X_1 | X_2 = 2)$  ?

- (A)  $N(1.5, 2.64)$
- (B)  $N(2.6, 1.84)$
- (C)  $N(3.5, 1.52)$
- (D)  $N(3.5, 2.84)$

145. Let  $X_1, X_2, \dots, X_n$  be *i.i.d.* Bernoulli ( $\theta$ ). Let the loss function for the problem of estimation of  $\theta$  be given by

$$L(\theta, a) = \frac{(\theta - a)^2}{\theta(1 - \theta)}$$

Let the prior for  $\theta$  ( $\pi(\theta)$ ) be given by  $\pi(\theta) = 1$   $0 \leq \theta \leq 1$ .

Let  $\bar{X}$  be the sample mean. Then :

- (A)  $\bar{X}$  is Bayes but not minimax
- (B)  $\bar{X}$  is not Bayes but it is minimax
- (C)  $\bar{X}$  is both Bayes and minimax
- (D)  $\bar{X}$  is not admissible

146. Let  $X_1, X_2, \dots, X_n$  be iid Bernoulli ( $\theta$ ) random variables. We wish to follow the Bayes procedure for estimation of  $\theta$ . Then, a conjugate prior for  $\theta$  :

- (A) is  $\pi(\theta) = 1 \quad 0 < \theta < 1$
- (B) is  $\pi(\theta) \propto \theta^{\alpha - 1} (1 - \theta)^{\beta - 1}$   
 $\alpha > 0, \beta > 0$
- (C) is  $\pi(\theta) \propto \frac{e^\theta}{1 + e^\theta}$
- (D) Cannot be decided since the loss function is not given

147. Suppose  $X_1, \dots, X_n$  are independent and identically distributed random variables each having  $U(-\theta, \theta)$  distribution. Then the minimal sufficient statistic for  $\theta$  is given by :

- (A)  $-X_{(1)}$
- (B)  $X_{(n)}$
- (C)  $\max \{-X_{(1)}, X_{(n)}\}$
- (D)  $\min \{-X_{(1)}, X_{(n)}\}$

148. Suppose  $\{X_1, X_2, \dots, X_n\}$  is a random sample from uniform  $U(\theta - 1/2, \theta + 1/2)$  distribution,  $\theta \in \mathbf{R}$ . Then :

- (A) Sample mean is unbiased for  $\theta$
- (B) Sample median is unbiased for  $\theta$
- (C)  $X_{(1)} + 1/2$  is unbiased for  $\theta$
- (D)  $X_{(n)} - 1/2$  is unbiased for  $\theta$

149. Suppose  $\{X_1, X_2, \dots, X_n\}$  are independent and identically distributed random variables each having  $U(-\theta, \theta)$  r.v.s. Then the maximum likelihood estimator of  $\theta$  is :

- (A)  $X_{(n)}$
- (B)  $(X_{(1)} + X_{(n)})/4$
- (C)  $-X_{(1)}$
- (D)  $\max \{|X_1|, \dots, |X_n|\}$

150. A random variable  $X$  follows  $N(\mu, 1)$  distribution. A conventional test based on sample mean is used to test the hypothesis  $H_0 : \mu = \mu_0$  against  $H_1 : \mu = \mu_1 > \mu_0$ . Consider the following statements :

- (i) If  $H_0$  is rejected at  $\alpha = 0.05$ , it is also rejected at  $\alpha = 0.10$ .
- (ii) If  $H_0$  is accepted at  $\alpha = 0.05$ , it is also accepted at  $\alpha = 0.01$ .

Then :

- (A) (i) is correct but (ii) is wrong
- (B) (i) and (ii) both are wrong
- (C) (i) and (ii) both are correct
- (D) (i) is wrong but (ii) is correct

151. Suppose  $\{X_1, X_2, \dots, X_n\}$  is a random sample of size  $n$  from uniform  $U(\theta - 1, \theta + 1)$  distribution. Then which of the following is *not* true ?

- (A)  $X_{(n)}$  is consistent for  $\theta$
- (B) Sample median is consistent for  $\theta$
- (C) Sample mean is consistent for  $\theta$
- (D)  $X_{(n)} - 1$  is consistent for  $\theta$

152. Suppose  $X_1, X_2, \dots, X_n$  are independent and identically distributed random variables each having exponential distribution with mean  $\theta$ . Then the asymptotic distribution of  $\sqrt{n} \left( \frac{\bar{X}}{S} - 1 \right)$  where

$S^2 = \frac{1}{n} \sum_{i=1}^n X_i^2 - (\bar{X})^2$  is :

- (A)  $N(0, 1)$
- (B)  $N(0, \theta)$
- (C)  $N(1, 1)$
- (D)  $N(0, \theta^2)$

153. Let  $X_n$  be asymptotically normally distributed with mean  $\mu$  and variance  $\sigma_n^2$ , such that  $\sigma_n^2 \rightarrow 0$  as  $n \rightarrow \infty$ . Which of the following statements is *not* true ?

- (A) For  $\mu \neq 0$ ,  $1/X_n \sim AN(1/\mu, \sigma_n^2/\mu^2)$
- (B) For any  $\mu$ ,  $e^{X_n} \sim AN(e^\mu, e^{2\mu} \sigma_n^2)$
- (C) For  $\mu = 0$ ,  $\log |X_n| \sigma_n \stackrel{d}{\sim} \log |N(0, 1)|$
- (D) For  $\mu \neq 0$ ,  $\log |X_n| \sim AN(\log |\mu|, \sigma_n^2/\mu^2)$

154. Let  $X_1, X_2, \dots, X_n$  be iid  $U(0, \theta)$ .  
 Consider  $T_1 = \text{sample median} (= M_n)$   
 and  $T_2 = 2\bar{X}$ , where  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ .

Then, ARE  $(T_1, T_2)$  is :

- (A) 3/4
- (B) 2/3
- (C) 1
- (D) 4/3

155. Let  $X_1, X_2, \dots, X_n$  be iid observations  
 from a continuous distribution  $F$ .  
 Define :

$$F_n(x) = \begin{cases} 0 & x < X_{(1)} \\ k/n & X_{(k)} \leq x < X_{(k+1)} \\ 1 & x \geq X_{(n)} \end{cases}$$

Then for any fixed  $x$ ,

$\sqrt{n} (F_n(x) - F(x)) \xrightarrow{d} Z$ , where  $Z$   
 is a normal r.v. with mean 0 and  
 variance :

- (A)  $F^2(x) / 2$
- (B)  $(1 - F(x))^2$
- (C)  $F^2(x)$
- (D)  $F(x) (1 - F(x))$

156. Let  $(X, Y) \sim N(\underline{0}, \Sigma)$  where

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad \rho \neq 0 \quad \text{and}$$

$$W = \frac{X - Y}{X + Y} \sqrt{\frac{1 + \rho}{1 - \rho}}$$

Then which of the following  
 statements is *not* correct ?

- (A)  $W^2 \sim F_{1,1}$
- (B)  $1/W^2 \sim F_{1,1}$
- (C)  $(X - Y)$  and  $(X + Y)$  are  
 independently distributed
- (D)  $W \sim \chi_1^2$

157. Let  $\underline{X} \sim N_n(\underline{\mu}, \sigma^2 I)$  and  $A$  be a  
 $n \times n$  matrix. Then  $\underline{Y} = AX$  are  
 uncorrelated normally distributed if  
 and only if which of the following  
 conditions hold ?

- (A)  $A$  is any non-singular matrix
- (B)  $A$  is an idempotent matrix with  
 rank  $< n$
- (C)  $A$  is an orthogonal matrix
- (D) All elements of  $A$  are equal  
 to 1

158. If  $S_1, S_2, \dots, S_5$  are the corrected sum of squares and sum of products matrices based on five independent samples of sizes 100 each coming from a four variate normal population. The distribution of

$$\sum_{i=1}^5 S_i.$$

- (A) is Wishart with  $k = 496$
- (B) is Wishart with  $k = 495$
- (C) is Wishart with  $k = 499$
- (D) cannot be decided on the basis of given information

159. Let  $M \sim W_p(n, \Sigma)$  and  $CC' = \Sigma$  where  $C$  is a non-singular matrix. Then  $C^{-1}MC^{-1}$  is distributed as :

- (A)  $W_p(n, C)$
- (B)  $W_p(n, \Sigma^{-1})$
- (C)  $W_p(n, I_p)$
- (D)  $W_p(n, C^{-1})$

160. The characteristic function of a vector  $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$  is given by :

- (A)  $e^{\underline{\mu}'t - \frac{1}{2}t'\Sigma t}$
- (B)  $e^{i\underline{\mu}'t - \frac{1}{2}t'\Sigma t}$
- (C)  $e^{\underline{\mu}'t - \frac{i}{2}t'\Sigma t}$
- (D)  $e^{i\underline{\mu}'t - \frac{i}{2}t'\Sigma t}$

161.  $y_1, y_2, y_3, y_4$  are four uncorrelated random variables with common variance  $\sigma^2$  and  $E(y_1) = E(y_2) = \theta_1 + 2\theta_2 + \theta_3$ ,  $E(y_3) = E(y_4) = \theta_1 + \theta_3$ . Hence :

- (A)  $\theta_2$  is not estimable
- (B)  $2\theta_2 + 3\theta_3$  is not estimable
- (C)  $\theta_1 + \theta_3$  is not estimable
- (D)  $\theta_1 + \theta_2 + \theta_3$  is not estimable

162. In a simple linear regression set up  $y = \beta_a e \beta, X e \varepsilon$ , the prediction of response variable  $y$  :

- (A) does not depend on regressor  $X$
- (B) will have same variation irrespective of  $X$
- (C) will have larger variation for larger values of  $X$
- (D) will have larger variation if the distance between  $X$  and mean of regressors increases

163. In a multiple linear regression set up  $\underline{Y} = X\underline{\beta} e \underline{\varepsilon}$  under the assumption that the random errors  $\varepsilon_1, \dots, \varepsilon_n$  are uncorrelated and homoscedastic the residuals  $e_1, \dots, e_n$  are :

- (A) correlated and homoscedastic
- (B) correlated and heteroscedastic
- (C) uncorrelated and homoscedastic
- (D) uncorrelated and heteroscedastic

164. In a multiple linear regression model  $\underline{Y} = X\underline{\beta} + \underline{\varepsilon}$  with  $E(\underline{\varepsilon}) = 0$ ,  $\text{Var}(\underline{\varepsilon}) = \sigma^2 I$  :
- (A) the regression sum of squares and error sum of squares are independent
  - (B) the regression sum of squares and error sum of squares are positively correlated
  - (C) the regression sum of squares and error sum of squares are uncorrelated
  - (D) the regression sum of squares and error sum of squares are negatively correlated
165. Suppose  $Y_i = \beta_1 X_{i_1} + \beta_2 X_{i_2} + \varepsilon_i$  where  $E(\varepsilon_i) = 0$ ,  $\text{Var}(\varepsilon_i) = \sigma^2$   $i = 1, 2, 3$  and  $X_{i_1} = i - 4$ ,  $i = 1, 2, 3$   $X_{i_2} = i - 2$ ,  $i = 1, 2$  and  $X_{i_2} = i$  for  $i = 3$ . A least-squares fit is carried out and sum of squares are calculated then :
- (A) the sum of squares due to  $\beta_1$  and  $\beta_2$  are uncorrelated with each other
  - (B) the sum of squares due to  $\beta_1$  and  $\beta_2$  are independent of each other
  - (C) the sum of squares due to  $\beta_1$  and  $\beta_2$  are correlated with each other
  - (D) the sum of squares due to  $\beta_1$  and  $\beta_2$  are uncorrelated with total sum of squares
166. Let  $N = 4$ . Two units are selected for inclusion in the sample based on the following design. The  $i$ -th unit is selected with probability  $p_i$  where  $p_1 = 1/6$ ,  $p_2 = 1/6$ ,  $p_3 = 2/3$ . The second unit is selected randomly from among the remaining three units in the population. Then, the first unit is included in the sample with probability :
- (A) 2/9
  - (B) 4/9
  - (C) 6/9
  - (D) 7/9
167. The regression estimator of the sample mean based on a SRSWOR :
- (A) is always better than the sample mean
  - (B) and the sample mean have the same efficiency
  - (C) is worse than the sample mean
  - (D) is better than the sample mean if  $n$ , the sample size, is sufficiently large



168. Let  $p$  be the observed proportion of units with the attribute A in a SRSWOR of size  $n$  from a finite population size  $N$ . Let  $P$  be the population proportion and  $Q = 1 - P$ . Then,  $\text{Var}(p)$  equals :

- (A)  $\frac{PQ}{n} \frac{N-n}{N-1}$   
 (B)  $\frac{PQ}{n}$   
 (C)  $\frac{PQ}{n} \frac{1}{N-1}$   
 (D)  $\frac{PQ}{N} \frac{N-n}{N-1}$

169. A block design with  $v$  treatments and  $b$  blocks is connected. Hence the rank of  $c$  matrix is :

- (A)  $v - 1$   
 (B)  $v$   
 (C)  $b - 1$   
 (D)  $b$

170. For a BIBD with parameters  $(v, b, r, k, \lambda)$  to exist, the condition  $b \geq v$  is :

- (A) neither necessary nor sufficient  
 (B) not necessary but sufficient  
 (C) necessary but not sufficient  
 (D) necessary and sufficient

171. A row-column design with 7 treatments arranged in 3 rows and 7 columns is given below :

1	2	3	4	5	6	7
2	3	4	5	6	7	1
4	5	6	7	1	2	3

Hence the design is :

- (A) Latin Square Design  
 (B) Youden Square Design  
 (C) Quasi-Latin Square Design  
 (D) Split-Plot design

172. In a  $2^3$  factorial design with three treatments A, B, C each at two levels, blocks of 4 plots are available. The design is replicated twice and the allocation of treatment combination is :

Replication 1			Replication 2		
Block	I	II	Block	I	II
	1	b		1	a
	a	c		b	c
	bc	ab		ac	ab
	abc	ac		abc	bc

Hence the two confounded treatment combinations in replication 1 and 2 respectively are :

- (A) AB and ABC
- (B) AB and AC
- (C) BC and ABC
- (D) BC and AC

173. Let  $\{X_t\}$  be a stationary time series with autocovariance function  $r(h)$ . Then, which of the following statements is *not* true about  $r(h)$  ?

- (A)  $\gamma(0) = \text{Var}(X_t)$
- (B)  $\gamma(k) = \gamma(-k)$
- (C)  $|\gamma(k)| \geq \gamma(0)$
- (D)  $\sum_i \sum_j \alpha_i \alpha_j \gamma(|t_i - t_j|) \geq 0$  for every set of time points  $\{t_1, t_2, \dots, t_n\}$  and real members  $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$

174. Given two time series,

$$X_t = 0.3 X_{t-1} - 0.2 X_{t-2} + Z_t$$

$$Y_t - 0.4 Y_{t-1} = Z_t + 1.2 Z_{t-1}$$

where  $\{Z_t\}$  iid  $WN(0, \sigma^2)$ , which of the following statements is *true* ?

- (A)  $\{Y_t\}$  is not causal, but invertible
- (B)  $\{X_t\}$  is not causal
- (C)  $\{Y_t\}$  is causal, but not invertible
- (D)  $\{X_t\}$  is not invertible, but causal

175. Let  $\{X_t\}$  be a stationary AR(1) model with  $X_t = \mu + \phi X_{t-1} + Z_t$ , where  $Z_t \sim WN(0, \sigma^2)$ . Then, which of the following statements is *true* ?

- (i)  $E(X_t) = \mu$
- (ii)  $V(X_t) = \sigma^2 / 1 - \phi^2$
- (iii)  $\text{Corr}(X_t, X_{t+s}) = \phi^s$
- (iv)  $V(X_t) = 1 - \phi^2$
- (A) (ii) and (iii)
- (B) (i) and (iii)
- (C) (i) and (ii)
- (D) (iii) and (iv)

176. Consider a Markov chain on  $S = \{1, 2, 3, 4, 5\}$  with its transition probability matrix given by :

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} \frac{3}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix} \end{matrix}.$$

Then :

- (A) there exists a unique stationary distribution
- (B) there does not exist any stationary distribution
- (C) all states are transient
- (D) there exist infinitely many stationary distributions

177. Let  $\{X(t), t \geq 0\}$  be a time-homogeneous Poisson process with rate  $\lambda$ . Let  $T$  be a positive continuous r.v. distributed independently of  $\{X(t), t \geq 0\}$ . Then :

- (A)  $\text{Cov}(X(T), T) = \frac{1}{2}$
- (B)  $\text{Cov}(X(T), T) = \lambda / (\lambda + E(T))$
- (C)  $\text{Cov}(X(T), T) = \lambda \text{Var}(T)$
- (D)  $\text{Cov}(X(T), T) < 0$

178. Let the off-spring probability distributions of the three Branching processes be given by :

$$\text{I. } p_0 = \frac{1}{4} \quad p_1 = \frac{1}{4} \quad p_2 = \frac{1}{2}$$

$$\text{II. } p_0 = \frac{1}{4} \quad p_1 = \frac{1}{2} \quad p_2 = \frac{1}{4}$$

$$\text{III. } p_1 = \frac{1}{4} \quad p_2 = \frac{3}{4}$$

Then, the extinction probabilities of the three Branching processes are respectively given by :

- (A) 0, 0, 1
- (B)  $\frac{1}{2}$ , 0, 1
- (C)  $\frac{1}{2}$ , 1, 0
- (D)  $\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}$

179. If all states of a Markov chain are persistent non-null :

- (A) it has a unique stationary distribution
- (B) it does not have a stationary distribution
- (C)  $\lim_{n \rightarrow \infty} p_{ij}^{(n)} > 0$  for all  $i$  and  $j$
- (D) it has at least one stationary distribution

180. The method of age-standardisation of fertility rates is used to :

- (A) exclude the effect of age distribution in measuring fertility
- (B) exclude effects of differences in fertility levels
- (C) compare fertility in two population groups
- (D) compare birth rates in two populations

181. A column  $L_x$  in the life table denotes :

- (A) the number of years lived by the persons in the age interval  $(x, x + 1)$
- (B) the number of individuals surviving to age  $x$
- (C) the number of individuals surviving to age  $x + 1$
- (D) the average number of years of life remaining after age  $x$

182. The following sampling plan is to be applied for acceptance or rejection of large lots.

*“Select two items from the lot and inspect both the items. If both are good, accept the lot, if both are defective, reject the lot; otherwise, select one more item from the lot and accept the lot if and only if this item is good.”*

If the incoming lot quality is 0.8, then the average outgoing quality (AOQ) is equal to :

- (A) 0.512
- (B) 0.896
- (C) 0.800
- (D) 0.640

183. Identify the correct match between the terms in Column-I below with the definitions given in Column-II.

**Column-I**

- (i) AOQ
- (ii) AQL
- (iii) ATI
- (iv) RQL

**Column-II**

- (α) Number of items per lot that will be inspected on average
  - (β) A lower bound on the incoming quality for which a lot is to be rejected
  - (γ) Proportion of defective items remaining in the lot after inspection
  - (δ) An upper bound on the incoming quality for which a lot to be accepted
- |     |      |       |      |     |
|-----|------|-------|------|-----|
| (i) | (ii) | (iii) | (iv) |     |
| (A) | (γ)  | (δ)   | (α)  | (β) |
| (B) | (γ)  | (β)   | (α)  | (δ) |
| (C) | (β)  | (δ)   | (α)  | (γ) |
| (D) | (α)  | (δ)   | (γ)  | (β) |

184. If the demand distribution is exponential with mean  $1/\alpha$ , then the optimal inventory level  $S^*$  is :

- (A)  $\alpha \log \frac{h+p}{h+c}$
- (B)  $\frac{1}{\alpha} \log \frac{h+c}{h+p}$
- (C)  $\frac{1}{\alpha} \log \frac{h+p}{h+c}$
- (D)  $\alpha \log \frac{h+c}{h+p}$

185. As the variance of service time increases, the waiting time of a customer in  $M|G|1$  queue :

- (A) increases
- (B) decreases
- (C) remains the same
- (D) cannot be determined

186. Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates servers 1 and 2 are respectively 8 and 10. A customer at server 1 is equally likely to join server 2 upon completion of service or leave the system. Similarly a customer at server 2, upon completion of the service will go to server 1 with probability 1/4. Then the expected number of customers in the queue waiting for service is :

- (A) 7/9
- (B) 1/4
- (C) 1/5
- (D) None of the above

187. The dynamic programming approach to an optimization problem gives the recurrence equation for optimal solution as :

$$f_k(b_k) = \min_{0 < x \leq b} \{x^2 + f_{k-1}(b_{k-1})\}$$

for  $k > 1$ ; and  $f_1(b_1) = (b_2 - x)^2$ .

Then,  $f_3(15) =$

- (A) 25
- (B) 112.5
- (C) 75
- (D) None of the above

188. Which of the following characteristics of dynamic programming is *not true* ?

- (A) The problem can be subdivided into stages
- (B) Every stage consists of a number of states
- (C) Linear programming problems cannot be solved using dynamic programming
- (D) The optimal solution to the problem is obtained using optimal solutions to sub-problems

189. Consider the LPP :

$$\text{Minimize } Z = 4x_1 + 6x_2 + 18x_3$$

$$\text{Subject to : } x_1 + 3x_2 \geq 3,$$

$$x_2 + 2x_3 \geq 5$$

$$\text{and } x_1, x_2, x_3 \geq 0.$$

Then, which of the following is not a constraint in the corresponding dual problem ? (Assume that  $w_1$  and  $w_2$  are the dual variables.)

- (A)  $w_1 \leq 4$
- (B)  $w_2 \leq 9$
- (C)  $3w_1 + w_2 \geq 6$
- (D)  $w_1 \geq 0, w_2 \geq 0$

190. In critical path analysis, CPM is :

- (A) event oriented
- (B) deterministic in nature
- (C) dynamic in nature
- (D) none of the above

**SEP - 30221/II—A**

**ROUGH WORK**

**SEP - 30221/II—A**

**ROUGH WORK**