Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

A

Paper-II
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

JUN - 30220

Time Allowed : 2 Hours] **[Maximum Marks : 200**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **190**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **190** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. Use only Blue/Black Ball point pen.
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 190 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.
   (*iii*) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरिक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

Mathematical Science
Paper II

Time Allowed : 120 Minutes] **[Maximum Marks : 200**

Note : This Paper contains **One Hundred Ninety (190)** multiple choice questions in THREE (3) sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

Section I : Q. Nos. 1 to 10, **Section II : Q. Nos. 11 to 100,** **Section III : Q. Nos. 101 to 190.**

SECTION I

1. Let (X_n) be a sequence of real numbers and $x \in \mathbf{R}$. Then which of the following is *true* ?

(A) If $\lim\limits_{n\to\infty} x_n \leq x$, then $x_n \leq x$ for all n

(B) If $\lim\limits_{n\to\infty} x_n \neq x$, then there exist m and n such that $x_m < x < x_n$

(C) If $\lim\limits_{n\to\infty} x_n$ does not exist, then (x_n) has no subsequence converging to x

(D) If $\lim\limits_{n\to\infty} x_n < x$, then $x_n < x$ for infinitely many numbers n

2. Define a function $f : [0, 1] \to \mathbf{R}$ as $f(x) = x^2 + e^x - 2$. Then :

(A) $f(x) = 0$ has no solution in $[0, 1]$

(B) $f'(x) = 0$ has a solution in $[0, 1]$

(C) f is increasing

(D) f is not one-one

3. Let z, w be two complex numbers such that $z\,w \in \mathbf{R}$ and $z + w \in \mathbf{R}$. Then :

(A) $z = w$

(B) $z = \bar{w}$

(C) $z, w \in \mathbf{R}$

(D) $z = \bar{w}$ or $z, w \in \mathbf{R}$

4. Define $f : \mathbf{C} - \{0\} \to \mathbf{C}$ by $f(z) = \frac{1}{z}$.

Then the power series expansion of $f(z)$ about $z = 1$ in the disc $\{z : |z - 1| < 1\}$ is :

(A) $1 - (z - 1) + (z - 1)^2 - (z - 1)^3 + \ldots\ldots\ldots\ldots$

(B) $1 + (z - 1) + (z - 1)^2 + (z - 1)^3 + \ldots\ldots\ldots\ldots$

(C) $1 - z + z^2 - z^3 + \ldots\ldots\ldots\ldots$

(D) $1 + z + z^2 + z^3 + \ldots\ldots\ldots\ldots$

5. If $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear map, then it is represented by a matrix of order :

(A) $m \times n$

(B) $m \times m$

(C) $n \times n$

(D) $n \times m$

6. Let V be a finite dimensional vector space over $\mathbf{R}$ and $T \in L(V)$. If λ is an eigenvalue of T, then which of the following statements is *false* ?

(A) $\ker(T - \lambda I) \neq \{0\}$

(B) $\ker(T - \lambda I) = \{0\}$

(C) $\text{rank}(T - \lambda I) < \dim V$

(D) $\det(T - \lambda I) = 0$

7. Suppose X is a continuous random variable having the following probability density function :

$$f(x) = C(4x - 2x^2), \quad 0 < x < 2$$
$$= 0, \qquad \text{otherwise}$$

What is the value of $P[X > 1]$?

(A) $\frac{2}{3}$

(B) $\frac{C}{2}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

8. The moment generating function of a random variable X is given by $Mx(t) = \exp[2e^t - 2]$. Then $E[X] =$

(A) 0

(B) 1

(C) 2

(D) ∞

9. Consider the LP problem :

Max Z = $10x_1 + 6x_2$

Subject to the constraints :

$5x_1 + 3x_2 \leq 30;$

$x_1 + 2x_2 \leq 18$ and

$x_1, x_2 \geq 0.$

Then the LP has :

(A) No feasible solution

(B) Multi-feasible solutions

(C) Unique feasible solution

(D) Cannot say anything about solution

10. Which of the following statement/s is/are *true* for an LP problem ?

(I) Every feasible solution is a basic feasible solution.

(II) If there is a feasible solution, then there is a basic feasible solution.

(A) only (I) is true

(B) only (II) is true

(C) both are true

(D) neither (I) nor (II) is true

SECTION II

11. Let the middle number of the three distinct real numbers a, b and c be mid$\{a, b, c\}$. Then mid$\{a, b, c\}$ =

(A) $\min\{\max\{a, b\}, \min\{b, c\} \min\{a, c\}\}$

(B) $\min\{\max\{a, b\}, \max\{b, c\} \max\{a, c\}\}$

(C) $\max\{\max\{a, b\}, \max\{b, c\} \min\{a, c\}\}$

(D) $\max\{\max\{a, b\}, \max\{b, c\} \max\{a, c\}\}$

12. Let (x_n) be a sequence of real numbers. Then(x_n) has a subsequence which is :

(A) monotone

(B) bounded

(C) cauchy

(D) convergent

13. Consider the following statements :

(a) There is a continuous map from [0, 1] onto (0, 1)

(b) There is a continuous map from (0, 1) onto [0, 1]

Then :

(A) only (a) is true

(B) only (b) is true

(C) both (a) and (b) are true

(D) both (a) and (b) are false

14. Consider the function

$$f(x)=\begin{cases} x\sin\left(\dfrac{1}{x}\right), & \text{if } 0 < x \le 1 \\ 0 & , \text{if } x = 0 \end{cases}$$

Then which of the following statements is *true* ?

(A) f is Riemann integrable

(B) f is not continuous at $x = 0$

(C) f is of bounded variation

(D) f is differentiable at $x = 0$

15. Consider **R** with the metric defined by :

$$d(x, y) = \begin{cases} 1, & \text{if } x \neq y \\ 0, & \text{if } x = y \end{cases}$$

Which of the following statements is *true* for (**R**, d) ?

(A) (0, 1) is not closed

(B) $(0, \infty)$ is unbounded

(C) [0, 1] is not compact

(D) {1, 2, ..., 100} is not open

16. For a complex number z, the inequality $|z - i| < |z + i|$ is :

(A) never true

(B) always true

(C) true if and only if $\mathrm{Im}(z) > 0$

(D) true if and only if $\mathrm{Re}(z) > 0$

17. Which of the following sets is mapped onto the right half plane under the map $f(z) = \sin z$?

(A) $\left\{z : 0 < \mathrm{Re}(z) \le \dfrac{\pi}{2}\right\}$

(B) $\left\{z : \dfrac{-\pi}{2} \le \mathrm{Im}(z) \le \dfrac{\pi}{2}\right\}$

(C) $\left\{z : 0 < \mathrm{Im}(z) \le \dfrac{\pi}{2}\right\}$

(D) $\left\{z : \dfrac{-\pi}{2} \le \mathrm{Re}(z) \le \dfrac{\pi}{2}\right\}$

18. Consider the following statements :

(a) Every non-negative harmonic function on **C** is constant.

(b) Every harmonic function on **C** – {0} has a harmonic conjugate

Then :

(A) only (a) is true

(B) only (b) is true

(C) both (a) and (b) are true

(D) both (a) and (b) are false

19. The value of the integral

$$\frac{1}{2\pi i} \int_{|z|=1} z^3 \cos\left(\frac{1}{z}\right) dz$$

is :

(A) 0

(B) 1

(C) $\frac{-1}{6}$

(D) $\frac{1}{24}$

20. Consider the function

$$f(z) = \frac{1}{z^2 \sin z}.$$

Then :

(A) f has a pole of order 1 at $z = 0$

(B) f has a pole of order 2 at $z = 0$

(C) the residue of f at $z = 0$ is $\frac{1}{3}$

(D) the residue of f at $z = 0$ is $\frac{1}{6}$

21. Let $\phi : G \to G'$ be a surjective group homomorphism. Then :

(A) If G is cyclic, then G′ is cyclic

(B) If G′ is cyclic, then G is cyclic

(C) G is cyclic if and only if G′ is cyclic

(D) If G is not cyclic, then G′ is not cyclic

22. Let n be a square free natural number. Then the quotient ring $Q[X]/\langle X^2 - n \rangle$ is :

(A) not a field

(B) an integral domain which is not a field

(C) not an integral domain

(D) an integral domain

23. Every finite group is isomorphic to a subgroup of :

(A) a cyclic group

(B) $\mathbf{Z}_n$ for some $n \geq 1$

(C) S_n for some $n \geq 1$

(D) $(\mathbf{R}, +)$

24. Let σ be a permutation in Sn. Let $\sigma = \sigma_1, \sigma_2 .... \sigma_k$, where $\sigma_1, \sigma_2, ..., \sigma_k$ are disjoint cycles of lengths $l_1, l_2, ..., l_k$ respectively. Then the order of σ is equal to :

(A) $l_1 + l_2 + ... + l_k$

(B) $l_1 \,.\, l_2 .... l_k$

(C) $gcd(l_1, ..., l_k)$

(D) $lcm(l_1, ..., l_k)$

25. Which of the following is a principal ideal of $\mathbf{Z}[x]$?

(A) $(2, x)$

(B) $(3, x)$

(C) $(x, x^2 - x, x + 4)$

(D) $(2x, 3x, x^2)$

26. T_1, T_2 and T_3 are three maps defined on $\mathbf{R}^3$ as :

$$T_1\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+1 \\ y \\ z \end{bmatrix}, \; T_2\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xy \\ z \end{bmatrix}$$

$$T_3\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y+z \\ z+x \end{bmatrix}.$$

Which of these maps are linear ?

(A) T_1, T_2, T_3

(B) T_1, T_2,

(C) T_3

(D) T_1, T_3

27. Let T be a linear operator on $\mathbf{R}^4$ having the minimal polynomial $(x + 1)^3$. Then which of the following statements is *true* ?

(A) T is diagonalizable over $\mathbf{R}$

(B) T is nilpotent

(C) T is triangulable over $\mathbf{R}$

(D) characteristic polynomial of T is $(x + 1)^3$

28. Let V be a finite dimensional vector space over K and W be a subspace of V. Then which of the following statements is *true* ?

(A) $\dim V = \dim W + \dim \frac{W}{V}$

(B) $\dim V = \dim W - \dim \frac{V}{W}$

(C) $\dim V = \dim W + \dim \frac{V}{W}$

(D) $\dim V = \dim \frac{V}{W}$

29. Let T be a linear operator on $\mathbf{R}^3$, given by :

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x + 2y + z \\ 2y + z \\ 2x + 3y + z \end{bmatrix}$$

Then the matrix representation of T with respect to standard basis of $\mathbf{R}^3$ is :

(A) $\begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

30. Let A and B be real $n \times n$ matrices. Which of the following statements is *false* ?

(A) If A^{-1} and B^{-1} are congruent, then so are A and B

(B) If A^t and B^t are congruent, then so are A and B

(C) If both A and B are congruent to the identity matrix, then they are congruent

(D) If both A and B have the same rank, then they are congruent

31. The partial differential equation

$$u_t + u_x = u^2$$

can be classified as :

(A) Linear

(B) Non-linear

(C) Semilinear

(D) None of the above

32. The complete integral of the partial differential equation

$$f(p, q) = 0$$

is :

(A) $z = \phi(a)\, x + y + b$

(B) $z = \phi(a)\, x^2 + ay + b$

(C) $z = \phi(a)\, x + ay + b$

(D) $z = x^2 + ay + b$

33. Consider the following two statements :

(I) The set $\{x, |x|\}$ is linearly independent on $(-\infty, \infty)$

(II) The Wronskian $W(x, |x|) = 0$ on $(-\infty, \infty)$

Then :

(A) Both (I) and (II) are true

(B) Neither (I) nor (II) is true

(C) Only (I) is true

(D) Only (II) is true

34. Solution of the differential equation

$$xdy - ydx = 0,$$

represents a :

(A) straight line

(B) hyperbola

(C) parabola

(D) circle

35. The order and degree of the differential equation

$$\left[1 + 3\frac{dy}{dx}\right]^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$$

is :

(A) (3, 2) (B) (3, 4)

(C) (3, 3) (D) (2, 3)

36. A dairy firm has three plants P_1, P_2, P_3 located in a state. They daily milk production at each plant (in thousand liters) is $P_1 \to 6$, $P_2 \to 1$, $P_3 \to 10$. Each day, the firm must fulfil the needs of its four distribution centers D_1, D_2, D_3, D_4. The minimum requirement at each center (in thousand liters) is $D_1 \to 7$, $D_2 \to 5$, $D_3 \to 3$, $D_4 \to 2$. Cost in hundred of rupees of shipping one thousand liters from each plant to each distribution center is given below :

Distribution Center

Plant	D_1	D_2	D_3	D_4
P_1	2	3	11	7
P_2	1	0	6	1
P_3	5	8	15	9

Then the initial basic feasible by North-West Corner method is :

(A) Non-degenerate and cost associated with this solution is ₹ 11,600

(B) degenerate and cost associated with this solution is ₹ 12,000

(C) non-degenerate and cost associated with this solution is ₹ 12,000

(D) degenerate and cost associated with this solution is ₹ 11,600

37. Consider the following LP program :

Max

$$Z = -3x_1 + 2x_2$$

Subject to the constraints :

$$x_1 \leq 3$$

$$x_1 - x_2 \leq 0$$

$$x_1, x_2 \geq 0$$

Then the above LP problem has :

(A) no feasible solution

(B) feasible solution

(C) unbounded solution

(D) none of the above

38. Consider the following statements :

(I) Dual of the dual is primal.

(II) Every solution of an LP problem is a basic feasible solution.

Then which of the following is *true* ?

(A) Only (II) is true

(B) Only (I) is true

(C) Both are true

(D) Neither (I) nor (II) is true

39. Consider the LP problem :

Max $Z = 4x_1 + 2x_2$

Subject to the constraints :

$$-x_1 - x_2 \leq -3$$

$$-x_1 + x_2 \geq -2 \text{ and}$$

$$x_1, x_2 \geq 0.$$

Then the dual of the LP problem is :

Min. $W = ay_1 + by_2$

$$-py_1 + qy_2 \geq 4$$

$$-y_1 - y_2 \geq 2 \text{ and } y_1, y_2 \geq 0$$

where the values of a, b, p, q are :

(A) $a = -3, b = -2, p = -1, q = -1$

(B) $a = 3, b = 2, p = -1, q = -1$

(C) $a = -3, b = 2, p = 1, q = 1$

(D) $a = 3, b = 2, p = -1, q = 1$

40. For an LP problem which of the following statements is *false* ?

(A) If the feasible region is a polyhedron, then at least one basic feasible solution is optimal

(B) If an optimal solution exists, then the feasible region is bounded

(C) If the half spaces corresponding to the constraints do not intersect, the problem has no feasible solution

(D) The set of all feasible solutions to the problem is a convex set

41. Let $f : \mathrm{D} \to \mathbf{R}$ be defined as $f(x, y, z) = xyz$; where $\mathrm{D} = \{(x, y, z) \in \mathbf{R}^3 / x^2 + y^2 + z^2 < 1\}$. Let P denote the plane $\{(x, y, z)/2x + 3y + 5z = 0\}$. Then $\int_{\mathrm{D} \cap \mathrm{P}} f\, dx\, dy\, dz$ is :

(A) Undefined

(B) 0

(C) π

(D) 2π

42. Let $\mathrm{A} = \begin{pmatrix} 1 & 2 \\ 3 & 6 \\ 1 & 2 \end{pmatrix}$ and $f : \mathbf{R}^2 \to \mathbf{R}^3$ be defined as $f(x) = \mathrm{A}x$. Then which of the following statements is *false* ?

(A) f is continuous function

(B) f is differentiable function

(C) f maps open neighbourhood of origin to an open neighbourhood

(D) the directional derivative of f in direction of (1, 0) is $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$

43. The vector normal to the gradient of the function $f : \mathbf{R}^3 \to \mathbf{R}$ defined by

$$f(x, y, z) = xy + y^2 + z$$

at the point P = (1, –1, 1) is :

(A) (–3, –3, 0)

(B) (3, 0, 3)

(C) (–3, 0, 3)

(D) (1, 0, 0)

44. Let f be a non-constant analytic function on the unit disc. Then which of the following is impossible ?

(A) $f(\mathrm{G}) \subseteq \{z : \mathrm{Re}(z) \geq 1\}$

(B) $f(\mathrm{G}) = \{z : |z| < 2\}$

(C) $f(\mathrm{G}) = \{z : |z| > 1\}$

(D) $f(\mathrm{G}) = \{z : |z| = 1\}$

45. Let $a, b \in \mathbf{C}$ and D be a disc in $\mathbf{C}$ and $f(z) = ae^z + be^{-z}$ for all $z \in \mathbf{C}$. If $f(z) = 0$ for all $z \in \mathrm{D}$, then :

(A) $a = 0$ and $b = 0$

(B) $a = 0$ and $b = 1$

(C) $a = 1$ and $b = 0$

(D) $a = 1$ and $b = 1$

46. Let $f : \mathbf{C} \to \mathbf{C}$ be a non-constant analytic such that $|f'(z)| < f(z)|$ for all $z \in \mathbf{C}$. Then :

(A) $\dfrac{1}{f}$ is not analytic

(B) f is a polynomial

(C) $f'(z) = 0$ for some z

(D) $f'' = cf'$ for some constant c

47. Let $\mathbf{R}[X]$ be the polynomial ring in one indeterminate over the field of real numbers. Then in the quotient ring $\mathbf{R}[X]/<X^8 + 1>$ number of maximal ideals is :

(A) 1

(B) 2

(C) 3

(D) 4

48. Let p be an odd prime number and $\mathbf{Z}\left[\sqrt{-p}\right] = \{a + b\sqrt{-p}\} \mid a, b \in \mathbf{Z}\}$ $(\subseteq \mathbf{C})$. Then the ring $\mathbf{Z}\left[\sqrt{-p}\right]$ is :

(A) not a Euclidean domain, but is a PID (Principal ideal domain)

(B) Not a PID, but is an UFD

(C) Neither Euclidean domain, nor a PID, but is an UFD

(D) not an UFD

49. Let $\Omega \subseteq \mathbf{C}$ be a region and $H(\Omega)$ be the set of all analytic functions on Ω, then :

(A) $H(\Omega)$ is an integral domain which does not satisfy ascending chain condition on ideals

(B) $H(\Omega)$ is a unique factorisation domain

(C) $H(\Omega)$ is a finite dimensional vector space over $\mathbf{C}$

(D) $H(\Omega)$ has only finitely many maximal ideals

50. For $x, y \in \mathbf{R}$, let $d(x, y) = |x - y|$. Then :

(A) $(-1, 1)$ is complete in d

(B) $(-1, 1)$ is not complete in d, but there is a complete metric on $(-1, 1)$ inducing the same topology on $(-1, 1)$ as of d

(C) every metric on $(-1, 1)$ inducing the same topology as that of d is complete

(D) no metric on $(-1, 1)$ inducing the same topology as that of d is complete

51. Let $X = \{(x, y) \in \mathbf{R}^2 : |x| \ |y| = 1\}$ and $Y = \{(x, y) \in \mathbf{R}^2 : |x| + |y| = 1\}$. Then :

(A) X and Y are compact

(B) X is compact, but Y is not compact

(C) Y is compact, but X is not compact

(D) neither X nor Y is compact

52. Let E be a measurable subset of $\mathbf{R}$ such that $m(k) = 0$ for all compact subsets k of E. Then :

(A) E is bounded

(B) For $\varepsilon > 0$ there is an open set U in $\mathbf{R}$ such that $E \subset U$ and $m(U) < \varepsilon$

(C) E is a G_δ-set

(D) E is an F_σ-set

53. Let G be a group of prime power order. Then :
   (A) G is abelian
   (B) G is simple
   (C) G is solvable
   (D) G has a Sylow subgroup which is abelian

54. Which of the following rings is *not* noetherian ?
   (A) $\mathbf{Q}[x_1, x_2, x_3, \ldots\ldots..]$
   (B) $\mathbf{Z}[x]$
   (C) $\mathbf{Z}[\sqrt{2}]$
   (D) $\mathbf{R}[x, y]/(x^2 + y^2 + 1)$

55. Let K/F be a Galois extension of fields. If Gal(K/F) is isomoprhic to S_3, the number of fields L such that $F \subseteq L \subseteq K$ is :
   (A) 6
   (B) 4
   (C) 10
   (D) 8

56. Let X and Y be Banach spaces and $T : X \to Y$ be a bijective continuous linear map. If $\| . \|_1$ and $\| . \|_2$ denote the norms on X and Y respectively, define $\| x \|_3 = \| Tx \|_2$ and $\| y \|_4 = \| x \|_1$, where $Tx = y$, for all $x \in X$ and $y \in Y$. Then :
   (A) $\| . \|_1$ and $\| . \|_3$ are equivalent, but $\| . \|_2$ and $\| . \|_4$ are not always equivalent
   (B) $\| . \|_2$ and $\| . \|_4$ are equivalent, but $\| . \|_1$ and $\| . \|_3$ are not always equivalent
   (C) $\| . \|_1$ is equivalent to $\| . \|_3$ and $\| . \|_2$ is equivalent to $\| . \|_4$
   (D) neither $\| . \|_1$ is always equivalent to $\| . \|_3$, nor $\| . \|_2$ is always equivalent to $\| . \|_4$

57. Let $T : l^2 \to l^2$ be defined by $Te_n = 0$ if n is odd and $Te_n = e_{n/2}$ if n is even. Then :
   (A) T is normal
   (B) T is self-adjoint
   (C) T is unitary
   (D) $T^*T = I$ = the identity map

58. Let $C'[0, 1] = \{f : [0, 1] \to \mathbf{R} : f$ is differentiable on $[0, 1]$ and f' is continuous$\}$ and $C[0, 1] = \{f : [0, 1] : f$ is continuous on $[0, 1]$ with $\|f\| = \sup\{|f(t)| : t \in [0, 1]\}$. Let $D : C^1[0, 1] \to C[0, 1]$ be defined as $Df = f'$ and $I : C[0, 1] \to C[0, 1]$ be defined as $I(f)(x) = \int_0^x f(t)\, dt$. Then :

(A) D and I are continuous linear maps

(B) D is continuous, but I is not

(C) I is continuous, but D is not

(D) both D and I are not continuous

59. Let X and Y be topological spaces and $f : X \to Y$ be continuous. (1) $f(\bar{A}) \subset \overline{f(A)}$, (2) $f(\mathring{A}) \subset f(\mathring{A})$ and (3) $f(\partial A) \subset \partial f(A)$. For every subset A of X :

(A) (1), (2) and (3) are true

(B) (1) and (2) are true, but not (3)

(C) (1) and (3) are true but not (2)

(D) (1) is true and (2) and (3) are not

60. Let $X = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 < 1\} \cup \{(x, y) \in \mathbf{R}^2 : (x - 2)^2 + y^2 < 1\}$ and $Y = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 < 1\}$ U $\{(x, y) \in \mathbf{R}^2 : (x - 3)^2 + y^2 < 1\}$ and $Z = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 < 1, x \neq 0\}$ be subspaces of the Euclidean space $\mathbf{R}^2$. Then :

(A) X and Y are homeomorphic to Z

(B) X is homeomorphic to Y, but not to Z

(C) X is homeomorphic to Z, but not to Y

(D) none of X or Y or Z is homeomorphic to the other two

61. Let $\{f : \mathbf{R} \to \mathbf{R} : f$ is continuous such that $f(t) = 0$ for all t in a subfield of $\mathbf{R}\} = X$. Then X contains :

(A) uncountable number of elements

(B) countably infinite number of points

(C) finitely many elements ≥ 2

(D) only one element

62. Consider the following statements about a lattice L :

(*i*) Every ideal of L is a prime ideal.

(*ii*) L is a chain.

Then which of the following is *correct* ?

(A) (I) implies (II) but not conversely

(B) (II) implies (I) but not conversely

(C) (I) if and only if (II)

(D) Neither (I) implies (II) nor (II) implies (I)

63. Let G be a connected graph with at least 3 vertices such that removal of any edge of G disconnects the graph. Then which of the following is *not* true ?

(A) G has no circuit

(B) There is a unique path in G between any pair of distinct vertices

(C) G has at least two pendant vertices

(D) G has at least two spanning trees

64. Given a group of n married women and their n husbands at least how many people be chosen from this group of $2n$ people to guarantee the set contains a married couple ?

(A) $n - 1$

(B) n

(C) $n + 1$

(D) $n + 2$

65. Let u_1 and u_2 be solution of the equation :

$$\Delta u = 1 \qquad \text{in D}$$

$$u = 0 \qquad \text{on } \partial \text{D}$$

where $D = \{(x, y) \in \mathbf{R}^2/(x - 2)^2 + (y - 1)^2 \leq 1/4\}$

and $\partial D = \{(x, y) \in \mathbf{R}^2/(x - 2)^2 + (y - 1)^2 = 1/4\}$ is the boundary of D.

If $u_1(2, 1) = u_2(2, 1)$; then :

(A) u_1 is a constant function

(B) u_2 is a constant function

(C) $u_1 \equiv u_2 \equiv$ a constant function

(D) $u_1 \equiv u_2$ in D

66. Consider the function

$$f(x) = |x| \quad -1 \leq x \leq 1$$

Then the Fourier series for f is :

(A) $\frac{1}{2} - \frac{4}{\pi^2}\left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + ...\right)$

(B) $\frac{1}{2} - \frac{4}{\pi^2}\left(\frac{\sin \pi x}{1^2} + \frac{\sin 3\pi x}{3^2} + ...\right)$

(C) $\frac{1}{2} - \frac{4}{\pi^2}\left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + ...\right) - \frac{4}{\pi^2}\left(\frac{\sin \pi x}{1^2} + \frac{\sin 3\pi x}{3^2} + ...\right)$

(D) $\frac{1}{2} + \frac{4}{\pi^2}\left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + ...\right) + \frac{4}{\pi^2}\left(\frac{\sin \pi x}{1^2} + \frac{\sin 3\pi x}{3^2} + ...\right)$

67. The equation $3u_{xx} - x^2u_y + 5u = 0$ transforms under the change of variables to polar coordinates is :

(A) a non-linear equation

(B) a hyperbolic equation

(C) an elliptic equation

(D) a parabolic equation

68. The linear diophantine equation $7x - 8y = 5$ has :

(A) exactly one integer solution

(B) exactly two integer solutions

(C) infinitely many integer solutions and the difference between any two values of x in the solutions is divisible by 8.

(D) infinitely many integer solutions and the difference between any two values of x in the solutions is divisible by 7

69. Which of the following is a quadratic residue modulo 43 ?

(A) -1

(B) 40

(C) 2

(D) 3

70. If μ is the Mobius function and n is a positive integer, then $\mu(n)\,\mu(n + 1)\,\mu(n + 2)\,\mu(n + 3)$ is equal to :

(A) 4

(B) 0

(C) 1

(D) 6

71. A dynamical system consists of three particles m_1, m_2, m_3 in motion in space. The distance between m_1 and m_2, m_2 and m_3, m_1 and m_3 remain constant throughout the motion. The motion will be governed by n Euler Lagrange equations. Then :

(A) $n = 7$

(B) $n = 6$

(C) $n = 9$

(D) $n = 3$

72. A particle of mass m is moving in a two-dimensional space. Its kinetic energy in terms of polar co-ordinates is given by :

(A) $\frac{1}{2}m(r^2 + r^2\dot{\theta}^2)$

(B) $\frac{1}{2}m(\dot{r}^2 + r\dot{\theta}^2)$

(C) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$

(D) $\frac{1}{2}m(r^2 + r\dot{\theta}^2)$

73. Let the Hamiltonian of a system be $H = \frac{p^2}{2} + \frac{k^2q^2}{2}$. If C_1, C_2 denote arbitrary constants the general solution of q is :

(A) $q(t) = C_1 \cos kt + C_2 \sin kt$

(B) $q(t) = kt + C_1$

(C) $q(t) = C_1e^{kt} + C_2e^{-kt}$

(D) $q(t) = C_1 \cos k^2t + C_2 \sin k^2t$

74. Consider a single operator equation $\bar{L} = I\bar{W}$ representing the relation between angular momentum vector and angular velocity :

(A) The operator equation represents linear transformation

(B) The vectors $\bar{L}$ and $\bar{W}$ have the same dimensions

(C) The vectors $\bar{L}$ and $\bar{W}$ are two physically different vectors having different dimensions

(D) The operator I acting upon the vector $\bar{W}$ represents orthogonal rotation

75. If velocity components are time independent, then :
   (A) Only stream lines and path lines are coincident
   (B) Only stream lines and streak lines are coincident
   (C) Only path lines and streak lines are coincident
   (D) Stream lines, path lines and streak lines are coincident

76. Velocity potential function for a steady flow exists, if :
   (A) flow is only incompressible
   (B) flow is inviscid, incompressible and irrotational
   (C) flow is inviscid, incompressible and rotational
   (D) flow is only inviscid

77. If $\phi = (x - t)(y - t)$ represents the velocity potential of an incompressible two-dimensional fluid then the stream lines at time t are the curves :
   (A) $(x - t)^2 - (y - t)^2 = \text{constant}$
   (B) $(x - t)^2 + (y - t)^2 = \text{constant}$
   (C) $x^2 - y^2 = t^2$
   (D) $x^2 + y^2 = t^2$

78. In a uniform stream with complex potential Uz, a two-dimensional source of strength m is situated at the point $(a, 0)$. The stagnation point is at :
   (A) $z = m + \frac{a}{U}$
   (B) $z = 0$
   (C) $z = a + \frac{U}{m}$
   (D) $z = a + \frac{m}{U}$

79. Let the first and second fundamental forms associated with a surface patch be respectively $E du^2 + 2F\, du dv + G dv^2$ and $L du^2 + 2M\, du dv + N dv^2$.
   If $S = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$ and $T = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$, then :
   (A) S and T are non-singular
   (B) S is non-singular and T may not be
   (C) T is not-singular and S may not be
   (D) S and T are singular

80. Let U be an open subset of $\mathbf{R}^2$ and S be a smooth surface in $\mathbf{R}^3$. If $\sigma : \mathrm{U} \to \mathrm{S}$ is a regular surface patch such that $\det\begin{pmatrix} \sigma_u . \sigma_u & \sigma_u . \sigma_v \\ \sigma_u . \sigma_v & \sigma_v . \sigma_v \end{pmatrix} = 1$

Then :

(A) σ is an isometry

(B) σ is conformal, but not an isometry

(C) σ preserves area, but need not be conformal

(D) σ preserves area and conformal

81. Let $\mathrm{S} = \{(x, y, f(x, y)) \in \mathbf{R}^3 \,|\, (x, y) \in \mathrm{U}\}$ where U is open in $\mathbf{R}^2$ and $f : \mathrm{U} \to \mathbf{R}$ is smooth. If f admits a local maximum at p and a local minimum at q, then the Gaussian curvatures k_1 and k_2 to S at the points $(p, f(p))$ and $(q, f(q))$ respectively satisfy :

(A) $k_1, k_2 \geq 0$

(B) $k_1 \geq 0, k_2 \leq 0$

(C) $k_1 \leq 0, k_2 \geq 0$

(D) $k_1, k_2 \leq 0$

82. The functional $\int_0^{\pi/4} (y^2 - y'^2)\, dx$ satisfying $y(0) = 0$ attains its extremal on :

(A) $y = x$

(B) $y = 0$

(C) $y = \sin x$

(D) $y = \cos x$

83. Let $\mathrm{J}(y) = \int_{-1}^{1} (y'e^y + xy^2)\, dx$ be a functional defined on $\mathrm{C}^1[-1, 1]$. Then the variation of the integral $\mathrm{J}[y]$ is :

(A) $\int_{-1}^{1} (y''e^y + e^y y' + 2xyy')\, dx$

(B) $\int_{-1}^{1} (y'^2 e^y + y^2 + 2xyy')\, dx$

(C) $\int_{-1}^{1} [[y'e^y + 2xy]\,\delta y + e^y \delta y']\, dx$

(D) $\int_{-1}^{1} [[y'e^y + 2xy + y^2]\,\delta y + e^y \delta y']\, dx$

84. Consider the functional

$$J[y] = \int_a^b F(x, y, y')\, dx$$

where $F(x, y, y') = \dfrac{1 + y^2}{y'^2}$

for admissible function $y(x)$. Which of the following are extremals for J ?

(A) $y(x) = A \sin x$ where A is constant

(B) $y(x) = A \sinh x + B \cosh x$, A, B are constants

(C) $y(x) = \sinh (Ax + B)$ where A, B are constants

(D) $y(x) = A \sin x + B \cos x$, where A, B are constants

85. The shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$ is :

(A) 0

(B) 5

(C) $\dfrac{19\sqrt{2}}{8}$

(D) $\sqrt{5}$

86. For the Fredholm integral equation

$$u(t) = \lambda \int_a^b k(t, s)\, u(s)\, ds$$

with separable kernel, Fredholm determinant $D(\lambda) \neq 0$. Then the integral equation has :

(A) unique zero solution

(B) unique non-zero solution

(C) infinitely many solutions

(D) no solution

87. Eigen value and eigen function of the homogeneous integral equation

$$u(t) = \lambda \int_0^1 e^{t+s}\, u(s) ds$$

is :

(A) $\lambda = \dfrac{2}{e^2 - 1}$, $u(t) = e^{t^2}$

(B) $\lambda = \dfrac{2}{e^2 - 1}$, $u(t) = e^t$

(C) $\lambda = \dfrac{e^2 - 1}{2}$, $u(t) = e^t$

(D) $\lambda = \dfrac{e^2 - 1}{2}$, $u(t) = e^{t^2}$

88. The solution of the Volterra integral equation of first kind

$$f(t)=\int_0^t e^{t-s}\,u(s)\,ds,$$

where $f(t)$ is known function, is :

(A) $tf(t) - f'(t)$

(B) $f'(t) - f(t)$

(C) $f'(t) - tf(t)$

(D) $t\left(f'(t) - f(t)\right)$

89. Which of the following is *not* correct ?

(A) $E = 1 + \Delta$

(B) $\Delta = \nabla(1 - \nabla)^{-1}$

(C) $\nabla = E^{-1}\,\Delta$

(D) $1 + \Delta = (E + 1)\nabla^{-1}$

90. If $u_1 = 1$, $u_3 = 17$, $u_4 = 43$ and $u_5 = 89$, then value of u_2 is :

(A) 3

(B) 5

(C) 13

(D) 15

91. Let $y = f(x)$ takes the values $y_0, y_1, \ldots\ldots, y_n$, for $a = x_0, x_1 = x_0 + h, \ldots\ldots, x_n = x_0 + nh = b$. Then the value of $\int_a^b f(x)dx$ by Trapezoidal rule is :

(A) $\frac{h}{2}[2(y_0 + y_1) + (y_2 + \ldots\ldots + y_n)]$

(B) $\frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots + y_{n-1})]$

(C) $\frac{h}{2}[2(y_0 + y_n) + (y_1 + y_2 + \ldots\ldots + y_{n-1})]$

(D) $\frac{h}{2}[(y_0 + y_1) + 2(y_2 + \ldots\ldots + y_n)]$

92. Laplace transform of $J_0(a\ t)$ where J_0 denotes the Bessel function of zero order is :

(A) $\dfrac{1}{s^2 + a^2}$

(B) $\dfrac{1}{\sqrt{s^2 + a^2}}$

(C) $\dfrac{1}{s^2 - a^2}$

(D) $\dfrac{1}{\sqrt{s^2 - a^2}}$

93. Fourier cosine transform of $f(t) = e^{-t^2}$ is :

(A) $\frac{1}{\sqrt{2}} e^{-4s^2}$

(B) $\frac{1}{\sqrt{2}} e^{-2s^2}$

(C) $\frac{1}{\sqrt{2}} e^{-\frac{s^2}{2}}$

(D) $\frac{1}{\sqrt{2}} e^{-\frac{s^2}{4}}$

94. If L{$f(t)$} = F(s) and L{$g(t)$} = G(s), then L^{-1}\{(F(s)G(s)\} is :

(A) $\int_0^t f(x)\, g(t-x)\, dx$

(B) $\int_0^t f(x)\, g(x)\, dx$

(C) $\int_0^t f(x)\, g(t+x)\, dx$

(D) $\int_0^t f(x-t)\, g(x-t)\, dx$

95. Consider the non-linear programming problem

Max. Z = $10x_1 - x_2^2$

Subject to the constraints :

$x_1 + x_2 \leq 14$

$-x_1 + x_2 \leq 6$ and $x_1, x_2 \geq 0$

Then the Lagrangian function is :

(A) $(10x_1 - x_2^2) - \lambda_1(x_1 + x_2 + r_1^2 - 14) - \lambda_2(-x_1 + x_2 + r_2^2 - 6)$

(B) $(10x_1 - x_2^2) + \lambda_1(x_1 + x_2 + r_1 - 14) + \lambda_2(-x_1 + x_2 + r_2 - 6)$

(C) $(10x_1 - x_2^2) + \lambda_1(-x_1 + x_2 - r_1 - 14) + \lambda_2(x_1 - x_2 - r_2 + 6)$

(D) $(10x_1 + x_2^2) + \lambda_1(x_1 + x_2 - r_1^2 + 14) + \lambda_2(x_1 - x_2 - r_2^2 + 6)$

96. The number of positive integers, not greater than 100 which are not divisible by 2, 3 or 5, is :

(A) 20

(B) 25

(C) 26

(D) 30

97. Let k, k' and δ be the vertex connectivity, edge connectivity and minimum degree respectively of graph G. Then :

(A) $\delta \leq k'$

(B) $k' \leq k$

(C) $k = k'$, whenever G is 3-regular

(D) $\delta = k'$, whenever G is regular

98. Consider ($\mathbf{R}$, μ) be Lebesgue measure space. Define

$$\nu(A) = \int_A f(x)\, d\mu \quad \text{where}$$

$$f(x) = \begin{cases} \sin x & 0 \le x \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

Then :

(A) ν is a measure

(B) ν is a signed measure

(C) ν is both measure and signed measure

(D) ν is neither a measure nor a signed measure

99. Let ν be a signed measure on (X, m). Let P be a positive set and N be a negative set. Then :

(A) $P \cap N$ is a positive set

(B) $P \cap N$ is a negative set

(C) $P \cap N$ is a null set

(D) $P \cap N$ is not measurable

100. Which of the following subsets of $\mathbf{R}$ has non-zero Lebesgue measure ?

(A) $\left\{n + \frac{1}{n} : n \in \mathbf{N}\right\}$

(B) The cantor set

(C) $\{a + b\sqrt{2} : a, b \in \mathbf{Q}\}$

(D) The set of limit points of $\mathbf{Q}$ in $\mathbf{R}$

SECTION III

101. For a data-set of n observations the sample mean, median and mode are 63, 60 and 58 respectively. Hence the data are :

(A) Symmetric about sample median

(B) Skewed to the left

(C) Skewed to the right

(D) Bimodal

102. The regression equation of Y on X is given by $y = 2 - 1.2x$ and that of X on Y is given by $x = -1 - 0.3y$. Hence the correlation coefficient between X and Y is :

(A) + 0.36

(B) + 0.6

(C) – 0.6

(D) – 0.36

103. The height of adult men in a population follows normal distribution with mean 1.65 meters and standard deviation σ. If 40% of the men are shorter than 1.5 meters, the percentage of population which is taller than 1.80 meters is :

(A) 80%

(B) 60%

(C) 40%

(D) Cannot be determined based on the given information

104. Suppose $E[X] = 5$. Which of the following is always *false* ?

(A) $E[X^2] < (E[X])^2$

(B) If $E[X^2] = 25$, then $X = 5$ wp 1

(C) $E[X^2] \geq 25$

(D) $E[\exp|X|] > \exp(E[|X|])$

105. Let E and F be two disjoint events such that $0 < P(E) < 1$ and $0 < P(F) < 1$. Which of the following is always *true* ?

(A) $P(E|F) = P(E)$

(B) $P(E|F) = P(F)$

(C) $P(E|F) \neq P(E)$

(D) $P(E^C|F) = 1$

106. Suppose X and Y are independent r.v.s. each with mean 1.

Let $\phi_X(t)$ denote the characteristic function of a random variable. Which of the following is *false* ?

(A) $\phi_{X+Y}(t) = \phi_X(t) \cdot \phi_Y(t)\ \forall t$

(B) $E[XY|X] = 1$

(C) $\phi_{X^2+Y^2}(t) = \phi_{X^2}(t) \cdot \phi_{Y^2}(t) \forall t$

(D) $(1 - X^2)$ and $\exp(Y)$ are independent

107. Let the joint distribution of (X, Y) be specified by :

		X = 1	X = 2	X = 3	X = 4
Y	1	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$
	2	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$	$\frac{3}{16}$

Then $P[X = 2Y]$ is :

(A) 0

(B) $\frac{9}{32}$

(C) $\frac{11}{32}$

(D) $\frac{1}{2}$

108. Which of the following matrices is a covariance matrix ?

(A) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$

109. If a random variable X has Cauchy distribution with location μ and scale σ, then the distribution of $Y = \frac{1}{X}$ is :

(A) Cauchy with location μ and scale $\frac{1}{\sigma}$

(B) Cauchy with location $\frac{\mu}{\mu^2 + \sigma^2}$ and scale $\frac{\sigma}{\mu^2 + \sigma^2}$

(C) Cauchy with location μ and scale σ

(D) not Cauchy

110. In a certain communication system, on the average, there is 1 transmission error per 10 seconds. Suppose the distribution of errors is Poisson. The probability of at least one error in a communication of half a minute duration is :

(A) 1 – 2 exp (–1)

(B) 1 – exp (–3)

(C) 1 – 4 exp (–3)

(D) 1 – exp (–1)

111. Suppose X is an absolutely continuous non-negative real-valued random variable whose distribution function is $F(x)$ and probability density function is $f(x)$. Suppose it is given that $f(x)/[1 - F(x)] = k$, a non-negative constant. Then which of the following statements is *true* ?

(A) $f(x) = 1,\ 0 < x < \infty$

(B) $f(x) = k\ e^{-kx},\ x > 0$

(C) $f(x) = k\ e^{-kx} + C,\ x > 0$, C a constant

(D) $f(x) = \begin{cases} 0 & x < k, \\ 1 & x \geq k. \end{cases}$

112. Suppose Y_1, Y_2 and Y_3 are independent and identically distributed p-variate random vectors having normal distribution. Then the distribution of $a_1Y_1 + a_2Y_2 + a_3Y_3$, where a_1, a_2, a_3 are vectors of order $1 \times p$, is :

(A) univariate normal

(B) four variate normal

(C) trivariate normal

(D) not normal

113. Let $X_1, X_2, ..., X_n$ be a random sample from $U(\theta, \theta + 1)$. If $X_{(1)} < X_{(2)} < .... < X_{(n)}$ are corresponding order statistics, then :

(A) $(X_{(1)}, X_{(n)})$ is jointly sufficient for θ

(B) $X_{(1)}$ is sufficient for θ

(C) $X_{(n)}$ is sufficient for θ

(D) $(X_{(1)}, X_{(n)}, + 1)$ is not jointly sufficient for θ

114. Let $Y_{(1)}$ and $Y_{(2)}$ be order statistics based on a sample of size 2 from a $U(\theta, 2\theta)$ distribution. Then $[Y_{(1)}, Y_{(2)}]$ is a confidence interval for θ with confidence coefficient :

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{1}{4}$

(D) 1

115. Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$, μ is unknown and $a^2 > 0$. Let $q_{n, \alpha}$ be the $(1 - \alpha)$th quantile of $\chi^2_{(n)}$ distribution. Consider the tests for testing $H_0 : \sigma^2 \leq 1$ *Vs.* $H_1 : \sigma^2 > 1$: T_1 : Reject H_0 if and only if $\sum_{i=1}^{n} (X_i - \bar{X})^2 > q_{n-1, \alpha}$ and T_2 : Reject H_0 if and only if $\sum_{i=1}^{n} (X_i - \bar{X})^2 > q_{n, \alpha}$ then which of the following statements is *correct* ?

(A) T_1 is a uniformly most powerful (UMP) level α test

(B) T_2 is a UMP level α test

(C) Both T_1 and T_2 are level α tests

(D) T_1 is level α test but T_2 is not

116. Let $\tau(\theta)$ be a parametric function and T be an estimator of $\tau(\theta)$. Let $V_\theta(T)$ denotes variance of T. The Cramer-Rao bound gives a :

(A) Lower bound for $V_\theta(T)$ for any arbitrary estimator T of $\tau(\theta)$

(B) Upper bound for $V_\theta(T)$ for any arbitrary estimator T of $\tau(\theta)$

(C) Lower bound for $V_\theta(T)$ among all unbiased estimators T of $\tau(\theta)$

(D) Upper bound for $V_\theta(T)$ among all unbiased estimators T of $\tau(\theta)$

117. Let $\{X_1, ..., X_n\}$ be a random sample from $N(\mu_1, \sigma^2)$ and $\{Y_1, ..., Y_m\}$ be a random sample from $N(\mu_2, \sigma^2)$. Let

$$T_1 = \sum_{i=1}^{n} X_i^2 + \sum_{j=1}^{m} Y_i^2, \; T_2 = \sum_{i=1}^{n} X_i,$$

$$T_3 = \sum_{i=1}^{m} Y_i, \; T_4 = \sum_{i=1}^{n} X_i^2, \; T_5 = \sum_{i=1}^{m} Y_i^2.$$

Then which of the following is a jointly sufficient complete statistic for (μ_1, μ_2, σ^2) :

(A) (T_2, T_3, T_4, T_5)

(B) (T_3, T_4, T_5)

(C) (T_2, T_3, T_1)

(D) (T_2, T_3, T_5)

118. Suppose $\{X_1, X_2, ..., X_m\}$ is a random sample from the distribution of X with mean μ_1 and variance σ_1^2, $\{y_1, ..., y_n\}$ is a random sample from the distribution of Y with mean μ_2 and variance σ_2^2. The test statistic to test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 = \sigma_2^2$ is $T_n = \frac{S_X^2}{S_Y^2}$, where S_X^2 and S_Y^2 are sample variances. Then the null distribution of T_n is :

(A) F if X and Y are independent and have normal distributions

(B) F if X and Y have normal distributions

(C) F if X and Y are independent

(D) F if X and Y have normal distributions with equal mean

119. In the method of constructing confidence interval for an unknown parameter based on pivotal quantity Q, which of the following statements is *correct* ?

(A) The distribution of Q is known and depends on the unknown parameters

(B) The distribution of Q is known and free from the unknown parameters

(C) The distribution of Q is not known but is free from the unknown parameters

(D) The distribution of Q is known and normal

120. To test $H_0 : p = p_0$ against the alternative $H_1 : p \neq p_0$, where p is the population proportion and p_0 is a specified value of p. Suppose P_n denotes the sample proportion based on a random sample of size n from the population. The test statistic for testing $H_0 : p = p_0$ against $H_1 : p \neq p_0$ is given by :

(A) $\dfrac{\sqrt{n}(P_n - p_0)}{\sqrt{p_0(1 - p_0)}}$

(B) $\sqrt{n}(P_n - p_0)$

(C) $\dfrac{P_n - p_0}{\sqrt{p_0(1 - p_0)}}$

(D) $\dfrac{P_n - p_0}{p_0(1 - p_0)}$

121. To test the equality of r independent multinomial distributions, multinomial (k, p_{i_1}, p_{i_k}) $i = 1 \ldots. r$ the null hypothesis is $H_0 : P_{ij} = P_j$ $\forall j = 1 \ldots\ldots k$ where p_j are unknown $j = 1 \ldots\ldots. k$, a χ^2 test statistic is used. The degrees of freedom of the χ^2 test are :

(A) $k(r - 1)$

(B) $r(k - 1)$

(C) $rk - 1$

(D) $(r - 1)(k - 1)$

122. X and Y are two continuous random variables independent of each other. The combined sample based on two independent samples of size 5 and 4 on X and Y respectively has the following pattern :

$y_4 < x_1 < x_5 < y_3 < x_2 < y_1 < y_2 < x_4 < x_3$

Hence the value of Mann-Whitney-Wilcoxon test statistic is :

(A) 6

(B) 8

(C) 10

(D) 12

123. There are four jobs J_1, J_2, J_3, J_4 that need to be processed on two machines M_1 and M_2 in sequence. The processing times are given in the following table :

	J_1	J_2	J_3	J_4
M_1	6	10	4	7
M_2	4	8	9	2

The total processing time to complete all the jobs on both the machines is :

(A) 30

(B) 32

(C) 33

(D) 35

124. Re-order level for an item is always :

(A) Less than minimum stock

(B) More than minimum stock

(C) Same as the minimum stock

(D) More than the average stock

125. Consider the following two M/M/1 queueing systems where :

In Q_1 : arrival rate λ, service rate μ

In Q_2 : arrival rate 2λ, service rate 2μ

Let W_1 and W_2 be the average waiting times of an arriving customer in Q_1 and Q_2 respectively. Then :

(A) $W_1 < W_2$

(B) $W_1 = W_2$

(C) $W_1 > W_2$

(D) The relation between W_1 and W_2 cannot be decided

126. Consider a population of $N = nk$ units with $k > 1$. A sample of size n is obtained by the following method :

(*i*) A unit is drawn from the set of units labelled $\{k, 2k, ..., N\}$ with probability $1/n$ and is included in the sample.

(*ii*) A SRSWOR of size $n - 1$ is then obtained from the remaining $N - 1$ units in the population and these units are included in the sample.

Then,

(A) The sampling method corresponds to a systematic sample

(B) There is no unbiased estimator of the population total

(C) Inclusion probability is not the same for all the population units

(D) The sample mean is an unbiased estimator of the population mean

127. Let V_{SY}^2 be the variance of the usual estimator in a systematic sampling design and V_{SRS}^2 the variance of the sample mean in a SRSWOR design. It is known that the population has a linear trend, i.e., y_i is approximately equal to ic, for a positive c, $i = 1, 2, ..., N$. Then :

(A) $V_{SY}^2 = V_{SRS}^2$

(B) $V_{SY}^2 < V_{SRS}^2$

(C) $V_{SY}^2 > V_{SRS}^2$

(D) It is not possible to compare the two estimators based on the given information

128. Under one way fixed effects ANOVA which of the following parametric functions are estimable, where the model is :

$$y_{ij} = \mu + \alpha_i + \in_{ij} \quad \begin{matrix} i = 1 \,.........\, p \\ j = 1 \,........\, n_i, \end{matrix}$$

$\in_{ij}$ are independent identically distributed with zero mean and finite variance.

(A) $\mu + \alpha_1 - \alpha_2$

(B) $\alpha_1 + \alpha_3$

(C) $2\alpha_1 - \alpha_2$

(D) $\alpha_1 + \alpha_p - 2\alpha_3$

129. Under a randomized block design with V treatments and r replication which of the following is most *correct* ?

(A) All pairwise contrasts $\alpha_i - \alpha_j$ among the unknown treatment effects α. $i = 1 \ldots\ldots V$ are estimable and are estimated with common variance

(B) All contrasts of the type $\sum_{i=1}^{V} C_i\alpha_i, \sum_{i=1}^{V} C_i = 0, \sum_{i=1}^{V} C_i^2 = 1$ are estimable and are estimated with common variance

(C) The best linear unbiased estimators $\hat{\alpha}_i - \hat{\alpha}_j \ 1 \leq i \neq j \leq V$ are all uncorrelated with each other

(D) All the above (A), (B), (C) are correct

130. Under an unconfounded 2^3 factorial experiment with factors, A, B, C in two replicates arranged in a randomized block design which of the following statements is *not* correct ?

(A) All factorial effects are estimated with same precision

(B) Best linear unbiased estimators of the 7 factorial effects are mutually uncorrelated with each other

(C) The error sum of squares has an additional degree of freedom than that of the treatment sum of squares

(D) Total number of degrees of freedom equal twice those carried by the treatment sum of squares

131. Let **B** be the σ-field of subsets of R generated by $\{(a, b) | a, b \in R\}$. Which of the following is *not true* ?

(A) Every countable subset of R ∈ **B**

(B) All closed intervals ∈ **B**

(C) All uncountable subsets of R ∈ **B**

(D) Intervals of the form $[a, b) \in$ **B**

132. Consider the following set function μ on a σ-field $\mathfrak{I}$

μ(A) = 0 if A is empty

= 1 if A is non-empty and finite

= ∞ if A is infinite

Then :

(A) μ is not monotone

(B) μ is not σ-additive

(C) μ is finite

(D) μ is a measure in $\mathfrak{I}$

133. Let R be the set of real numbers, **B** the Borel σ-field. Let 'm' denotes the Lebesgue measure and μ the measure defined by the function $F(x) = (1 - e^{-x})$ if $x > 0$ and $F(x) = 0$ elsewhere.

Consider the statements :

(I) $\mu << m$, (i.e. μ is absolutely continuous w.r.t. m)

(II) $m << \mu$

(III) $\frac{d\mu}{dm}(x) = e^{-x}$ $x > 0$, $\frac{d\mu}{dm}(x) = 0$, $x \leq 0$

Which of the above statements are *true* ?

(A) (I) and (II) only

(B) (I) and (III) only

(C) (II) and (III) only

(D) All the three

134. Let $\{X_n, n \geq 1\}$ be a sequence of independent identically distributed random variables with mean 0 and finite variance $\sigma^2 > 0$:

Let $S_n = \dfrac{X_1 + X_2 + ... + X_{n^2}}{n^2}$

Then, which of the following is *false* ?

(A) $P\left[\lim_{n\to\infty} S_n e^{S_n} = 0\right] = 1$

(B) As $n \to \infty$, $S_n e^{S_n}$ converges to a normal random variable with mean '0' and variance σ^2

(C) As $n \to \infty$, S_n converges to '0' in probability

(D) As $n \to \infty$, e^{S_n} converges to 1 with probability 1

135. Let $\{X_n, n \geq 1\}$ be a sequence of i.i.d. random variables with mean 3 and variance 11. Which of the following is *not* always correct ? As $n \to \infty$.

(A) $\dfrac{1}{n}\sum_{k=1}^{n} X_k^2$ converges in probability to 2

(B) $\left(\dfrac{1}{n}\sum_{k=1}^{n} X_k\right)^2$ converges in probability to 9

(C) $\dfrac{1}{n}\sum_{k=1}^{n} (X_k - 3)^2$ converges in probability to '0'

(D) $\sum_{k=1}^{n}\left(\dfrac{X_k}{n}\right)^2$ converges in probability to '0'

136. Let $\{X_n, n \geq 1\}$ be a sequence of random variables such that $P\left[\lim_{n\to\infty} X_n = X\right] = 1$ for some r.v. X. Then which of the following is *not* always *correct* ?

(A) $\lim_{n\to\infty} E\left[\cos(X_n)\right] = E[\cos(X)]$

(B) $\lim_{n\to\infty} E\left[\exp(it\, X_n)\right] = E[\exp(it\, X)]$

(C) $P\left[\lim_{n\to\infty} \exp(X_n) = \exp(X)\right] = 1$

(D) $\lim_{n\to\infty} E[\exp(X_n)] = E[\exp(X)]$

137. Let $\{X_n\}$ be a sequence of independent and uniformly bounded random variables, and $E[X_n] = 0$, and $Var(X_n) = \frac{1}{(2n)}$ for each n.

Let $S_n = X_1 + ... + X_n$ and $\sigma_n^2 = Var(S_n)$

Then which of the following is *not* true ?

(A) $\frac{S_n}{\sigma_n}$ does not converge in distribution

(B) $\frac{S_n}{\sigma_n}$ converges in distribution to a normal random variable

(C) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sigma_n^3} E[|X_k|^3] = 0$

(D) $\lim_{n\to\infty} \frac{S_n}{\sigma_n^2} = 0$ in probability

138. Let $\{X_n\}$ be a sequence of independent random variables such that $E[X_n] = 0$ and $Var(X_n) = \frac{1}{n^2}$, $n = 1, 2, ...$ Let $E = \left\{\omega \mid \sum_n X_n(w)\right.$ $\left. \text{converges}\right\}$.

Then which of the following is always *correct* ?

(A) $0 < P(E) < 1$

(B) $P(E) = \frac{1}{2}$

(C) $P(E) = 0$

(D) $P(E) = 1$

139. Let Y_1, Y_2 .... be a sequence of independent random variables with mean zero and let $\mathbf{F}_n = \sigma(Y_1, ..., Y_n)$.

Define $X_n = \sum_{k=1}^{n} Y_k$; $W_1 = Y_1$ and $W_{n+1} = W_n + Y_{n+1} W_n^2$ and $T_n = \sum_{k=1}^{n} (Y_k + 1)$. Then :

(A) Both $\{X_n\}$ and $\{W_n\}$ are $\mathbf{F}_n$-martingales

(B) Only $\{X_n\}$ is a $\mathbf{F}_n$-martingale

(C) Both $\{X_n\}$ and $\{T_n\}$ are $\mathbf{F}_n$-martingales

(D) All the three sequences are $\mathbf{F}_n$-martingales

140. Suppose a distribution function $F : \mathbf{R} \to [0, 1]$ of a random variable X is as follows :

$$F(x) = \begin{cases} 0, & \text{if} \quad x < -2, \\ 1/3, & \text{if} \quad -2 \le x < 0, \\ 1/2, & \text{if} \quad 0 \le x < 5, \\ 1/2 + (x-5)^2/2, & \text{if} \quad 5 \le x < 6, \\ 1, & \text{if} \quad x \ge 6. \end{cases}$$

Then E(X) is :

(A) $\dfrac{11}{6}$

(B) $\dfrac{1}{6}$

(C) $\dfrac{125}{6}$

(D) $\dfrac{13}{6}$

141. Which of the following statements is *correct* ?

The set of points of continuities of a distribution function is :

(A) uncountable

(B) countable

(C) finite

(D) empty

142. Suppose X and Y are two random variables whose third order moments exist. Then $(E(|X + Y|^3))^{1/3}$ is less than or equal to :

(A) $(E(|X|^3))^{1/3} \ (E(|Y|^3))^{1/3}$

(B) $(E(|X|^3))^{1/3} + (E(|Y|^3))^{1/3}$

(C) $(E(|X|) + E(|Y|))^{1/3}$

(D) $(E|X| \ . \ E|Y|)^{1/3}$

143. Suppose X is an absolutely continuous random variable with probability density function $f(.)$ and characteristic function $\phi(.)$. Suppose $\phi(.)$ is a Riemann integrable on R. Then :

(A) $f(x) = \dfrac{1}{2\pi} \displaystyle\int_R e^{iux} \phi(u) du$

(B) $f(x) = \displaystyle\int_R e^{-iux} \phi(u) du$

(C) $f(x) = \dfrac{1}{2\pi} \displaystyle\int_R e^{-iux} \phi(u) du$

(D) $f(x) = \dfrac{1}{2\pi} \displaystyle\int_0^\infty e^{-iux} \phi(u) du$

144. It is given that the distribution of Y given X = x is Normal (x, x^2). Suppose X ~ U(0, 1). Then, which of the following statements is *not* true ?

(A) E(Y) = 1/2

(B) Var(Y) = 5/12

(C) Cov(X, Y) = 3/12

(D) Corr(X, Y) = 1

145. Let $\phi_{\underline{X}}(\underline{t})$ be the characteristic function of $\underline{X}$ ~ $N_3(\underline{\mu}, \Sigma)$. Then $EX_1X_2^2X_3$ is given by :

(A) $\left.\dfrac{(-1)^4 \partial^4 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2^2 \partial t_3}\right|_{\underline{t}=\underline{0}}$

(B) $\left.\dfrac{(-1)^3 \partial^3 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2 \partial t_3}\right|_{t_1=1,\, t_2=2,\, t_3=1}$

(C) $\left.\dfrac{(-1)^3 \partial^4 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2^2 \partial t_3}\right|_{t_1=1,\, t_2=2,\, t_3=1}$

(D) $\left.\dfrac{\partial^4 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2^2 \partial t_3}\right|_{t_1=1,\, t_2=2,\, t_3=1}$

146. Let $f(x\,|\,\theta) = e^{-(x-\theta)}\ x \geq \theta$ and let the prior for θ be given by $\pi(\theta) = e^{-\theta}$ $0 < \theta < \infty$. Then :

(A) the posterior distribution for θ does not exist

(B) the posterior distribution for θ is U(0, x).

(C) the posterior distribution for θ is exponential distribution

(D) the posterior distribution for θ exists but its mean is not finite

147. Suppose X_1, ..., X_n are independent random variables with probability density function

$$f(x_i, \theta) = \begin{cases} \dfrac{1}{2i\theta}, & \text{if } -i\theta < x_i < i\theta \\ 0, & \text{otherwise.} \end{cases}$$

Then the minimal sufficient statistic for θ is :

(A) $\{X_{(1)}, \ldots\ldots\ X_{(n)}\}$

(B) $\max\{|X_1|, |X_2|/2, \ldots.. |X_n|/n\}$

(C) $\max\{|X_1, X_2/2, X_3/3, \ldots.. X_n/n\}$

(D) $\min\{|X_1, X_2/2, X_3/3, \ldots.. X_n/n\}$

148. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample of size n from the $N(\mu, \sigma^2)$ distribution, $\mu \in \mathbf{R}$, $\sigma^2 > 0$. Suppose $\hat{\sigma}_n^2$ and T_n denote the maximum likelihood estimator and the UMVUE of σ^2 respectivley. Which of the following statements is *correct* ?

(A) $\hat{\sigma}_n^2$ has smaller variance than that of T_n

(B) $\hat{\sigma}_n^2$ has smaller MSE than that of T_n

(C) $\hat{\sigma}_n^2$ has larger variance than that of T_n

(D) $\hat{\sigma}_n^2$ and T_n have the same MSE

149. Suppose $f(x, \theta)$ is a probability density function of X for which differentiation under integral sign is permissible.

Then $E\left(\frac{d}{d\theta}\log f(X, \theta)\right)$ is :

(A) equal to the Fisher information

(B) less than zero

(C) less than 1

(D) equal to zero

150. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample Poisson distribution with mean θ. Then the uniformly most powerful test for testing $H_0 : \theta = 1$ against $H_1 : \theta > 1$:

(A) rejects H_0 if $\bar{X} < c_1$ or $\bar{X} > c_2$ for some c_1, c_2

(B) rejects H_0 if $\bar{X} < c$ for some c

(C) rejects H_0 if $\bar{X} > c$ for some c

(D) rejects H_0 if $c_1 < \bar{X} < c_2$ for some c_1, c_2

151. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample of size n from the $U(0, \theta)$ distribution. Then :

(A) Sample mean $\bar{X}_n$ is consistent for θ

(B) Sample median is cosistent for θ

(C) $2\bar{X}_n$ is consistent for θ

(D) $X_{(1)}$ is consistent for θ

152. Suppose $\{X_1, X_2, ..., X_n\}$ are independent and identically distributed random variables each having the probability density function :

$$f(x) = \frac{\theta}{x^2}, \; x \geq \theta$$

Then :

(A) $\sqrt{n}\left(X\left(\left[\frac{n}{4}\right]+1\right) - \frac{4\theta}{3}\right) \underset{\to}{L} z \sim N(0, 16\;\theta/27)$

(B) $\sqrt{n}\left(X\left(\left[\frac{n}{4}\right]+1\right) - \frac{4\theta}{3}\right) \underset{\to}{L} z \sim N(0, \theta/3)$

(C) $\sqrt{n}\left(X\left(\left[\frac{n}{4}\right]+1\right) - \frac{4\theta}{3}\right) \underset{\to}{L} z \sim N(0, 16\;\theta^2/27)$

(D) $\sqrt{n}\left(X\left(\left[\frac{n}{4}\right]+1\right) - \frac{4\theta}{3}\right) \underset{\to}{L} z \sim N(0, 16\;\theta/27)$

153. Let $X_1, X_2, ..., X_n$ be *iid* $f(x, \theta, \alpha)$, where :

$$f(x; \theta, \alpha) = \frac{e^{-x/\theta}\, x^{\alpha-1}}{\Gamma(\alpha)\, \theta^{\alpha}}.$$

Then, $\sqrt{n}(\log \bar{X} - \log \alpha\theta) \underset{\to}{d} z$, where z is distributed as :

(A) $N(0, 1)$

(B) $N(0, 1/\alpha)$

(C) $N(0, \alpha^2/\theta^2)$

(D) $N(0, \theta^2/\alpha^2)$

154. Let $X_1, X_2, ..., X_n$ be *iid* $U(0, \theta)$

Define $T_1 = \bar{X}$ and $T_2 = (X_{(1)} + X_{(n)})/2$. Where $X_{(1)} = \min(X_1, ... X_n)$, $X_{(n)} = \max(X_1 .... X_n)$. Then :

(A) both T_1 and T_2 are equally efficient when $n = 2$

(B) $ARE(T_1, T_2) > 1$ when $n > 2$

(C) $ARE(T_1, T_2) < 1$ for all n

(D) $ARE(T_1, T_2) \not\to 0$ as $n \to \infty$

155. For a sample correlation coefficient r, it is known that :

$$\sqrt{n}\,(r - \rho) \underset{\to}{d} N(0, (1 - \rho^2)^2),$$

where ρ is the population correlation coefficient. For what transformation $g(.)$,

$$\sqrt{n}(g(r) - g(\rho)) \underset{\to}{d} N(0, 1) ?$$

(A) $\log \frac{1+\rho}{1-\rho}$

(B) $\frac{1}{2} \log \frac{1-\rho}{1+\rho}$

(C) $\frac{1}{2} \log(1+\rho)$

(D) $\frac{1}{2} \log \frac{1+\rho}{1-\rho}$

156. Let $\underline{X} \sim N_4(\underline{\mu}, \Sigma)$ where eigen values of Σ are given by $\lambda_1 = 6$, $\lambda_2 = 4$, $\lambda_3 = 3$, $\lambda_4 = 2$. Let Y_i be the principal component corresponding to λ_i, $i = 1, 2, 3, 4$. Then which of the following statements is *not* correct ?

(A) Y_i, $i = 1, ..., 4$ are mutually independently distributed

(B) Y_1, Y_2, Y_3, Y_4 are identically normally distributed

(C) The percentage of variation explained by Y_1 and Y_2 is greater than 60%

(D) Y_4 explains the smallest portion of variance

157. Let $\underline{X} \sim N_n(\underline{\mu}, \Sigma)$ where Σ is a +ve definite matrix. Then which of the following is a necessary condition for $\underline{Y} = A\underline{X}$ to have independently distributed marginal distributions ?

(A) A is any non-singular matrix

(B) A is an idempotent matrix

(C) A is an orthogonal matrix

(D) Columns of A form orthonormal eigen vectors of Σ

158. Let $\underline{X} \sim N(\underline{0}, \Sigma)$ where Σ is a +ve definite matrix, and A is an idempotent matrix of rank $k < P$. Let $Z = X^1\Sigma^{-1}AX$. Then mean and variance of z respectively equal :

(A) n, $2n$

(B) k, $2k$

(C) k, k

(D) n, n

159. Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ with

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{bmatrix}$$

Then the conditional distribution of (X_1, X_2) given $X_3 = x_3$ is $N_2(\underline{\mu}^*, \Sigma^*)$ where $\underline{\mu}^*$ and Σ^* are respectively given by :

(A) $\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$

(B) $\begin{bmatrix} \mu_1 + \rho^2(x_3 - \mu_3) \\ \mu_2 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$

(C) $\begin{bmatrix} \mu_1 + \rho^2(x_3 - \mu_3) \\ \mu_2 \end{bmatrix}, \begin{bmatrix} 1-\rho^4 & \rho \\ \rho & 1 \end{bmatrix}$

(D) $\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} 1-\rho^4 & \rho \\ \rho & 1 \end{bmatrix}$

160. Let $\underline{X} \sim N_n(\underline{0}, \sigma^2 I_n)$.

Let $Y = \sum_{i=1}^{n} X_i^2/\sigma^2$ and $Q_i = \underline{X}' A_i \underline{X}$ with rank $(A_i) = r_i, i = 1, 2, ..., k$. Let $Y = \sum_{i=1}^{k} Q_i$. Then $Q_1, Q_2, ..., Q_k$ are independently distributed if and only if :

(A) $r_i > 0 \ \forall \ i$ and $\sum_{i=1}^{k} r_i = n$

(B) $r_i > 0 \ \forall \ i$ and $\prod_{i=1}^{k} r_i = n$

(C) $A_1, ..., A_n$ are all positive semidefinite matrices and $\sum_{i=1}^{n} r_i = n$

(D) Each $Q_1, Q_2, ..., Q_k$ have χ^2 distribution and $\sum_{i=1}^{n} r_i \le n$

161. y_1, y_2, y_3 are three uncorrelated random variables with common variance σ^2 and $E(y_1) = \theta_1 + \theta_3$ $E(y_2) = \theta_2 + \theta_3$, $E(y_3) = \theta_1 - \theta_3$. Hence :

(A) Best Linear Unbiased Estimator (BLUE) of θ_2 is y_2 with variance σ^2

(B) BLUE of θ_2 is $y_2 - \frac{(y_1 - y_3)}{2}$ with variance $\frac{3}{2}\sigma^2$

(C) BLUE of θ_1 is $\frac{y_1 + y_3}{2}$ with variance σ^2

(D) BLUE of θ_1 is $\frac{y_1 + y_2}{2}$ with variance $\frac{3\sigma^2}{2}$

162. In a simple linear regression analysis the fitted line using least squares theory will :

(A) definitely pass through the point $(\bar{x}\ \bar{y})$ when the intercept is present

(B) definitely pass through the point $(\bar{x}\ \bar{y})$

(C) never pass through the point $(\bar{x}\ \bar{y})$

(D) definitely pass through the point $(\bar{x}\ \bar{y})$ if slope is positive

163. In a multiple linear regression set up $\underline{y} = \mathrm{X}\underline{\mathrm{B}} + \underline{\mathrm{E}}$ under the assumption E($\underline{\mathrm{E}}$) = 0, Var($\underline{\mathrm{E}}$) = σ^2. In the response variables y_1 .... y_n are :

(A) identical and independent

(B) non-identical uncorrelated

(C) non-identical and independent

(D) identical and uncorrelated

164. Consider a multiple linear regression model $\underline{y} = \mathrm{X}\underline{\mathrm{B}} + \underline{\mathrm{E}}_{n+1}$ and suppose $\hat{y}_i$ and e_i i = 1, ..., n are the fitted values of response variable y and residuals respectively. Then :

(A) $\sum_{i=1}^{n} e_i = 0$

(B) $\sum_{i=1}^{n} e_i^2 = n\sigma^2$

(C) $\sum_{i=1}^{n} e_i y_i = 0$

(D) $\sum_{i=1}^{n} e_i y_{\hat{i}} = 0$

165. Consider a two-way classification model $y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$ i = 1 ... v, j = 1, ..., b with E(ε_{ij}) = 0, Var(ε_{ij}) = σ^2, ε_{ij} uncorrelated where τ_i δ = 1 ..... v are fixed effects and β_j j = 1, ..., b are random effects with mean 0 and variance σ_β^2. Hence the hypothesis regarding the insignificance of random effects is :

(A) $H_0 : \beta_1 = \beta_2 = ... = \beta_b$

(B) $H_0 : \sigma_\beta^2 = 0$

(C) $H_0 : \tau_1 = \tau_2 = ... = \tau_v$

(D) $H_0 : \sigma^2 = 0$

166. A systematic sample of size 2 is drawn from a finite population with N = 4 and k = 2. The samples are drawn independently of each other. The probability that the two units (2, 4) are included in sample equals :

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{1}{4}$

167. The variance of the usual estimator in a stratified sampling design is given by :

$$V^2 = \sum_{h=1}^{L} \frac{W_h^2 S_h^2}{n_h}(1 - f_h)$$

in the standard notations. We wish to obtain an allocation that minimizes the sample size. Then,

(A) such an optimal allocation does not exist

(B) proportional allocation is optimal

(C) the optimal allocation is the one for which n_h is proportional to S_h

(D) the optimal allocation is the one for which n_h is proportional to $W_h S_h$

168. Let us consider the two sampling strategies, both based on a sample of size n : I. SRSWOR II post-stratification. To estimate the population total :

(A) If the stratum sizes are sufficiently large for each stratum and if the error effects in W_h can be ignored, post-stratification is better than SRSWOR

(B) SRSWOR is always better than post-stratification

(C) If n is sufficiently large, post-stratification is better than SRSWOR

(D) Post-stratification is always better than SRSWOR

169. The incidence matrix of a general block design is :

$$N = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Hence the number of estimable orthogonal block contrasts is :

(A) 2

(B) 1

(C) 3

(D) 4

170. Consider a BIBD with parameter (v, b, r, k, λ). If $v = b = 11$ and $r = 5$, then :

(A) $k = 5, \lambda = 2$

(B) $k = 5, \lambda = 1$

(C) $k = 2, \lambda = 5$

(D) $k = 11, \lambda = 1$

171. In a M/M/K queueing system the departure rate μn when there are n customers in the system is :

(A) μ

(B) $n\mu$

(C) $n\mu$ if $n \leq k$ and 0 if $n > k$

(D) $n\mu$ if $n \leq k$ and nk if $n > k$

172. In a 2^4 factorial design with two blocks of 8 plots each the treatment combinations are allotted as below :

Block I	**Block II**
a	*1*
b	*ab*
ac	*c*
bc	*abc*
d	*ad*
abd	*bd*
cd	*acd*
abcd	*bcd*

Hence the confounded treatment combination is :

(A) ABD

(B) ACD

(C) BCD

(D) ABCD

173. Let $\{X_t\}$ be a stationary AR(1) time series given by the recursive equation $X_t = \phi\ X_{t-1} + Z_t$, where Z_t ~ W N $(0, \sigma^2)$. Let P_3X_4 represesents the forecast of X_4 given the observations (X_1, X_2, X_3). If $C(\phi)$ is defined as :

$$C(\phi) = \frac{E(X_4 - P_3X_4)^2}{E(X_4 - E(X_4))^2},$$

then, which of the following is *not true* ?

(A) $C(0) = 1$

(B) $0 \leq C(\phi) \leq 1$

(C) $C(\phi) > 0$ when $\phi > 0$

(D) $C(\phi) < 0$ when $\phi > 0$

174. Let $\{X_i, i = 1, 2, ... 10\}$ be a time series with mean zero, variance one and equicorrelation $\frac{1}{2}$. Let $\bar{X}_{10} = \frac{1}{10}\sum_{i=1}^{10} X_i$, then variance of $\bar{X}_{10}$ is :

(A) 0.1

(B) 0.325

(C) 0.2

(D) 0.125

175. Let $\{X_t\}$ be a stationary time series model given by :

$X_t = \mu + \phi X_{t-1} + Z_t$, Z_t ~ *iid* normal $(0, \sigma^2)$. Then, which of the following statements is *not* true if $P_{X_t}(X_{t+k})$ is the forecast of X_{t+k}. Given $X_1, X_2 ..... X_t$?

(A) The forecast mean square error of $P_{X_t}(X_{t+k})$ approaches to $\mu^2 + \sigma^2/(1 - \phi^2)$

(B) The forecast mean square error of $P_{X_t}(X_{t+k})$ approaches to $\sigma^2/(1 - \phi^2)$

(C) $P_{X_t}(X_{t+k}) \rightarrow E(X_t)$ as $k \rightarrow \infty$

(D) The forecast $P_{X_t}(X_{t+k})$ is a weighted average of $E(X_t)$ and the latest observation X_t

176. Let $\{X_n\}_0^\infty$ be a Markov chain with $P[X_0 = 0] = P[X_0 = 1] = \frac{1}{4}$ and $P[X_0 = 2] = \frac{1}{2}$. The t.p.m. is given by :

$$p = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \end{array}$$

Then $P[X_1 = 1]$ is given by :

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{5}$ (D) $\frac{1}{6}$

177. Let $\{X(t), t \geq 0\}$ be a time homogeneous Poisson process with rate λ. Let $X(T) = n > 0$ and let $T_1 < .... < T_n < T$ be times at which these n events occur. Then, $E(T_1 | X(T) = n)$ is given by :

(A) $\frac{T}{n+1}$ (B) $T.\lambda^n$

(C) $\frac{n}{n+1} - T$ (D) $\frac{1}{2T}$

178. Consider a Branching process with μ as the mean of the off-spring distribution. Let $\mu = \frac{1}{2}$ and $X_0 = 1$. Then $E\left(\sum_0^\infty X_n\right)$ equals :

(A) 1 (B) $1\frac{1}{2}$

(C) 2 (D) ∞

179. Consider a Markov chain on $S = \{0, 1, 2, ...\}$ with $p_{01} = 1$, $p_{i0} = \frac{3}{4}$, $p_{i(t)} = \frac{1}{4}$ $\forall$ $i = 1, 2$ .... then :

(A) all the states are persistent non-null

(B) all the states are persistent

(C) all the states are transient

(D) all the states do not communicate with each other

180. The denominator in infant mortality rate is :

(A) total number of live births in a given period

(B) total number of live births and still births in a given period

(C) total number of pregnancies in a given period

(D) total population in a given period

181. In terms of life table functions, the chance that a child just born to a mother of age 31 and a father of age 33 will be alive till 40 years but will be orphaned by both parents is :

(A) $\left(\frac{l_{40}}{l_0}\right)\left(\frac{l_{71}-l_{31}}{l_0}\right)\left(\frac{l_{73}-l_{33}}{l_0}\right)$

(B) $p_{40}q_{31}.q_{33}$

(C) $\left(\frac{L_{40}}{l_0}\right)\left(\frac{L_{71}-L_{31}}{l_0}\right)\left(\frac{L_{73}-L_{33}}{l_0}\right)$

(D) $\left(\frac{l_{40}}{l_0}\right)\left(\frac{l_{71}-l_{31}}{l_{31}}\right)\left(\frac{l_{73}-l_{33}}{l_{33}}\right)$

182. For a production process, the sample ranges are found to be 1.2, 1.5, 1.1, 1.4 and 1.5. Suppose that the subgroup size is 5. Then, what will be the process standard deviation, given that $A_2 = 0.577$ and $d_2 = 2.326$?

(A) 0.511

(B) 2.463

(C) 2.322

(D) 0.576

183. The respective failure rate functions of two components are $r_1(t) = \sqrt{t}$ and $r_2(t) = 1\sqrt{t}$. Then the life time distribution of a series system with the above components is :

(A) IFR

(B) DFR

(C) Neither IFR nor DFR

(D) Both IFR and DFR

184. In the inventory model $S = Q - u.t$ with $Q = 1000$, $u = 40$ and $t = 10$, the current stock level is :

(A) 400

(B) 1200

(C) 1000

(D) 600

185. Which of the following is an example of the control of the queue discipline ?

(A) Customer is allowed to leave the facility

(B) The number of servers are varied depending on the queue length

(C) Separate server is allocated for special category customers

(D) The mean arrival rate is adjusted

186. Let D_n be the waiting time of the nth customer in G|M|1 queue. Consider the following recurrence equation :

$$D_{n+1} = \begin{cases} D_n + V_n - U_n & \text{if } D_n + V_n - U_n \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Which of the following interpretations of the variables in the above equation are valid ?

(*i*) U_n is the interarrival time between nth and $(n + 1)$th customer

(*ii*) $D_n + V_n$ is the departure time of the nth customer from the system

(A) Both are not valid

(B) Both are valid

(C) Only (*i*) is valid

(D) Only (*ii*) is valid

187. Suppose that R is the shortest route from city 1 to city 10 passing through some other cities. The principle of optimality implies that :

(*i*) For every city j on R, the subpath in R from city j to city 10 must be the shortest.

(*ii*) For every city j on R, there must be a unique path from city j to city 10.

Which of the above statements is *correct* ?

(A) (*i*) only

(B) (*ii*) only

(C) Both (*i*) and (*ii*)

(D) Neither (*i*) nor (*ii*)

188. By the principle of optimality, the minimum value of $y_1^2 + y_2^2$ s.t. $y_1 y_2 = c, y_1 \geq 0, y_2 \geq 0$ is :

(A) $2c^2$

(B) $2c$

(C) $\left(\frac{c}{2}\right)^2$

(D) $c^2/2$

189. Which of the following is *not* true regarding the Karmarkar's algorithm ?

(A) The algorithm starts with an interior point of the feasible region

(B) One of the basic conditions is that $x = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ satisfies the equation Ax = 0.

(C) Constraints are assumed to be non-homogeneous

(D) The algorithm reduces the number of iterations

190. What is the failure rate function of the series system of two independent components when the failure rate functions of the two components are respectively $r_1(t)$ and $r_2(t)$?

(A) $\min(r_1(t), r_2(t))$

(B) $r_1(t) + r_2(t)$

(C) $r_1(t) \times r_2(t)$

(D) $\max(r_1(t), r_2(t))$

ROUGH WORK

ROUGH WORK