Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

A

Paper-II
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

JUN - 30219

Time Allowed : 2 Hours] **[Maximum Marks : 200**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **190**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **190** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. Use only Blue/Black Ball point pen.
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 190 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.
   (*iii*) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरिक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

Mathematical Science

Paper II

Time Allowed : 120 Minutes] **[Maximum Marks : 200**

Note : This Paper contains **One Hundred Ninety (190)** multiple choice questions in THREE (3) sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

Section I : Q. Nos. 1 to 10, **Section II : Q. Nos. 11 to 100,** **Section III : Q. Nos. 101 to 190.**

SECTION I

1. Let for $n \in \mathbf{N}$, $I_n = \left(0, \frac{1}{n}\right)$, $J_n = \left[0, \frac{1}{n}\right]$, $K_n = (n, \infty)$ and $L_n = [n, \infty)$. Which of the following sets is non-empty ?

(A) $\bigcap\limits_{n=1}^{\infty} I_n$

(B) $\bigcap\limits_{n=1}^{\infty} J_n$

(C) $\bigcap\limits_{n=1}^{\infty} K_n$

(D) $\bigcap\limits_{n=1}^{\infty} L_n$

2. Let $f(x, y) = |\sin x - \sin y|$ for $x, y \in \mathbf{R}$. Then :

(A) $f(x, y) \leq |x|$ for all x, y

(B) $f(x, y) \leq |x - y|$ for all x, y

(C) $f(x, y) \neq 0$ for $x \neq y$

(D) $f(x, y) \geq |y|$ for all x, y

3. Limsup of the sequence $\left\{-2, 2, -\frac{3}{2}, \frac{3}{2}, -\frac{4}{3}, \frac{4}{3} ....\right\}$ is :

(A) 3/2

(B) 2

(C) 1

(D) 0

4. One of the values of i^i is :

(A) $e^{-\pi/2}$

(B) $e^{-i\pi/2}$

(C) $e^{\pi/2}$

(D) $e^{i\pi/2}$

5. Let V denote the vector space of $n \times n$ symmetric complex matrices, over $\mathbf{R}$. Then dim V as a vector space over $\mathbf{R}$ is :

(A) n^2

(B) $\frac{n^2 + n}{2}$

(C) $n^2 + n$

(D) $n^2 - n$

6. Let $T : \mathbf{R}^n \to \mathbf{R}^n$ be defined as $T(\bar{e}_1) = \bar{0}$, $T(\bar{e}_j) = \bar{e}_{j-1}, j = 2, ..., n$ where $\{\bar{e}_1, \bar{e}_2, ..., \bar{e}_n\}$ is the standard basis of $\mathbf{R}^n$. Then :

(A) T is non-linear

(B) T is idempotent

(C) Ker T = {0}

(D) T is nilpotent

7. A coin is biased so that head is twice as likely to occur as a tail. If a coin is tossed four times, what is the probability of getting two tails and two heads ?

(A) $\frac{3}{8}$

(B) $\frac{8}{27}$

(C) $\frac{1}{8}$

(D) $\frac{4}{27}$

8. Suppose A and B are mutually exclusive events having probabilities P(A) = 0.25, P(B) = 0.35. What is the probability that A occurs but B does not ?

(A) 0.1

(B) 0.25

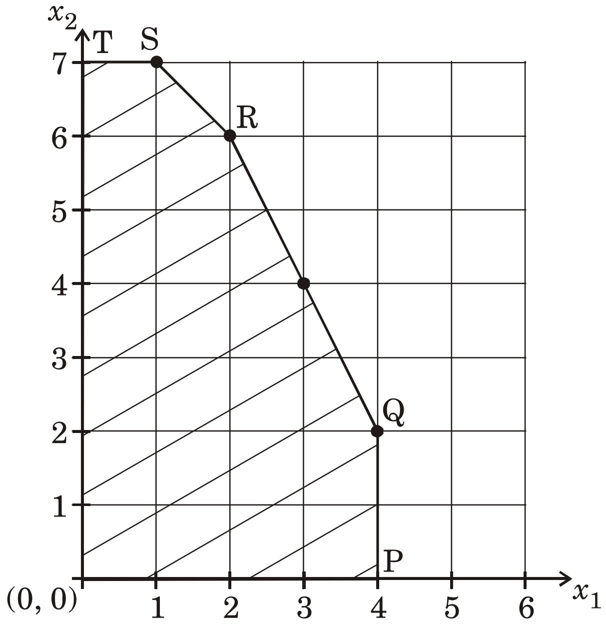
(C) 0.4

(D) 0.6

9. The graph given below shows the bounded feasible region (in shaded portion) for the problem :

Max. $z = 2x_1 + x_2 - 12,$

$x_1, x_2 \geq 0.$



The objective function attains its maximum when :

(A) $x_1 = 1$ and $x_2 = 7$

(B) $x_1 = 3$ and $x_2 = 4$

(C) $x_1 = 3$ and $x_2 = 3$

(D) $x_1 = 3$ and $x_2 = 5$

10. Which of the following statements is true with respect to the optimal solution of an LP problem ?

(A) Every LP problem has an optimal solution.

(B) Optimal solution of an LP problem occurs only at an extreme points of the convex set of feasible solutions.

(C) If optimal solution exists, then there will be always at least one at the corners of the set of feasible solutions.

(D) Every feasible solution is an optimal solution

SECTION II

11. Let X and Y be bounded subsets of **R** and let Z = X $\cup$ Y. Then :

(A) Z need not be bounded

(B) Sup Z = max {sup X, sup Y}

(C) Z need not have supremum

(D) Z has maximum

12. Let $f : [0, 1] \to \mathbf{R}$ be continuous and $\int_0^x f = \int_x^1 f$ for all $x \in [0, 1]$. Then :

(A) f is strictly monotonic

(B) f is constant

(C) $f(t) = t$ for all $t \in [0, 1]$

(D) $f(t) = t^2$ for all $t \in [0, 1]$

13. Suppose $f(x)$ is defined on [0, 1] as follows :

$$f(x) = \begin{cases} 0, & \text{if } x \text{ irrational} \\ 1/q, & \text{if } x \text{ rational} \end{cases}$$

and $x = \dfrac{p}{q}$ with $(p, q) = 1$. Then f is :

(A) Riemann integrable

(B) Continuous at rational points

(C) Discontinuous at irrational points

(D) Discontinuous everywhere

14. Consider the function :

$$f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } 0 < x \le 1 \\ 0, & \text{if } x = 0 \end{cases}$$

Then :

(A) f is not continuous at $x = 0$

(B) f is continuous but not differentiable at $x = 0$

(C) f is differentiable but not of bounded variation

(D) f is of bounded variation

15. Let A and B be two non-empty disjoint subsets of **R** and let $d(A, B) = \inf\{|a - b| : a \in A, b \in B\}$. Then $d(A, B) > 0$ if :

(A) A and B are open

(B) A and B are closed

(C) A is closed and B is compact

(D) A or B is singleton

16. The number of Mobius transformations which map the real line onto the unit circle is :

(A) 0

(B) 1

(C) 2

(D) Infinitely many

17. The set $\{z \in \mathbf{C} : |e^z| = 2019\}$ represents :
(A) the line $x = \log(2019)$
(B) the line $y = 2019$
(C) a circle
(D) a countable set

18. For $z = x + iy$, let $f(z) = x^2 + y^2 + 2xyi$. Then :
(A) f is everywhere analytic
(B) f is nowhere analytic
(C) f is analytic only at $z = 0$
(D) f is analytic at every point on the real axis and nowhere else

19. Let $\nu : [0, 1] \to \mathbf{C}$ be defined by $\nu(t) = e^{4\pi it}$. Then :
$$\frac{1}{2\pi i}\int_{\nu} \frac{e^z}{z^3}dz =$$
(A) 0
(B) 1
(C) 2
(D) $4\pi i$

20. Let $f(z) = \operatorname{cosec}\left(\frac{1}{z-1}\right)$ for $z \neq 1$, $z \in \mathbf{C}$. Then at the point $z = 1$, f has :
(A) no singularity
(B) a removable singularity
(C) a pole
(D) a non-isolated singularity

21. Let G be a finite group of order $2n$. Then the number of elements of order 2 in G is :
(A) $2r$ for some $1 \leq r \leq n$
(B) $2r + 1$ for some $1 \leq r \leq n - 1$
(C) n if n is even
(D) $n + 1$ if n is odd

22. Let H be a proper subgroup of the additive group (Q, +) of rational numbers. Then the quotient group Q/H must be :
(A) finite of even order
(B) finite of odd order
(C) finite
(D) infinite

23. Let G be a group of order 61. Then the number of subgroups of G is :
(A) 61
(B) 2
(C) 7
(D) 1

24. Let G be the group of invertible 2×2 matrices over the field $\mathbf{Z}_2 = \{0, 1\}$. Then the number of elements in G is :
(A) 6
(B) 2
(C) 4
(D) 8

25. Which of the following is a PID ?

(A) $\mathbf{Z}[x]$

(B) $\mathbf{R}(x)\ [y]$

(C) $\mathbf{C}[x, y]$

(D) $\mathbf{Z}_{30}$

26. Let W be the subspace of $\mathbf{R}^4$ spanned by the vectors $[1\ 2\ 0\ 0]^t$, $[0\ 1\ 2\ 0]^t$ and $[1\ 1\ -2\ 0]^t$. The dimension of the quotient space $\mathbf{R}^4/\mathrm{W}$ is :

(A) 1

(B) 2

(C) 3

(D) 4

27. Which of the functions defines an inner product on $\mathbf{C}^2$? Let $x = [x_1, x_2]^t$, $y = [y_1, y_2]^t$?

(*i*) $(x, y) = x_1\bar{y}_2$

(*ii*) $(x, y) = x_1\bar{y}_1 + x_2\bar{y}_2$

(*iii*) $(x, y) = x_1y_1 + x_2y_2$

(A) (*ii*) and (*iii*)

(B) (*i*)

(C) (*ii*)

(D) (*iii*)

28. If $\mathrm{T} : \mathbf{R}^3 \to \mathbf{R}^2$ is given by :

$$\mathrm{T}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+z \\ y-z \end{pmatrix},$$

then the matrix representation of T with respect to standard bases of $\mathbf{R}^3$ and $\mathbf{R}^2$ is :

(A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{pmatrix}$

29. If a real matrix A has characteristic polynomial $(x - 1)(x^2 + 1)$, then which of the following statements about A is *true* ?

(A) A is diagonalizable over $\mathbf{R}$

(B) A is triangulable over $\mathbf{R}$

(C) A is nilpotent

(D) A is invertible

30. Let V, W be two complex vector spaces and $T \in L(V, W)$. If T has matrix representation :

$$\begin{bmatrix} 2 & 1+i & 3 \\ 4+i & 1-i & i \end{bmatrix}$$

Which of the following matrices represents the adjoint map T^* ?

(A) $\begin{pmatrix} 2 & 1-i & 3 \\ 4-i & 1+i & -i \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 4+i \\ 1+i & 1-i \\ 3 & i \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 4-i \\ 1-i & 1+i \\ 3 & -i \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 1+i & 3 \\ 4+i & 1-i & i \end{pmatrix}$

31. A set of all surfaces of revolution with z-axis as the axis of revolution is characterized by the partial differential equation :

(A) $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$

(B) $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = 0$

(C) $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$

(D) $y\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y} = 0$

32. The solution of the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

is :

(A) $z = x\phi_1(x+y) + y\phi_2(x-y)$

(B) $z = x\phi_1(x+y) + \phi_2(x-y)$

(C) $z = \phi_1(x+y) + \phi_2(x-y)$

(D) $z = \phi_1(x+y) + x\phi_2(x-y)$

33. An nth order linear ordinary differential equation :

(A) has exactly n linearly independent solutions

(B) has at most n linearly independent solutions

(C) has less than n independent solutions

(D) has minimum n linearly independent solutions

34. The solution of the differential equation

$$y^1 - 2xy = xy^2$$

is :

(A) $\left[-\frac{1}{2} + ce^{-x^2}\right]^2$

(B) $\left[-\frac{1}{2} + ce^{-x^2}\right]^{-1}$

(C) $\left[-\frac{1}{2} + ce^{x^2}\right]^{-1}$

(D) $\left[-\frac{1}{2} + ce^{x^2}\right]^2$

35. Let D be the rectangle :

$$\left\{(x, y) \in \mathbf{R}^2 \,\middle|\, |x| \le 1, |y| \le 1\right\}$$

and h and g are functions defined on D given by :

$h(x, y) = xy^2$ and

$g(x, y) = y^{2/3}$.

Then :

(A) Only h satisfies Lipschitz condition on D

(B) Only g satisfies Lipschitz condition on D

(C) Both h and g satisfy Lipschitz condition on D

(D) Neither h nor g satisfies Lipschitz condition on D

36. The assignment cost of assigning any one operator to any one machine is given in the following table :

Machines	Operators I	II	III	IV
U	10	5	13	15
V	3	9	18	3
W	10	7	3	2
X	5	11	9	7

The optimal assignment is :

(A) U $\to$ II, V $\to$ III, W $\to$ I, X $\to$ IV

(B) U $\to$ II, V $\to$ IV, W $\to$ III, X $\to$ I

(C) U $\to$ III, V $\to$ IV, W $\to$ II, X $\to$ I

(D) U $\to$ IV, V $\to$ II, W $\to$ III, X $\to$ I

37. Consider the following LP problem :

Max. : $Z = x_1 + x_{2/2}$

Subject to the constraints :

$$3x_1 + 2x_2 \le 12$$
$$5x_1 = 10$$
$$x_1 + x_2 \ge 8$$
$$-x_1 + x_2 \ge 4 \text{ and}$$
$$x_1, x_2 \ge 0.$$

Then the LP problem has :

(A) Feasible solution

(B) No feasible solution

(C) Degenerate feasible solution

(D) Non-degenerate feasible solution

38. Let the primal maximization LP problem has m constraints and n non-negative variables. Then consider the following two statements about it :

(I) The dual have n constraints and m non-negative variables.

(II) The dual is a minimization problem.

Which of the following is *true* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both are true

(D) Neither (I) nor (II) is true

39. Consider the LP problem :

Max. : $Z = 4x_1 + 2x_2$

Subject to the constraints :

$$-x_1 - x_2 \leq -3,$$
$$-x_1 + x_2 \geq -2 \text{ and}$$
$$x_1, x_2 \geq 0.$$

Then the dual of the above LP problem is :

Min. : $W = py_1 + qy_2$

Subject to the constraints :

$$ry_1 + sy_2 \geq 4$$
$$-y_1 - y_2 \geq 2$$
$$y_1, y_2 \geq 0,$$

where the values of p, q, r, s are :

(A) $p = 3, q = -2, r = -1, s = -1$

(B) $p = -3, q = -2, r = 1, s = -1$

(C) $p = 3, q = -2, r = 1, s = -1$

(D) $p = -3, q = 2, r = -1, s = 1$

40. If the dual problem has an unbounded solution, then primal has :

(A) No feasible solution

(B) Unbounded solution

(C) Feasible solution

(D) None of the above

41. Let $f : [a, b] \to \mathbf{R}$ be a continuous function. Then which of the following statements is *not* true for the image set of f, $I_m f$?

(A) $I_m f$ is unbounded

(B) $I_m f$ has maximum

(C) $I_m f$ is an interval

(D) $I_m f$ has minimum

42. The function $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $f(x, y) = (2xy, x^2 - y^2)$ is one-one on the set :

(A) $\left\{(x, y) \in \mathbf{R}^2 / y \geq 0\right\}$

(B) $\left\{(x, y) \in \mathbf{R}^2 / x^2 + y^2 \leq 1\right\}$

(C) $\left\{(x, y) \in \mathbf{R}^2 / \frac{1}{2} < x < 1, |y| < \frac{1}{4}\right\}$

(D) $\mathbf{R}^2$

43. Let $f : \mathbf{R}^3 \to \mathbf{R}^2$ be defined by the equation $f(x, y, z) = (xy, y + z)$ and $g : \mathbf{R}^2 \to \mathbf{R}^2$ be defined as $g(x, y) = (x + y, y)$. Then the derivative $D(g \circ f)\ (0, 0, 0) =$

(A) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

44. Let $f : \mathbf{C} \to \mathbf{C}$ be a bijective analytic function. Then :

(A) f must be a linear polynomial

(B) $f'(z) = 0$ for some z

(C) $f^{-1}(z)$ may not be continuous

(D) $f^{-1}(z)$ is continuous but may not be analytic

45. Which of the following is *not* possible for an analytic function f on $D = \{z : |z| < 1\}$?

(A) $f(D) = \{z : |z| \leq 1\}$

(B) $f(D) = D$

(C) $f(D) = \left\{z : |z| < \frac{1}{2}\right\}$

(D) $f(D) \subseteq \mathbf{R}$

46. Let $f(z)$ and $g(z)$ be two non-constant analytic functions on a region G. Then which of the following is possible ?

(A) $f - \bar{g}$ is analytic

(B) $f + \bar{g}$ is analytic

(C) $\bar{f} + \bar{g}$ is analytic

(D) $fg \equiv 0$

47. Let p be a prime number and $\mathbf{F}_p$ be a finite field with p elements. Then the unit group $\mathbf{F}_p^x = \mathbf{F}_p \setminus \{0\}$ of $\mathbf{F}_p$ is simple if and only if :

(A) $p = 2$

(B) $p = 3$

(C) $p \geq 5$

(D) p is of the form $2^n + 1$ for some $n \geq 2$.

48. Let x be the image of X in the quotient ring $A := \mathbf{R}[X, Y]/\langle X^2 + Y^2 - 1\rangle$. Then the element x is :
   (A) not irreducible in A
   (B) irreducible, but not prime in A
   (C) prime in A
   (D) a unit in A

49. The quadratic form associated to the trace form $\mathrm{Tr}_{\mathbf{R}}^{\mathbf{C}} : \mathbf{C} \times \mathbf{C} \to \mathbf{R}$, $(x, y) \mapsto \mathrm{Tr}(\lambda_{XY})$, where $\lambda_{XY} : \mathbf{C} \to \mathbf{C}$ is the **R**-linear map defined by $\lambda_{XY}(z) = xyz$, is :
   (A) X^2
   (B) Y^2
   (C) $X^2 + Y^2$
   (D) $X^2 - Y^2$

50. With the induced topology by the metric $d(x, y) = |x - y|$,
   (A) **R** and **Q** are of second category
   (B) **R** is of second category and **Q** is not
   (C) **Q** is of second category and **R** is not
   (D) Both **R** and **Q** are not of second category

51. Let L be a countable subset of **R** and M be a Lebesgue measurable subset of **R** with $m(M) > 0$. If $N = L \cup M$ and $O = (L \cup M) \setminus (L \cap M)$, then :
   (A) $m(M) = m(N) = m(O)$
   (B) $m(M) < m(N) = m(O)$
   (C) $m(M) = m(N) < m(O)$
   (D) $m(M) < m(N) < m(O)$

52. Let X be a metric space and $Y \subset X$:
   (A) If Y is dense in X, then $X \setminus Y$ is nowhere dense in X
   (B) If Y is nowhere in X, then $X \setminus Y$ is dense in X
   (C) If Y is dense in X, then $\mathrm{Int}(Y) \neq \phi$
   (D) If Y is dense in X, then $\mathrm{Int}(X \setminus Y) \neq \phi$

53. There is a non-abelian group of order :
   (A) 49
   (B) 41
   (C) 15
   (D) 12

54. The group $\mathbf{Z}_{72}$ is the direct product of groups as :

(A) $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_3 \times \mathbf{Z}_3$

(B) $\mathbf{Z}_{36} \times \mathbf{Z}_{36}$

(C) $\mathbf{Z}_9 \times \mathbf{Z}_8$

(D) $\mathbf{Z}_2 \times \mathbf{Z}_{36}$

55. Let p be a prime number and n, m natural numbers with m divides n. Then the finite extension $\mathbf{F}_p n/\mathbf{F}_p m$ of finite fields is a :

(A) Galois extension with abelian but non-cyclic Galois group of order $\frac{n}{m}$

(B) Galois extension with cyclic Galois group of order $\frac{n}{m}$

(C) Galois extension with abelian but non-cyclic Galois group of order nm

(D) Galois extension with cyclic Galois group of order nm

56. Let $a_n = (n!)^{\frac{1}{2n}}$ and $b_n = \left(\frac{1}{n^2}\right)^{\frac{1}{3n}}$.

Then the sequences :

(A) $(a_n)_n, (b_n)_n \in l^\infty$

(B) $(a_n)_n \in l^\infty$ and $(b_n)_n \notin l^\infty$

(C) $(a_n)_n \notin l^\infty$ and $(b_n)_n \in l^\infty$

(D) $(a_n)_n \notin l^\infty$ and $(b_n)_n \notin l^\infty$

57. For any sequence (α_n) of real numbers, define $T(\alpha_n) : l^2 \to l^2$ by $T(\alpha_n)(e_k) = \alpha_k e_k)$. The number of unitary operators of the form $T(\alpha_n)$ on l^2 is :

(A) finite, but more than 1

(B) countably infinite

(C) uncountably many

(D) zero

58. Let

$$V = \left\{ f \in C[0, 1] : f(t) = 0 \ \forall\ t \le \frac{1}{2} \right\}$$

and

$$W = \left\{ f \in C[0, 1] : f(t) = 0 \ \forall\ t > \frac{1}{2} \right\}.$$

If C[0, 1] is assigned the inner product

$$\langle f, g \rangle = \int_0^1 f(t)\, \overline{g(t)}\, dt,$$

then :

(A) C[0, 1] is the orthogonal direct sum of V and W

(B) C[0, 1] is the direct sum of V and W, but not the orthogonal direct sum

(C) C[0, 1] is a sum of V and W, but not a direct sum

(D) C[0, 1] is not the sum of V and W

59. Let τ be the topology generated by $\{(a, \infty) : a \in \mathbf{R}\}$ on $\mathbf{R}$. In this topology, the closure of $\{0\}$ in $\mathbf{R}$ is :

(A) $\{0\}$

(B) $(-\infty, 0]$

(C) $[0, \infty)$

(D) $\mathbf{R}$

60. Let

$X = \{(x, y) \in \mathbf{R}^2 : 0 \le |x| = |y| \le 1\}$

$Y = \{(x, y) \in \mathbf{R}^2 : |x| + |y| = 1\}$ and

$Z = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 1\}$

be subspaces of the Euclidean space $\mathbf{R}^2$. Then :

(A) X is homeomorphic to Y and Z

(B) X is homeomorphic to Y, but not Z

(C) Y is homeomorphic to Z, but not X

(D) Z is homeomorphic to X, but not Y

61. Let X be a second countable space and $f : X \to Y$ be continuous open surjective map. Then :

(A) Y is second countable

(B) Y is separable, but not second countable

(C) Y is first countable but not separable

(D) Y is separable, but not first countable

62. Consider the following statements :

(I) Every bounded lattice is complete.

(II) Every Boolean lattice is a distributive lattice.

(III) Every complete lattice is bounded.

Then which of the following is *true* ?

(A) Only (III) is true

(B) Only (II) and (III) are true

(C) Only (II) is true

(D) All are true

63. Let G be a simple graph with degree of every vertex an even number ≥ 2. Then G is :

(A) bipartite

(B) disjoint union of cycles

(C) Hamiltonian

(D) Without a cut vertex

64. Two boys and two girls are lined up randomly in a row. What is the probability that the girls and boys alternate ?

(A) 2/3

(B) 1/2

(C) 1/3

(D) 3/4

65. Let $u(r, \theta)$ be a harmonic function in the disc

$$D = \{(r, \theta)/0 \leq r < R, -\pi < \theta \leq \pi\}$$

such that u is continuous in closed disc $\bar{D}$ and satisfies :

$$u(R, \theta) = \cos, \ |\theta| \leq \pi/3$$

$$= 0, \quad \pi/3 < |\theta| \leq \pi.$$

The mean value theorem gives the value of $u(0, 0)$ as :

(A) $\sqrt{3}/2\pi$

(B) 0

(C) $\sqrt{3}$

(D) $\frac{1}{2\pi}$

66. The equation

$$3\Delta u + 4u_{xy} - u^2 = 1$$

is :

(A) Linear

(B) Hyperbolic

(C) Parabolic

(D) Elliptic

67. Let u_1, u_2 be two solutions of the Cauchy problem :

$u_{tt} - u_{xx} = x + t^2$

$u(x, 0) = \cos x$, $u_t(x, 0) = 3$

Then the solutions u_1 and u_2 satisfy :

(A) $u_1 \equiv 2u_2$

(B) $u_1 + u_2 \equiv 0$

(C) $u_1 - u_2 \equiv 0$

(D) $u_1 u_2 = 1$

68. Square of any integer is of the form :

(A) $3k$ or $3k - 1$

(B) $4k$ or $4k - 1$

(C) $5k$ or $5k + 1$

(D) $3k$ or $3k - 2$

69. The last two digits of 3^{123} are :

(A) 47

(B) 67

(C) 27

(D) 87

70. Which of the following natural numbers cannot be written as a sum of 2 squares ?

(A) 405

(B) 1111

(C) 117

(D) 164

71. If a system of n particles with k non-holonomic constraints has r degrees of freedom, then :

(A) $r = 3n - k$

(B) $r = n - k$

(C) $r > 3n - k$

(D) $r < 3n - k$

72. The Lagrangian of a particle of mass m in spherical polar co-ordinates is given by :

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2\right) - mgl\cos\theta$$

The quantity that is conserved is :

(A) $\frac{\partial L}{\partial \dot{r}}$

(B) $\frac{\partial L}{\partial \dot{\theta}}$

(C) $\frac{\partial L}{\partial \dot{\phi}}$

(D) $\frac{\partial L}{\partial r}$

73. If the Hamiltonian of the dynamical system is given by $H = pq$, then as $t \to \infty$.

(A) $q \to \infty$, $p \to 0$

(B) $q \to 0$, $p \to 0$

(C) $q \to \infty$, $p \to \infty$

(D) $q \to 0$, $p \to \infty$

74. Euler's equation of motion for a rigid body about a fixed point in the absence of any net torque and $I_{xx} = I_{yy}$ imply that the z-component of the angular velocity :

(A) satisfies simple harmonic motion

(B) is constant

(C) is always zero

(D) is a function of time

75. If $\bar{q}$ denotes velocity field of an incompressible fluid in motion, then the mass conservation will *not* imply :

(A) $\nabla \bar{q} = 0$

(B) $\dfrac{\partial \rho}{\partial t} + \nabla \bar{q} = 0$

(C) $\dfrac{\partial \rho}{\partial t} + \nabla (\rho \bar{q}) = 0$

(D) $\nabla \times \bar{q} = 0$

76. If the velocity components of a possible fluid motion are $u = 2cxy$, $v = c(a^2 + x^2 - y^2)$ where a, c are non-zero constants, then the stream function ϕ is :

(A) $-cx^2y$

(B) $-c\left(x^2y + a^2y - \dfrac{y^3}{3}\right)$

(C) $2cxy$

(D) $c(a^2 + x^2 - y^2)$

77. The velocity components for two-dimensional flow are given by $u = y^2 - x^2$, $v = 2xy$. The stream function ψ and the velocity potential ϕ of the flow are :

(A) $\phi = xy^2 + \dfrac{x^3}{3}$, $\psi = \dfrac{y^3}{3} + x^2y$

(B) $\phi = \dfrac{y^3}{3} - x^2y$, $\psi = xy^2 - \dfrac{x^3}{3}$

(C) $\phi = \dfrac{x^3}{3} - xy^2$, $\psi = x^2y - \dfrac{y^3}{3}$

(D) $\phi = \dfrac{y^3}{3} + x^2y$, $\psi = xy^2 + \dfrac{x^3}{3}$

78. In two-dimensional fluid flow consider a doublet of strength μ placed at $z = z_0$ and inclination α to the positive x-axis. The image of this doublet in a straight line is :

(A) a doublet of strength μ placed at $z = -z_0$ and inclination α to the positive x-axis.

(B) a doublet of strength μ placed at $z = -\bar{z}_0$ and inclination $\pi - \alpha$ to the positive x-axis.

(C) a doublet of strength μ placed at $z = -z_0$ and inclination $\pi - \alpha$ to the positive x-axis.

(D) a doublet of strength μ placed at $z = -\bar{z}_0$ and inclination α to the positive x-axis.

79. Let the first and second fundamental form of a surface patch are $\mathrm{E}du^2 + 2\mathrm{F}dudv + \mathrm{G}dv^2$ and $\mathrm{L}du^2 + 2\mathrm{M}dudv + \mathrm{N}dv^2$ respectively. Then the Gaussian curvature of the patch is :

(A) $\det\begin{pmatrix} \mathrm{E} & \mathrm{F} \\ \mathrm{F} & \mathrm{G} \end{pmatrix}$

(B) $\det\begin{pmatrix} \mathrm{L} & \mathrm{M} \\ \mathrm{M} & \mathrm{N} \end{pmatrix}$

(C) $\det\begin{pmatrix} \mathrm{E} & \mathrm{F} \\ \mathrm{F} & \mathrm{G} \end{pmatrix} \cdot \det\begin{pmatrix} \mathrm{L} & \mathrm{M} \\ \mathrm{M} & \mathrm{N} \end{pmatrix}$

(D) $\det\begin{pmatrix} \mathrm{L} & \mathrm{M} \\ \mathrm{M} & \mathrm{N} \end{pmatrix} \Big/ \det\begin{pmatrix} \mathrm{E} & \mathrm{F} \\ \mathrm{F} & \mathrm{G} \end{pmatrix}$

80. Let v be a unit speed smooth curve in $\mathbf{R}^3$ with tangent $\bar{t}$, normal $\bar{n}$, binormal $\bar{b}$. Then :

(A) $\bar{t}'$ is orthogonal to $\bar{t}$ and $\bar{b}$

(B) $\bar{t}'$ is orthogonal to $\bar{b}$ and $\bar{n}$

(C) $\bar{t}'$ is orthogonal to $\bar{t}$ and $\bar{n}$

(D) $\bar{t}'$ is not orthogonal to any of $\bar{t}$ or $\bar{n}$ or $\bar{b}$

81. The Gaussian curvature of the hyperbolic paraboloid $\frac{x^2}{2} - \frac{y^2}{3} - z = 0$ is :

(A) always < 0

(B) always > 0

(C) always 0

(D) positive at some points and negative at some points

82. The variational problem of extremizing the functional

$\mathrm{I}[y(x)] = \int_0^{2z} \left(y'^2 - y^2\right)dx$, $y(0) = 1$, $y(2z) = 1$

has :

(A) a unique solution

(B) exactly two solutions

(C) an infinitely many solutions

(D) no solution

83. Any function that gives an extremum of the functional

$$\iint_D \left\{z_x^2 + z_y^2 + 2zf(x, y)\right\} dx\, dy$$

must satisfy :

(A) Laplace equation

(B) Heat equation

(C) Wave equation

(D) Poisson equation

84. The curve which extremizes the functional

$$I[y(x)] = \int_0^{\pi/4} \left(y''^2 - y^2 + x^2\right) dx$$

under the conditions $y(0) = 0$, $y'(0) = 1$, $y(\pi/4) = y'(\pi/4) = \frac{1}{\sqrt{2}}$ is :

(A) $y(x) = 1 - \cos x$

(B) $y(x) = \tan x$

(C) $y(x) = \cos x$

(D) $y(x) = \sin x$

85. The shortest arc connecting two points on the surface of a sphere is called :

(A) a great circle arc

(B) a catenary

(C) a catenoid of revolution

(D) a cycloid

86. Consider the following statements :

(I) Every initial value problem can be reduced to a Fredholm integral equation.

(II) Every boundary value problem can be reduced to a Volterra integral equation.

Then :

(A) Only (I) is correct

(B) Only (II) is correct

(C) Both (I) and (II) are correct

(D) Both (I) and (II) are wrong

87. Iterated kernel $K_m(t, s)$ of the integral equation

$$u(t) = 1 + \lambda \int_0^t e^{t-s} u(s)\, ds$$

is :

(A) $\dfrac{(t-s)^{m-1}}{(m-1)!}$

(B) $\dfrac{e^{t-s}}{(m-1)!}$

(C) $\dfrac{(t-s)^{m-1}}{(m-1)!} e^{t-s}$

(D) $(t - s)e^{t-s}$

88. Solution of the integral equation

$$u(t) = a\sin t - 2\int_0^t \cos(t-s)u(s)ds$$

is :

(A) $u(t) = a \sin t$

(B) $u(t) = a \cos t$

(C) $u(t) = ate^{-t}$

(D) $u(t) = at^2e^{-t}$

89. Which of the following is *not correct* ?

(A) $(\Delta - \nabla) = \Delta\nabla$

(B) $(1+\Delta)(1-\nabla) = 1$

(C) $\mu^2 = 1 + \frac{1}{4}\delta^2$

(D) $\delta = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$

90. The first approximation of the root lying between 0 and 1 of the equation

$$x^3 + 3x - 1 = 0$$

by Newton-Raphson method with initial approximation $x_0 = 0$, is :

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

91. For a given initial value problem

$$y' = 1 + xy,\ y(0) = 2,$$

the value of $y(0.1)$ by Euler's method with $n = 0.1$ is :

(A) 2.1

(B) 2.0

(C) 2.2

(D) 2.4

92. Let $L\{f(t)\} = F(s)$ and $u(t-a)$ be a unit step function. Then $L\{f(t-a)\,u(t-a)\}$ is :

(A) $e^{-as}\ F(s)$

(B) $\frac{e^{-as}}{s}\ F(s)$

(C) $e^{as}\ F(s)$

(D) $\frac{e^{as}}{s}\ F(s)$

93. Fourier sine transform of

$$f(t) = \frac{e^{-at}}{t},\ a > 0$$

is :

(A) $\sqrt{\frac{2}{\pi}}\tan^{-1}\left(\frac{s}{a}\right)$

(B) $\sqrt{\frac{2}{\pi}}\sin^{-1}\left(\frac{s}{a}\right)$

(C) $\sqrt{\frac{2}{\pi}}\tan\left(\frac{s}{a}\right)$

(D) $\sqrt{\frac{2}{\pi}}\sin\left(\frac{s}{a}\right)$

94. Suppose that the function $y(t)$ satisfies the differential equation :

$$y'' - 4y' + 4y = 0$$

with initial condition $y(0) = 0$, $y'(0) = 3$. Then the Laplace transform of $y(t)$ is :

(A) $\dfrac{3}{(s+2)^2}$

(B) $\dfrac{3}{(s-2)^2}$

(C) $\dfrac{3.5}{(s+2)^2}$

(D) $\dfrac{3.5}{(s-2)^2}$

95. Consider the transportation problem

Min. $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to :

$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, ..., m,$

$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, ..., n$ and $x_{ij} \geq 0$

The existence of a feasible solution of the transportation problem is possible :

(I) if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

(II) if and only if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Then which of the following is *true* ?

(A) Only (I) is true
(B) Only (II) is true
(C) Both are true
(D) None of them is true

96. The number of non-isomorphic modular lattices on 5 elements is :

(A) 3
(B) 4
(C) 5
(D) 2

97. Consider the following statements :

(I) Every distributive lattice is complemented and complementation is unique.

(II) Every modular lattice is complemented and complementation is unique

Then which of the following is *true* ?

(A) (I) is true but (II) is not true
(B) (II) is true but (I) is not true
(C) Neither (I) is true nor (II) is true
(D) Both statements are true

98. Let $(\mathbf{R}, \mathrm{M}, \mu)$ be a Lebesgue measure space. Define :

$$\lambda(\mathrm{A}) = \int_{\mathrm{A}} \sin x \, d\mu$$

Which of the following statements is *true* ?

(A) λ defines a measure on $\mathbf{R}$
(B) λ defines a signed measure on $\mathbf{R}$
(C) λ does not define a signed measure
(D) λ defines a signed measure on $\mathbf{R}$ but not a measure

99. Let ($\mathbf{R}$, M, μ) denote the real line with Lebesgue measure μ. Define $\nu_1(A) = \int_A f(x)dx$ where :

$f(x) = e^{-x}$ for $x > 1$
$= 0$ otherwise

and $\nu_2(A) = \int_A g(x)dx$ where

$g(x) = e^{-1}$ for $0 \le x \le 1$
$= 0$ otherwise

Then :

(A) The measure ν_1 is absolutely continuous with respect to ν_2
(B) The measure ν_2 is absolutely continuous with respect to ν_1
(C) ν_1 and ν_2 are mutually singular measures
(D) μ is absolutely continuous with respect to both ν_1 and ν_2

100. Consider the following two statements for a Lebesgue measurable function $f : [0, 1] \to \mathbf{R}$.

(P) f is Lebesgue integrable.
(Q) $|f|$ is Lebesgue integrable.

Then which of the following is *true* ?

(A) (P) $\not\Rightarrow$ (Q)
(B) (Q) $\not\Rightarrow$ (P)
(C) (P) $\Leftrightarrow$ (Q)
(D) (P) $\not\Rightarrow$ (Q) and (Q) $\not\Rightarrow$ (P)

SECTION III

101. The 90^m sample percentile of 80 observations is 6. Suppose 6 is added to the 7 largest observations and 3 is subtracted from the remaining observations. The 90^m sample percentile of the modified 80 observations is :

(A) 12
(B) 3
(C) 6
(D) 9

102. For two random variables x and y covariance between $2x$ and y, $\text{cov}(2x, y) = 4$. Hence, $\text{cov}(5x - 2, 2y - 5)$ is :

(A) 40
(B) 0
(C) 10
(D) 20

103. Let x and y be two independent normal random variables with mean μ_1 and μ_2 respectively and common variance σ^2. For $0 < \alpha < 1$ if the 100αth quantile of x is larger than 100αth quantile of y, then :

(A) $\mu_1 = \mu_2$

(B) $\mu_1 > \mu_2$

(C) $\mu_1 < \mu_2$

(D) Nothing can be said about the relationship between μ_1 and μ_2

104. If $P(A|B) = P(B)$, then :

(A) $P(B|A) = P(A)$

(B) A and B are independent events

(C) $P^2(A) \leq P(B)$

(D) $P^2(B) \leq P(A)$

105. The probability mass function of a random variable X is given by :

k	0	1	2	3
$P[X = k]$	0.2	0.1	p	q

where $p \geq 0$ and $q \geq 0$.

Which of the following is feasible ?

(A) $E[X] = 1.2$

(B) $E[X] = 2.1$

(C) $E[X] = 2.8$

(D) $E[X] = 3.4$

106. Let X_1, X_2, X_3 be independent identically distributed random variables with probability mass function given below :

k	–1	0	1	2
$P[X = k]$	0.2	0.3	0.1	0.4

Then $P[X_1 + X_2 = 0/X_2 = 1, X_3 = 2]$ is :

(A) 0.11

(B) 0.13

(C) 0.2

(D) 0.3

107. Let $M_V(t)$ denote the mgf of a random variable V.

Suppose X and Y are two independent r.v.s. whose mgf exists. Let $Z = X + Y$ and $W = XY$. Then which of the following is true for all $t \in (-h, h)$ for some $h > 0$.

(A) $M_Z(t) = M_X(t)\ M_Y(t)$ and $M_W(t) = \int M_Y(tx)\ f_X(x)\ dx$

(B) $M_Z(t) = M_X(t)\ M_Y(t)$ and $M_W(t) = M_X(t) + M_Y(t)$

(C) $M_Z(t) = M_X(t) + M_Y(t)$ and $M_W(t) = M_X(t)\ M_Y(t)$

(D) $M_Z(t) = \int M_Y(t + x)\ f_X(x)\ dx$ and $M_W(t) = M_X(t)\ M_Y(t)$

108. Let X, Y be two random variables such that $E[Y/X = x] = x^2$ and X is standard normal r.v. Then which of the following is *false* ?

(A) $E[Y] = 1$

(B) $\text{cov}(X, Y) = 0$

(C) $\text{Corr}(X, Y) > 0$

(D) $E[X^2Y] = 3$

109. Suppose X_1, X_2, X_3 are independent and identically distributed random variables each having exponential distribution with mean θ. Suppose Y_1, Y_2, Y_3 is the corresponding order statistics. Then, $E(Y_1)$ is :

(A) $\frac{5\theta}{6}$

(B) $\frac{\theta}{6}$

(C) $\frac{\theta}{3}$

(D) θ

110. The pth quantile of a standard normal random variable is ξ_p. Then, the pth quantile of a chi-square random variable with 1 degree of freedom is :

(A) ξ_p^2

(B) ξ_{1+p}^2

(C) $\xi_{(1+p)/2}^2$

(D) 2ξ

111. Suppose (X, Y) is a two-dimensional random vector with range $(0, \infty) \times \{1, 2\}$, such that for any $A \subset (0, \infty)$ and $y = 1, 2$,

$$P(X \in A, Y = y) = \frac{1}{2}\int_A ye^{-yx}dx.$$

Then, which of the following statements is *false* ?

(A) The distribution of (X, Y) is neither discrete nor absolutely continuous.

(B) The marginal distribution function of X is $F(x) = 1 - (1/2)\,[e^{-x} + 2e^{-2x}]$, $0 < x < \infty$.

(C) The random variable X is having an absolutely continuous distribution with density function $(1/2)\,[e^{-x} + 2e^{-2x}]$.

(D) The marginal distribution of Y is $P(Y = y) = 1/2$, $y = 1, 2$.

112. Suppose a random vector $\underline{X}$ has normal distribution with mean vector $\underline{\mu}$ and dispersion matrix V.

Then which of the following has a chi-square distribution ?

(A) $(X - \underline{\mu})' V(X - \underline{\mu})$

(B) $(X - \underline{\mu})' V^{-1}(X - \underline{\mu})$

(C) $\exp\{(X - \underline{\mu})' V(X - \underline{\mu})\}$

(D) $\exp\{(X - \underline{\mu})' V^{-1}(X - \underline{\mu})\}$

113. Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$ where $\theta \subset [a, b]$ and $a < b$ are real numbers.

Then which of the following statements about the Maximum Likelihood Estimator (MLE) of θ is *correct* ?

(A) MLE of θ does not exist.

(B) MLE of θ is $\bar{X}$.

(C) MLE of θ exists but it is not $\bar{X}$.

(D) MLE of θ is an unbiased estimator of θ.

114. Let T_1 be a $100 \cdot \alpha\%$ lower confidence limit for θ and T_2 be a $100 \cdot \alpha\%$ upper confidence limit for θ. Let $P[T_1 < T_2] = 1$ and $\frac{1}{2} < \alpha < 1$. Then a $100(2\alpha - 1)\%$ confidence limit for θ is :

(A) $[T_1, T_2)$

(B) $\left[\frac{T_1}{\alpha}, \frac{T_2}{\alpha}\right]$

(C) $\left[\frac{T_2}{\alpha}, \frac{T_1}{\alpha}\right]$

(D) $[(2\alpha - 1)T_1, \alpha T_2]$

115. Suppose X has a distribution with probability mass function that of discrete uniform on {–1, 0, 1} under H_0 and U(–1, 1) under H_1. Then the test function

$$\phi(x) = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{if } x \geq 0 \end{cases}$$

has size :

(A) Not properly defined since H_0 is discrete and H_1 is continuous

(B) 1/3

(C) 0

(D) 1/2

116. Which of the following theorems is useful for obtaining uniformly minimum variance unbiased estimator of a parametric function ?

(A) Neyman-Pearson theorem

(B) Basu's theorem

(C) Rao-Blackwell theorem

(D) Rao-Blackwell-Lehmann-Scheffe theorem

117. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from uniform $(\theta, 5\theta)$. Define $X_{(1)} = \min\{X_1, \ldots\ldots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \ldots\ldots, X_n\}$. Maximum likelihood estimator of θ is :

(A) $\dfrac{X_{(1)}}{5}$

(B) $\dfrac{X_{(n)}}{5}$

(C) $\max\left\{\dfrac{X_{(n)}}{5}, X_{(1)}\right\}$

(D) $\min\left\{\dfrac{X_{(n)}}{5}, X_{(1)}\right\}$

118. Suppose $\{X_1, X_2, \ldots\ldots, X_n\}$ is a random sample from the distribution of X with mean μ and variance σ^2. The test statistic to test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ is given by

$$T_n = \frac{\Sigma\left(X_i - \bar{X}\right)^2}{\sigma_0^2}.$$ Then the null distribution of T_n is :

(A) χ^2_{n-1} if X has normal distribution

(B) χ^2_{n-1} irrespective of distribution of X

(C) t distribution with $(n - 1)$ degrees of freedom

(D) F distribution if X has normal distribution

119. X is a normal (μ, σ^2) random variable and a 95% confidence interval for μ is constructed based on a sample of size n. Suppose the length of the interval is L_1. If instead the variance of X is $\sigma_1^2 > \sigma^2$ the length of the 95% confidence interval constructed using the same technique will be :

(A) equal to L_1

(B) smaller than L_1

(C) larger than L_1

(D) larger or smaller than L_1

120. Suppose $R_{1.23}$ is a multiple correlation coefficient between X_1 and (X_2, X_3). Then which of the following statements *cannot* be true ?

(A) $R_{1.23}$ is a maximum correlation between X_1 and $(aX_2 + a_3X_3)$ where a_1 and a_2 are any real numbers.

(B) $R_{1.23}$ is a simple correlation between X_1 and $\hat{X}_1$, where $\hat{X}_1$ is a line of best fit based on X_2 and X_3.

(C) $R_{1.23} = -0.4$

(D) $R_{1.23} = 0.4$

121. A frequency data is classified in 9 classes and Gamma distribution is fitted to it after estimating the parameters. If a χ^2 goodness of fit test is to be used without combining the classes, the degrees of freedom associated with χ^2 test are :

(A) 9

(B) 8

(C) 7

(D) 6

122. (X_i, Y_i) $i = 1, ..., 9$ is a random sample from bivariate population where all X_i's are distinct and all Y_i's are distinct $i = 1, ..., 9$. If the number of concordance pairs is 9, the Kendall's sample correlation coefficient is :

(A) –1/4

(B) –1/2

(C) +1/2

(D) +1/4

123. The following failure rates have been observed for certain type of light bulbs :

Week	1	2	3	4	5
Probabilities of failing by the end of week	0.10	0.25	0.50	0.80	1

If the cost of failure replacement is Rs. 20 per bulb and the cost of group replacement is Rs. 500, what is the optimal replacement interval ? (Assume that the group has 1000 bulbs.)

(A) One week

(B) Two weeks

(C) Three weeks

(D) Four weeks

124. The Economic Order Quantity (EOQ), primarily :

(A) minimizes the set-up cost

(B) reduces shortages

(C) balances carrying and ordering costs

(D) All of the above

125. Consider the two M/M/1 queuing systems Q_1 and Q_2, where :

Q_1 : arrival rate λ, service rate μ

Q_2 : arrival rate λ^2, service rate μ^2.

It is known that $\lambda < \mu$. Let $L_s^{(i)}$ be the number of customers in the system in the equilibrium state, $i = 1, 2$. Then :

(A) $L_s^{(1)} < L_s^{(2)}$

(B) $L_s^{(1)} = L_s^{(2)}$

(C) $L_s^{(1)} > L_s^{(2)}$

(D) The two cannot be compared

126. Consider a finite population of $N = 2n$ units. A sample of size 2 is drawn as follows :

(*i*) The population is randomly divided into 2 groups each of size n.

(*ii*) From each of the two groups as obtained above, one unit is drawn with probability $1/n$.

These two units form the sample. Then, the *correct* statement is :

(A) The sample is a stratified sample with two strata.

(B) The sample mean is not an unbiased estimator of the population mean.

(C) The sample is a SRSWOR of size 2.

(D) The sample is not randomly obtained.

127. Consider the following sampling designs : a stratified sample scheme with SRSWOR within each stratum and a SRSWOR. Both have the same sample size. We are interested in estimating the population mean. Let V^2_{prop} and V^2_{SRS} be the corresponding variances of the usual estimators of the population mean. Then,

(A) $V^2_{\text{prop}} < V^2_{\text{SRS}}$

(B) $V^2_{\text{prop}} = V^2_{\text{SRS}}$

(C) $V^2_{\text{prop}} > V^2_{\text{SRS}}$

(D) $V^2_{\text{prop}} \leq V^2_{\text{SRS}}$

128. A completely randomized design has n plots and 10 treatments. While calculating the F-ratio for assessing equality of treatment effects, the experimenter forgot to divide the numerator and denominator sums of squares by corresponding degrees of freedom. But the statistician said that the calculated F-ratio is correct. Hence n is equal to :

(A) 10

(B) 20

(C) 30

(D) 19

129. Under completely randomized design with model :

$$Ey_{ij} = \mu + \alpha_i + \in_{ij} \quad i = 1, \ldots\ldots, p$$
$$j = 1, \ldots\ldots, n_i$$

$\in_{ij} \sim N(0, \sigma^2)$ and independently distributed, a parametric function $c\mu + \sum_{i=1}^{p} d_i\alpha_i$ where c and $d_1, \ldots\ldots., d_p$ are known constants is estimable if and only if :

(A) $c = 0$

(B) $\sum_{i=1}^{p} d_i = 0, \; c > 0$

(C) $c = 0, \; \sum_{i=1}^{p} d_i > 0$

(D) $\sum_{i=1}^{p} d_i = c$

130. Under a Latin-Square design with V treatments, which of the following statements is *not* correct ?

(A) It can accommodate three sources of heterogeneity

(B) It requires exactly V^3 experimental units

(C) The elementary (pairwise) contrasts among column effects, among row effects and among treatment effects are all estimated with common variance

(D) The degrees of freedom associated with error are equal to $(V - 2)(V - 1)$

131. Let **F** be a σ-field of subsets of Ω.

Let μ be a non-negative, finitely additive set function on **F**.

Suppose $\{A_n\}$ is a sequence of disjoint sets such that $\bigcup_{n=1}^{\infty} A_n \in \mathbf{F}$.

Then which of the following statements is always *correct* ?

(A) $\sum_{n=1}^{\infty} \mu(A_n) = \mu\left(\bigcup_{n=1}^{\infty} A_n\right)$

(B) $\sum_{n=1}^{\infty} \mu(A_n) \geq \mu\left(\bigcup_{n=1}^{\infty} A_n\right)$

(C) $\sum_{n=1}^{\infty} \mu(A_n^c) = \mu\left(\bigcup_{n=1}^{\infty} A_n^c\right)$

(D) $\sum_{n=1}^{\infty} \mu(A_n) \leq \mu\left(\bigcup_{n=1}^{\infty} A_n\right)$

132. Let $(\Omega, \mathbf{F}, \mu)$ be a measure space and let $f_n : \Omega \to \bar{\mathrm{R}}$, be a sequence of extended Borel measurable functions. Suppose $|f_n| \leq 100$, $\forall\, n = 1, 2, \ldots\ldots$, and $\lim_{n\to\infty} f_n = f$ a.e. μ. Then, which of the following statements is *false* ?

(A) $\int_{\Omega} |f|\, d\mu \leq \liminf_{n} \int |f_n|\, d\mu$

(B) $\lim_{n\to\infty} \int |f_n|\, d\mu = \int |f|\, d\mu$

(C) $\int \limsup_{n\to\infty} |f_n|\, d\mu = \int |f|\, d\mu$

(D) $\lim_{n\to\infty} \int \sum_{k=1}^{n} |f_k|\, d\mu = \int \sum_{k=1}^{\infty} |f_k|\, d\mu$

133. Let $\Omega = \{1, 2, ...\}$ the set of positive integers and $\mathbf{F} = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{1\}^c, \{2\}^c, \{1, 2\}^c, \Omega\}$,
where A^c denotes the complement of the set A. Let $R = (-\infty, \infty)$ and $\mathbf{B}$ the Borel σ-field of R.
Which of the following functions f from $(\Omega, \mathbf{F})$ to $(R, \mathbf{B})$ is measurable ?
(A) $f(k) = k,\ k = 1, 2, ...$
(B) $f(1) = 1,\ f(2) = 2,\ f(3) = 1,\ f(k) = 0,\ k \geq 4$
(C) $f(k) = 1$ if k is odd and $f(k) = 2$ if k is even
(D) $f(1) = 2,\ f(2) = 3,\ f(k) = 10,\ k \geq 3$

134. Let $\{X_n, n \geq 1\}$ be a sequence of independent identically distributed random variables. Define $W_n = \cos\left(X_n^2\right)$.
Then which of the following is always *true* ?
(A) Both the sequences $\{X_n\}$ and $\{W_n\}$ satisfy the strong law of large numbers.
(B) The sequence $\{X_n\}$ satisfies the strong law of large numbers.
(C) Neither the sequence $\{X_n\}$ nor $\{W_n\}$ satisfy the strong law of large numbers.
(D) The sequence $\{W_n\}$ satisfies the strong law of large numbers.

135. Let $\{X_n, n \geq 1\}$ be a sequence of independent identically distributed random variables with finite (non-zero) fourth moment.

Let $Y_n = \left(X_1^2 + ... + X_n^2\right)\Big/n$

Which of the following is *true* ?

As $n \to \infty$, $\sqrt{n}\left(Y_n - E\left[X_1^2\right]\right)$.

(A) Converges in distribution to a chi-square r.v. with 1 degree of freedom.

(B) Converges in distribution to a standard normal r.v.

(C) Converges in distribution to a normal r.v.

(D) Converges to a r.v. degenerate at zero.

136. The joint density of X and Y is given by :

$$f(x, y) = \begin{cases} \frac{y}{2} e^{-xy}, & 0 < x < \infty,\ 0 < y < 2 \\ 0 &, \text{otherwise} \end{cases}$$

Consider the statements :

(I) The conditional density of X given Y = 1,

$$f(x \mid 1) = \begin{cases} \frac{1}{2} e^{-x}, & 0 < x < \infty \\ 0 &, \text{otherwise} \end{cases}$$

(II) $E[X \mid Y = 1] = 1$

(III) $P[X \leq 2 \mid Y = 1] = 1 - e^{-2}$

Which of the above statements are *true* ?

(A) All the three

(B) (I) and (II) only

(C) (I) and (III) only

(D) (II) and (III) only

137. Suppose $\{X_n\}$ is a sequence of random variables and X is a random variable such that $P\left[\lim_{n\to\infty} X_n = X\right] = 1$.

Then which of the following may *not* hold ?

(A) $\lim_{n\to\infty} P\left[|X_n - X| > 1/2\right] = 0$

(B) $P\left[\lim_{n\to\infty} \exp(X_n) = \exp(X)\right] = 1$

(C) $\lim_{n\to\infty} E\left[|X_n - X|^2\right] = 0$

(D) $\lim_{n\to\infty} P\left[X_n \leq x\right] = P\left[X \leq x\right]$ at all continuity points x of $P[X \leq x]$

138. Let $\{X_n\}$ be a sequence of independent random variables. Define $A_n = [X_n \geq n^{-1}]$.

Let $E = \bigcap_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$

Then which of the following is always *correct* ?

(A) P(E) = 0

(B) P(E) is either 0 or 1

(C) P(E) = 1

(D) 0 < P(E) < 1

139. Suppose $\{X_n\}$ is a $\mathbf{F}_n$-martingale and $E\left[X_n^2\right] < \infty$ for all n.

Which of the following is *not* always *true* ?

(A) $E[X_n]$ is a constant for all n.

(B) $\left\{X_n^2\right\}$ is not a $\mathbf{F}_n$-martingale

(C) $W_n = \sum_{k=1}^{n} X_k$ is not a $\mathbf{F}_n$-martingale

(D) $E\left[X_n^2\right]$ is constant for all n.

140. Suppose a distribution function $F : \mathbf{R} \to [0, 1]$ of a random variable X is as follows :

$$F(x) = \begin{cases} 0, & \text{if} \quad x < 0, \\ 1/4, & \text{if} \quad 0 \le x < 1, \\ 1/2, & \text{if} \quad 1 \le x < 2, \\ 1/2 + (x-2)/2, & \text{if} \quad 2 \le x < 3, \\ 1, & \text{if} \quad x \ge 3. \end{cases}$$

Then E(X) is :

(A) 5/6

(B) 2/3

(C) 3/2

(D) 7/6

141. Suppose a distribution function of random variable X is :

$$F(x) = \begin{cases} 0, & \text{if} \quad x < 0, \\ x/2, & \text{if} \quad 0 \le x < 1, \\ 1, & \text{if} \quad x \ge 1. \end{cases}$$

Then, which of the following statements is *not correct* ?

(A) $P(-1 < X \le 1/2) = 1/4$

(B) $E(X) = 1/6$

(C) $E(X) = 3/4$

(D) $P(X = 1) = 1/2$

142. Suppose X is an arbitrary random variable and $g(\cdot)$ is a non-negative Borel function on R. If $g(\cdot)$ is even and non-decreasing on $[0, \infty)$, then for every $a > 0$.

(A) $P[|X| \le a] \le E(g(x))/g(a)$

(B) $P[|X| \le a] \ge E(g(x))/g(a)$

(C) $P[|X| \ge a] \le E(g(x))/g(a)$

(D) $P[|X| \ge a] \ge E(g(x))/g(a)$

143. Suppose X follows Cauchy distribution with location parameter μ and scale σ. Then the characteristic function of X is :

(A) $\exp\{it\mu - \sigma|t|\}$

(B) $\exp\left\{it\mu - \frac{1}{2}\sigma^2 t^2\right\}$

(C) $\exp\{it\mu - \sigma t\}$

(D) $\exp\left\{\sigma^2 - i(t-\mu)^2\right\}$

144. Suppose that X and Y are independent random variables having Poisson distribution with means λ_1 and λ_2 respectively. Then, the conditional distribution of X given X + Y and X + Y given X are respectively :

(A) Binomial $\left(x + y; \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$ and Poisson with parameter λ_2; but taking values X, (X + 1), ...,

(B) Negative binomial with parameters $\left(x + y; \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$ and Poisson with parameter $\lambda_1 + \lambda_2$.

(C) Poisson with mean $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ and Poisson with parameter λ_1

(D) Geometric with parameter $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ and Poisson with parameter $\lambda - 1 + \lambda_2$, but taking values X, (X + 1), ...,

145. Consider a statistical decision problem with $\Theta = \{\theta_1, \theta_2\}$ and $D = \{d_1, d_2, d_3, d_4\}$. The risk functions are given by :

θ / d	d_1	d_2	d_3	d_4
θ_1	4	1	5	2
θ_2	1	2	3	3

Then :

(A) d_1 is minimax

(B) d_2 is minimax

(C) d_3 is minimax

(D) d_4 is minimax

146. Let X be a Poisson random variable with parameter $\theta = E(X)$, $0 < \theta < \infty$. Consider the Bayesian procedure for estimation of θ. Then, the conjugate prior for θ :

(A) $\pi(\theta) \; \alpha \; e^{-\theta}$

(B) $\pi(\theta) \; \alpha \; \theta^{\alpha - 1} e^{-\theta^{\beta}} \quad \alpha, \beta > 0$

(C) $\pi(\theta) \; \alpha \; e^{-\alpha\theta} \theta^{\lambda - 1} \quad \alpha, \lambda > 0$

(D) $\pi(\theta) \; \alpha \; \frac{1}{1 + \theta^2}$

147. Suppose X_1 has exponential distribution with mean θ, X_2 has exponential distribution with mean $\theta/2$ and X_1 and X_2 are independent. Which of the following statements is *not correct* ?

(A) $X_1 + X_2$ is sufficient for θ

(B) $X_1 + 2X_2$ is sufficient for θ

(C) $X_1 + 2X_2$ is complete for θ

(D) $(X_1 + 2X_2)/2$ is unbiased for θ

148. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from Poisson $P(\theta)$ distribution, $\theta > 0$. Which of the following statements is *not correct* ?

(A) Sample mean $\bar{X}_n$ is unbiased for θ

(B) Sample variance S_n^2 is unbiased for θ

(C) $0.3\bar{X}_n + 0.7S_n^2$ is unbiased for θ

(D) $\bar{X}_n^2/S_n^2$ is unbiased for θ

149. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from $U(\theta, \theta + 1)$. Which of the following statistic is *not* a maximum likelihood estimator of θ ?

(A) $X_{(1)}$

(B) $X_{(n)} - 1$

(C) $X_{(n)}$

(D) $\dfrac{X_{(1)} + X_{(n)}}{2} - 0.5$

150. The distributions of X under H_0 and H_1 are given by :

x	1	2	3	4
H_0	0.25	0.25	0.25	0.25
H_1	0.17	0.17	0.17	0.32

The most powerful test at level $\alpha = 0.05$ is given by :

(A) reject H_0 if $X = 4$

(B) reject H_0 with probability 0.25 if $X = 4$

(C) reject H_0 with probability 0.2 if $X = 4$

(D) reject H_0 with probability 0.32 if $X = 4$

151. Suppose $\{X_1, X_2, ..., X_n\}$ are independent and identically distributed random variables with mean μ and variance σ^2. Suppose

$$S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}_n\right)^2.$$ Then

which of the following statements is *not true* ?

(A) S_n is unbiased for σ

(B) S_n^2 is unbiased for σ^2

(C) S_n is consistent for σ

(D) S_n^2 is consistent for σ^2

152. Suppose $\{X_1, X_2, ..., X_n\}$ are independent and identically distributed random variables with mean zero and variance σ^2. Then the asymptotic distribution of

$$T_n = \frac{\sqrt{n}\sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^2}$$

is :

(A) t distribution

(B) N(0, 1) distribution

(C) N(0, $1/\sigma^2$) distribution

(D) degenerate at 0

153. Let $S = S(X_1, X_2, ..., X_n)$ be an unbiased estimator of the parametric functional $\theta(F)$ based on a random sample $X_1, X_2, ..., X_n$ from F. Suppose U is the U-statistics corresponding to S. Then, which of the following statements is *true* ?

(A) $E_F(U) > E_F(S)$

(B) $E_F(U) < E_F(S)$

(C) $V_F(U) \leq V_F(S)$

(D) $V_F(U) > V_F(S)$

154. Let $X_1, X_2, ..., X_n$ be independent Poisson (λ) random variables and

$$T_1 = \frac{1}{n}\sum_{i=1}^{n} I\left(X_i = 0\right), \quad T_2 = e^{-\bar{X}}$$

Then, the asymptotic relative efficiency ARE(T_1, T_2) is :

(A) $e^{-\lambda}$

(B) $(e^{\lambda} - 1)$

(C) $(e^{\lambda} - 1)/(\lambda - 1)$

(D) $(e^{\lambda} + 1)$

155. Let T_n be a consistent estimator of θ such that $\sqrt{n}\left(T_n - \theta\right) \to N(0, 1)$ in distribution and $T_n \xrightarrow{p} \theta$. Let T_n^* be an estimator of θ be given by :

$$T_n^* = T_n + \frac{c}{\sqrt{n}} \qquad c > 0.$$

Then :

(A) T_n^* is not a consistent estimator of θ

(B) $\sqrt{n}\left(T_n^* - \theta\right) \xrightarrow{D}$ a normal distribution

(C) T_n^* is consistent for θ

(D) $E\left(T_n^* - \theta\right)^2 \to 0$ as $n \to \infty$

156. Let $X = (X_1, X_2, X_3, X_4) \sim N_4(\underline{0}, \Sigma)$ where

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix}$$

is positive definite.

Then which of the following statements is *true* ?

(A) $X_1 + X_2, X_2 + X_3, X_3 + X_4$ have all same distributions

(B) $\dfrac{(X_1 - X_2)^2}{(X_1 - X_3)^2} \sim F_{11}$

(C) $\dfrac{(X_1 - X_3)^2 + (X_2 - X_4)^2}{2} \sim \chi_2^2$

(D) $\Sigma X_i \sim N(0, 4)$

157. Let $X_1 \sim N(0, 1)$ and

$$X_2 = \begin{cases} -X_1 & -3 \le X_1 \le 3 \\ X_1 & \text{otherwise} \end{cases}$$

Then which of the following statements is *correct* ?

(A) X_1 and X_2 are negatively correlated

(B) X_1 and X_2 are positively correlated

(C) X_1 and X_2 are perfectly correlated

(D) X_1 and X_2 have joint normal distribution

158. Let $(X_1, X_2) \sim N_2(\mu, \Sigma)$ where Σ is a +ve definite matrix. Then which of the following has a singular normal distribution where $X_3 = X_1 - 2X_2$?

(A) $X_1 + X_3, X_1 - X_2$

(B) $X_1 - X_3, X_2 - X_3$

(C) X_1, X_3

(D) X_2, X_3

159. Let $U_1, U_2, ..., U_n$ be independent identically distributed random vectors with common distribution $N_p(0, \Sigma)$, $\Sigma = ((\sigma_{ij}))$ is a +ve definite matrix. Let $S = ((S_{ij})) = \sum_{j=1}^{n} U_j U'_j$.

Then which of the following statements is *not true* ?

(A) $\sum_{i=1}^{n} S_{ii} \sim \text{constant } \chi^2_n$

(B) $S_{11} + S_{22} \sim \text{constant } \chi^2_2$

(C) $S_{11} \sim \text{constant } \chi^2_1$

(D) $S_{11} + S_{12} \sim \text{constant } \chi^2_2$

160. Let $\phi_X(\underline{t})$ be the characteristic function of $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$. Then $EX_1X_2^2X_3$ is given by :

(A) $\left.\dfrac{(-1)^4 \partial^4 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2^2 \partial t_3}\right|_{\underline{t} = \underline{0}}$

(B) $\left.\dfrac{(-1)^3 \partial^3 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2 \partial t_3}\right|_{t_1 = 1,\, t_2 = 2,\, t_3 = 1}$

(C) $\left.\dfrac{(-1)^3 \partial^4 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2^2 \partial t_3}\right|_{t_1 = 1,\, t_2 = 2,\, t_3 = 1}$

(D) $\left.\dfrac{\partial^4 \phi_{\underline{X}}(\underline{t})}{\partial t_1 \partial t_2^2 \partial t_3}\right|_{t_1 = 1,\, t_2 = 2,\, t_3 = 1}$

161. Y_1, Y_2 and Y_3 are three uncorrelated random variables with common variance σ^2. Further $E(Y_1) = \theta_2 - \theta_1$, $E(Y_2) = E(Y_3) = \theta_2 + \theta_3$. Then :

(A) $\theta_1 - \theta_2$ is not estimable

(B) $\theta_1 + \theta_2$ is not estimable

(C) $\theta_1 + \theta_3$ is not estimable

(D) $\theta_2 + \theta_3$ is not estimable

162. In a simple linear regression of Y on X based on n observations. The fitted values of Y_i's are Y_i^n, $i = 1 \,.....\, n$ respectively. Then :

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} Y_i^n$$

(A) always

(B) only if the regression equation is $Y = B_0 + B_1X + \varepsilon$

(C) if and only if the regression equation is $Y = B_0 + B_1X + \varepsilon$

(D) if the regression equation is $Y = B_0 + B_1X + \varepsilon$

163. A least squares regression fit :

(A) may be used to predict the value of Y if the corresponding values of regressor X are given

(B) indicates a cause-effect relationship between response variable Y and regressors

(C) can be determined only if a satisfactory relationship exists between response variable Y and regression

(D) all the above statements are true

164. In a multiple linear regression model $Y_i = B_0 + B_1X_{i1} + ... + B_pX_{ip} + \varepsilon_i$ with $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$, $i = 1, ..., n$.

(A) Mean regression Sum of Squares (SS) is an unbiased estimator of σ^2

(B) Mean regression SS and Mean Error SS are unbiased and Mean total SS is a biased estimator of σ^2

(C) Mean total SS is unbiased estimator of σ^2

(D) Mean total SS and Mean regression SS are unbiased estimators of σ^2

165. Suppose $Y_i = B_0 + B_1(x_i - 2) + \varepsilon_i$, $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$ and where $x_i = i$, $i = 1, 2, 3$. The Best Linear Unbiased estimators of B_0 and B_1.
(A) do not exist
(B) exist and are uncorrelated with each other
(C) exist and are independent of each other
(D) exist and are positively correlated with each other

166. A probability proportional to size (PPS) without replacement sample of size 2 is to be drawn from a population of N = 3 units. Let $z_1 = 1/4$, $z_2 = 1/4$, $z_3 = 1/2$. The i-th unit is selected with probability z_i, $i = 1, 2, 3$. If the first unit selected is i, then the j-th unit is selected for inclusion in the sample with probability $z_j/(1 - z_i)$, $j \neq i$. Then, the probability that the third unit is the population is included in the sample is given by :
(A) 1
(B) 1/2
(C) 5/6
(D) 2/3

167. Let $r = \bar{y} / \bar{x}$ denote the ratio of the sample means of the variable y and of the auxiliary variable x. The sample is a SRSWOR sample of size n. Let $f = n/N$ and $\bar{X}$ be the population mean of the auxiliary variable x. Let $R = \bar{Y}/\bar{X}$. By considering $\mathrm{Cov}(r, \bar{x})$, the bias of r, as an estimator of R, is given by :
(A) $\frac{1-f}{n}$
(B) $\frac{1-f}{nR}$
(C) $\frac{1-f}{n}R$
(D) $-\frac{1}{X}\mathrm{Cov}(r, \bar{x})$

168. Let us consider the following methods of estimation, all based on SRSWOR of the same sample size. I Sample mean II Ratio estimator III Regression estimator. Let the corresponding variances be denoted by V_1^2, V_2^2 and V_3^2. Assuming that the sample size is sufficiently large, we have :

(A) $V_1^2 \leq V_3^2 \leq V_2^2$

(B) $V_3^2 \leq V_2^2 \leq V_1^2$

(C) $V_3^2 \leq V_1^2 \leq V_2^2$

(D) None of the above

169. The incidence matrix of a block design is given by :

$$N = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Hence the design is :

(A) connected and not orthogonal

(B) not connected and not orthogonal

(C) not connected and orthogonal

(D) connected and orthogonal

170. Suppose N is the incidence matrix of a BIBD with parameters (v, b, r, k, λ), then :

(A) rank (NN′) = v

(B) rank (NN′) = b

(C) rank (NN′) = $v - 1$

(D) rank (NN′) = $b - 1$

171. 8 treatments are arranged in a row-column design as given below :

1	2	3	4
5	6	7	8
3	8	1	6
7	4	5	2

Hence it is a :

(A) Latin Square Design

(B) Youden Square Design

(C) Quasi-Latin Square Design

(D) Incomplete Block Design

172. For a 2^4 factorial design with four treatments A, B, C, D each at two levels, the treatment combinations were allotted in two blocks of 8 plots each as below :

Block I	1	*a*	*c*	*ac*	*bd*	*abd*	*bcd*	*abcd*
Block II	*b*	*ab*	*bc*	*abc*	*d*	*ad*	*cd*	*acd*

Hence the treatment combination which is confounded is :

(A) ABCD

(B) ACD

(C) BCD

(D) ABD

173. Let $\{X_t\}$ be a time series defined as $X_t = A \sin(\omega t + B)$, where $E(A) = 0$, $Var(A) = 1$, $B \sim$ Uniform $(-\pi, \pi)$ and A and B are independent. Then, *h*-lag covariance function $r(h)$ is :

(A) $\frac{1}{2}\cos(\omega h)$

(B) $\frac{1}{2}\sin(\omega h)$

(C) $\frac{1}{2}\cos(\omega h + 4)$

(D) $\frac{1}{2}\sin(\omega h + 2)$

174. Given a time series $X_t = X_{t-1} + Z_t$, where X_0 is distributed like Z_t and Z_t's are iid $N(0, \sigma^2)$. Which of the following statements is *true* ?

(A) $V\left(X_t|\sqrt{t}\right) = 1$

(B) $\{X_t\}$ is stationary

(C) $E(X_t) = t$

(D) $Cov(X_t, X_s) = \sigma|t - s|$

175. What will be the variance of $(X_1 + X_2 + X_3)/3$, if X_1, X_2 and X_3 are from an AR(1) series $X_t = 1/2X_{t-1} + Z_t$, where $Z_t \sim$ iid normal (0, 1) ?

(A) 9/18

(B) 8/18

(C) 16/18

(D) 13/18

176. Consider a Markov chain on S = {1, 2, 3, 4} with the transition probability matrix given by :

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Then :

(A) $\lim_{n \to \infty} p_{ij}^{(n)}$ exists for all (i, j) and the limit is independent of i

(B) $\lim_{n \to \infty} p_{ij}^{(n)}$ does not exist for all (i, j) and there is a unique stationary distribution

(C) there does not exist any stationary distribution

(D) there exist infinitely many stationary distributions

177. Let $\{X(t), t \geq 0\}$ be a time-homogeneous Poisson process with rate λ. Then :

(A) $\text{Cov}(X(s), X(t)) = \dfrac{\min(s, t)}{\sqrt{st}}$

(B) $\text{Cov}(X(s), X(t)) = \dfrac{\lambda}{\lambda + 1}$

(C) $\text{Cov}(X(s), X(t)) = \dfrac{\max(s, t)}{\sqrt{s, t}}$

(D) $\text{Cov}(X(s), X(t)) = \dfrac{\min(s, t)}{st}$

178. Consider a Markov chain on S = {0, 1, 2, ...}. The transition probabilities are given by :

$$p_{00} = \frac{1}{2} \quad p_{01} = \frac{1}{2}$$

and for $i \geq 1$,

$$p_{ii-1} = \frac{1}{2} \quad p_{ii+1} = \frac{1}{2}$$

Then :

(A) all states are persistent non-null

(B) there exists a persistent non-null state

(C) there exists a unique stationary distribution

(D) all states are transient or persistent null

179. Consider a branching process $\{X_n, n \geq 0\}$. Let $X_0 = 1$ and assume that $E\left(\sum_{\theta}^{\infty} X_n\right) < \infty$. Then, the extinction probability :

(A) equals 0

(B) lies in (0, 1)

(C) does not exist

(D) 1

180. Death rates are standardised to :

(A) obtain an estimate of ideal rates

(B) eliminate the differential influence of one or more variables

(C) adjust them with registration of deaths

(D) obtain correct estimate of actual rates

181. Which one of the following population growth model is *not* specified correctly ?

(A) $P_t = P_0(1 + rt)$

(B) $P_t = P_0 e^{rt}$

(C) $P_t = K/(1 + e^{a + bt})$

(D) $P_t = BC^t$

182. The failure time of a component is exponentially distributed with mean life 200 hours. The design of the component is modified after which the mean life has increased to 400 hours. What is the amount of increase in reliability at 800 hours ? (Let $a = e^{-2}$.)

(A) a

(B) a^2

(C) $a(1 - a)$

(D) $(1 - a)/a$

183. If the failure rate function r of a component is $r(t) = \dfrac{t}{1 + t}$, $t \geq 0$; then its survival function is :

(A) e^{-t}

(B) te^{-t}

(C) $1 - e^{-t}$

(D) $(1 + t)e^{-t}$

184. If the lead time is m periods and usage rate is u, the re-order stock level is given by :

(A) Q/2

(B) $\dfrac{Q}{u} - m$

(C) m

(D) $m \cdot u$

185. In the model S = Q – ut, a stock out will occur when :

(A) $t = 0$

(B) $t = u/Q$

(C) $t = Q/u$

(D) Q = S

186. In a M/M/K queueing system the departure rate μn when there are n customers in the system is :

(A) μ

(B) $n\mu$

(C) $n\mu$ if $n \leq k$ and 0 if $n > k$

(D) $n\mu$ if $n \leq k$ and nk if $n > k$

187. For a certain dynamic programming problem, the recurrence equation for optimal solution is found to be

$f_1(c) = c$

$f_k(c) = \max\limits_{0 < x \leq c} x \cdot f_{k-1}(c - x)$ for

$k > 1$.

What is the value of $f_2(5)$?

(A) 25

(B) 25/2

(C) 25/4

(D) 125/27

188. When a positive integer is divided into 5 parts, the maximum value of their product is :

(A) 5K

(B) $(K/5)^5$

(C) $(5K)^5$

(D) 5 + K

189. Which of the following is *false* ?

(A) If x_0 is an optimal solution to the primal, then dual has a feasible solution.

(B) If x_0 is an optimal solution to the primal, then the optimal solution to the dual is given by $B^{-1} C_B$ where B is the optimal basis of the primal.

(C) If dual has an unbounded solution, then primal has an infeasible solution.

(D) Dual simplex method always leads to degenerate basic feasible solution.

190. Consider the LPP :

Maximize : $Z = 3x_1 + 5x_2$

Subject to : $x_1 \leq 4,$

$x_2 \leq 6,$

$3x_1 + 2x_2 \leq 18,$

$x_1, x_2 \geq 0$

The first stage in the dynamic programming algorithm to solve the above problem involves :

(A) Maximizing $3x_1$ subject to $x_1 \leq 4$

(B) Maximizing $3x_1$ subject to $3x_1 + 2x_2 \leq 18$

(C) Maximizing $5x_2$ subject to $3x_1 + 2x_2 \leq 18$

(D) Maximizing $5x_2$ subject to $x_2 \leq 6$ and $3x_1 + 2x_2 \leq 18$

ROUGH WORK

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