Test Booklet Code & Serial No.

प्रश्नपत्रिका कोड व क्रमांक

A

Paper-II

MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

JAN - 30218

Time Allowed : 1¼ Hours] **[Maximum Marks : 100**

Number of Pages in this Booklet : **28** Number of Questions in this Booklet : **84**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. (*a*) This paper consists of **Eighty Four (84)** multiple choice questions, each question carrying **Two (2)** marks.
   (*b*) There are *three* sections, **Section-I, II, III** in this paper.
   (*c*) Students should attempt all questions from **Sections I and II** *or* **Sections I and III**.
   (*d*) Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.
   (*e*) The OMR sheets with questions attempted from both the Sections viz. II & III, will not be assessed.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. (*a*) या प्रश्नपत्रिकेत एकूण **चौऱ्यांशी (84)** बहुपर्यायी प्रश्न दिलेले आहेत, प्रत्येक प्रश्नाला **दोन (2)** गुण आहेत.
   (*b*) या प्रश्नपत्रिकेत **खण्ड**-I, II, III असे तीन खण्ड आहेत.
   (*c*) विद्यार्थ्यांनी **खण्ड-I आणि II** किंवा **खण्ड I आणि III** यांचे सगळे प्रश्न सोडावे.
   (*d*) खाली दिलेल्या प्रश्नाचे चार पर्याय किंवा उत्तर दिलेले आहेत. प्रश्नाचे बहुपर्यायी उत्तरामधून केवळ एक 'बरोबर' आहे.
   (*e*) ओ.एम.आर. उत्तरपत्रिकेच्या क्रमश: दोन्ही खण्ड–II व III मधील सोडवलेले प्रश्नाची आकारणी नाही केली जाईल.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science
Paper II

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Note : Attempt all questions either from Sections I & II or from Sections I & III only. The OMR sheets with questions attempted from both the Sections viz. II & III, will not be assessed.
Section I : Q. Nos. 1 to 16, Section II : Q. Nos. 17 to 50, Section III : Q. Nos. 51 to 84.

Section I

1. The sequence $a_n = (-1)^n + \frac{6}{n^2}$ has :

(A) no limit point

(B) one limit point

(C) two limit points

(D) more than two limit points

2. The series :

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots, \ |x| < 1$$

represents the function :

(A) $\tan^{-1} x$

(B) $\tan x$

(C) $\sin^{-1} x$

(D) $\log(1 + x)$

3. The modulus and argument of $\frac{1}{-1+i}$ are :

(A) $\frac{1}{\sqrt{2}}, \frac{5\pi}{4}$

(B) $\frac{1}{\sqrt{2}}, \frac{3\pi}{4}$

(C) $\sqrt{2}, \frac{\pi}{4}$

(D) $\frac{1}{\sqrt{2}}, \frac{7\pi}{4}$

4. $\int \frac{dz}{z^2 - 1}$ is :

(A) $2\pi i$

(B) 0

(C) $4\pi i$

(D) πi

5. If

$$A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{pmatrix},$$

where $\omega = e^{2\pi i/3}$, then $A^2 =$

(A) I

(B) A

(C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

6. If V denotes the vector space of $n \times n$ real skew symmetric matrices, then $\dim V =$

(A) $n^2 - n$

(B) $n - 1$

(C) $\frac{n(n+1)}{2}$

(D) $\frac{n(n-1)}{2}$

7. If

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & -2 \\ 0 & -6 & 4 \end{pmatrix},$$

then the rank of the matrix AA^t is :

(A) 1

(B) 2

(C) 3

(D) 0

8. Matrix

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 4 & -2 \\ 0 & 3 & -1 \end{pmatrix}$$

is similar to :

(A) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

9. Let X be a r.v. with the following $F(x)$,

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ \frac{x}{4} & ; \quad 0 \le x < 2 \\ \frac{3}{4} & ; \quad 2 \le x < 3 \\ 1 & ; \quad x \ge 3 \end{cases}$$

Which of the following statements is *correct* ?

(A) $P[X = 2] = \frac{1}{3}$, $P[X = 3] = \frac{2}{3}$

(B) $f(x) = \frac{1}{3}$; $0 \le x < 3$

(C) $P[X = 2] = P[X = 3] = \frac{1}{2}$

(D) $f(x) = 1$; $0 < x < 1$

10. Let X be a discrete r.v. with the following pmf :

$$P[X = x] = \begin{cases} k; & x = 0, \pm j;\ j = 1, 2, \ldots.. n \\ 0; & \text{otherwise} \end{cases}$$

Let $Y = X^2$, then, $P[Y = 4]$ is :

(A) $\frac{1}{2n+1}$

(B) $\frac{2}{n+1}$

(C) $\frac{n}{2n+1}$

(D) $\frac{2}{2n+1}$

11. Let X_1 and X_2 be jointly distributed with pdf $f(x_1, x_2)$ given by :

$f(x_1, x_2) =$

$$\begin{cases} \frac{1}{4}\left\{1 + x_1 x_2 \left(x_1^2 - x_2^2\right)\right\}; & |x_1| < 1 \text{ and } |x_2| < 1 \\ 0 \qquad ; & \text{otherwise} \end{cases}$$

Then, the characteristic function of X_1 is :

(A) $\frac{\sin t}{t}$

(B) $\frac{\cos t}{t}$

(C) $\frac{\sin t}{t^2}$

(D) $\frac{t}{\sin t}$

12. Let X be a r.v. such that variance of X is $\frac{1}{2}$. Then, an upper bound for $P[|X - FX| > 1]$ as given by the Chebychev's inequality is :

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{4}$

13. Let X be a normal random variable with mean 1 and variance 1. Define the events :

$$A = \{-2 < X < 1\},$$

$$B = \{-1 < X < 1\},$$

$$C = \{0 < X < 2\}.$$

Which of the following statements is *correct* ?

(A) $P(C) < P(B) < P(A)$

(B) $P(A) = P(B) < P(C)$

(C) $P(A) = P(B) = P(C)$

(D) $P(B) < P(A) < P(C)$

14. The problem :

Max. : $Z = 3x_1 + 2x_2$

Subject to :

$$x_1 - x_2 \leq 1,$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

has :

(A) Feasible solution

(B) Optimum solution

(C) Feasible but not optimum solution

(D) Unbounded solution

15. The set

$$S = \left\{(x_1, x_2) : 3x_1^2 + 2x_2^2 \leq 6\right\}$$

is a :

(A) Concave

(B) Not concave

(C) Convex

(D) Not convex

16. If the set of feasible solutions of the system AX = B, X ≥ 0, is a convex polyhedron, then at least one of the extreme points gives a/an :

(A) Unbounded solution

(B) Bounded but not optimal

(C) Optimal solution

(D) Infeasible solution

Section II

17. Let :

$$f(x) = e^{-1/x} \quad \text{if} \quad x > 0$$
$$= 0 \quad \text{if} \quad x \leq 0,$$

(A) $f(x)$ is not continuous

(B) $f(x)$ is continuous but not C^1

(C) $f(x)$ is C^1 but not C^∞

(D) $f(x)$ is C^∞ but not analytic

18. The Taylor series for $\frac{z}{z^2+1}$ around 0 is :

(A) $z + z^3 + z^5 + ...$

(B) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - ....$

(C) $z - z^3 + z^5 - ....$

(D) $z - \frac{z^3}{3} + \frac{z^5}{5} - ....$

19. The matrix :

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

represents :

(A) rotation by angle $\pi/4$ around x-axis

(B) rotation by angle $\pi/2$ around y-axis

(C) rotation by angle $\pi/4$ around y-axis

(D) rotation by angle $\pi/8$ around z-axis

20. Let V denote vector space of $n \times n$ real matrices, having zero trace. Then dim V =

(A) $n - 1$

(B) $n^2 - 1$

(C) $n^2 - n$

(D) $\frac{n(n-1)}{2}$

21. Which of the following statements is *not true* ?

(A) Every closed and bounded subset of a matric space is compact

(B) Every compact subset of a metric space is closed and bounded

(C) A closed subset of a compact set is compact

(D) Cartesian product of compact sets is compact

22. The function $f(x) = |x|^3$, $x \in \mathbf{R}$ is :

(A) continuous but not differentiable

(B) differentiable but not c^1

(C) of class c^2

(D) of class c^3

23. $f(x) = \sin x,\ x \in \mathbf{R}$

(A) $f(x)$ is continuous but not uniformly continuous

(B) $f(x)$ is uniformly continuous but not Lipschitz

(C) $f(x)$ is Lipschitz and uniformly continuous

(D) $f(x)$ is neither Lipschitz nor uniformly continuous

24. Total variation of the function :

$$f(x) = \frac{x^3}{3} - x,\ x \in [2,\ 3]$$

is :

(A) 16/3

(B) 17/3

(C) 0

(D) 13/3

25. The maximum value of $\frac{\log x}{x}$, $x > 0$ is :

(A) e

(B) $\frac{1}{e}$

(C) $e + \frac{1}{e}$

(D) $\frac{-1}{e} + e$

26. The function $f(x)$ defined as :

$f(x) = 0$, if x is irrational

$= 1$, if x is rational,

is :

(A) Continuous on $\mathbf{R}$

(B) Continuous at rational points

(C) Continuous at irrational points

(D) Discontinuous at all points of $\mathbf{R}$

27. $\sin(x + iy)$ is equal to :

(A) $\sin x \sin y + i \cos x \cos y$

(B) $\sin x \cosh y + i \cos x \sinh y$

(C) $\sin x \cos y + i \cos x \sin y$

(D) $\sin x \sinh y + i \cos x \cosh y$

28. Which of the following complex numbers are collinear ?

(A) $1 + 2i,\ 2 + 5i,\ 4 + 11i$

(B) $1 - i,\ 2 + i,\ 1 + i$

(C) $i,\ -1 + 2i,\ 3 + 4i$

(D) $0,\ -3 + i,\ 7 + 8i$

29. Let $f(z)$ be an entire function, then $f(z)$ is also bounded if and only if :

(A) $f(z)$ is a polynomial function

(B) $f(z)$ is the reciprocal of a polynomial function

(C) $f(z)$ is a polynomial in $\sin z$ and $\cos(z)$

(D) $f(z)$ is a constant

30. Which of the following complex functions has a pole at $z = 0$?

(A) $f(z) = e^{1/z^3}$

(B) $f(z) = \sin\left(\frac{1}{z}\right)$

(C) $f(z) = \frac{1 + z + 2z^3}{z^4 - z^7}$

(D) $f(z) = z^3 + 7z + 1$

31. Let f be continuous in an open set D, then $\int_C f(z)\,dz = 0$ for each piecewise differentiable closed curve C in D if and only if :

(A) f is identically zero

(B) f is a polynomial function on D

(C) f is a periodic function on D

(D) f is an analytic function on D

32. The value of $\int_{|z|=1} \frac{\sin z}{z}\,dz$ is :

(A) $2\pi i$

(B) 0

(C) 2π

(D) $-2\pi i$

33. Which of the following rings need not have identity ?

(A) The ring of homomorphisms of a vector space to itself

(B) The ring of automorphisms of a vector space to itself

(C) The ring of $n \times n$ upper triangular matrices over **R**

(D) The ring of polynomials $P_0[\mathbf{R}]$ vanishing at origin

34. Let m be an even positive integer. What is the product of all mth roots of unity in **C** ?

(A) 1

(B) -1

(C) i

(D) $-i$

35. Let a be an element of a group G such that a^n = identity, then which of the following statements is *true* ?

(A) The order of G is finite

(B) The order of the element a is n

(C) Every subgroup of G of order n contains a

(D) The order of a is finite and divides n

36. Let G be a group of order n and $H = \{a \in G \mid a = a^{-1}\}$, then which of the following is *true* ?

(A) If the cardinality of H is odd then n is odd

(B) The cardinality of H divides n

(C) If the cardinality of H is even then n is odd

(D) The cardinality of H is a power of two

37. Let G be a non-abelian group with n elements, then which of the following values of n is possible :

(A) $n = 13$

(B) $n = 9$

(C) $n = 10$

(D) $n = 15$

38. Let F be a field with 128 elements, then which of the following is *true* ?

(A) F has a subfield with 8 elements

(B) F has no proper non-prime subfield

(C) Such a field F does not exist

(D) F has a subfield with 32 elements

39. Let T be a linear operator defined on vector space V. If there exists $v \in$ V such that v, Tv, ..., $T^{n-1}v$ are linear independent vectors and dim V = n, then :

(A) T is invertible

(B) T is nilpotent

(C) T is diagonalizable

(D) for T $\neq$ 0

40. If characteristic equation of matrix A is same as minimum polynomial which reads as $(x-1)^3 (x-4)$, then the Jordan canonical form of A is :

(A) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

41. Let T be a linear operator defined on a vector space V; $T : V \mapsto V$. If dim ker T > 0, then :

(A) 0 is an eigenvalue of T

(B) T is invertible

(C) T is nilpotent

(D) $I_m\ T = V$

42. If $T(x_1, x_2) = (-x_2, x_1)$, then matrix representation of T in the ordered basis $\{(1, 1)^t; (1, -1)^t\}$ is :

(A) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

43. If P is a 3×3 matrix of rank three and :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix},$$

then the rank of PA is :

(A) 1

(B) 2

(C) 3

(D) Depends upon matrix P

44. If A is a 2×2 real matrix with trace zero and determinant 1, then the eigenvalues of A are :

(A) real and distinct

(B) real and repeated

(C) complex with non-zero real part

(D) purely imaginary

45. The partial differential equation :

$$x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} + xu_x + yu_y = 0$$

is :

(A) is hyperbolic if $x > 0$, $y > 0$

(B) is hyperbolic on $\mathbf{R}^2$

(C) is elliptic on $\mathbf{R}^2$

(D) is parabolic on $\mathbf{R}^2$

46. The Pfaffian differential equation :

$$\overline{X} \cdot d\overline{r} = P(x, y, z)\, dx + Q(x, y, z)\, dy + R(x, y, z)\, dz = 0$$

is integrable if :

(A) $\overline{X} \cdot \overline{X} \neq 0$

(B) $\overline{X} \cdot \text{curl}\, \overline{X} = 0$

(C) $\overline{X} \cdot \text{curl}\, \overline{X} \neq 0$

(D) $\overline{X} \cdot \overline{X} = 0$

47. The differential equation obtained by eliminating the arbitrary constants A and ϕ from :

$$y = Ae^{-\alpha t} \sin(\omega t + \phi)$$

is :

(A) $\dfrac{d^2 y}{dt^2} + 2\alpha \dfrac{dy}{dt} + (\alpha^2 + \omega^2)y = 0$

(B) $\dfrac{d^2 y}{dt^2} + 2t \dfrac{dy}{dt} + (\alpha^2 - \omega^2)y = 0$

(C) $\dfrac{d^2 y}{dt^2} + 2\alpha t \dfrac{dy}{dt} + (\alpha^2 + \omega^2)y = 0$

(D) $\dfrac{d^2 y}{dt^2} + 2\alpha t \dfrac{dy}{dt} + (\alpha^2 - \omega^2)y = 0$

48. Let ϕ_1, ϕ_2 be linearly independent solutions of the linear differential equation :

$$y'' + a_1 y' + a_2 y = 0,$$

where a_1, a_2 are constants, then the Wronskian $W(\phi_1, \phi_2)$ is constant if and only if :

(A) $a_1 = 0$

(B) $a_1 \neq 0$

(C) $a_2 = 0$

(D) $a_2 \neq 0$

49. The differential equation :

$$(x^3 + xy^4)\ dx + 2y^3\ dy = 0$$

will become exact on multiplication by :

(A) e^{x^2}

(B) e^x

(C) e^{-x}

(D) $e^{x^2 + x}$

50. The differential equation :

$$Y^{(4)} + \sin xy = 0$$

(A) has exactly 4 linearly independent solutions

(B) has at most 4 linearly independent solutions

(C) has less than 4 linearly independent solutions

(D) has more than 4 linearly independent solutions

Section III

51. The mean height of 10000 children of age 6 years is 41.26" and the standard deviation is 2.24". Then the odds against the possibility that the mean of a random sample of 100 is greater than 41.7" is :

(A) 1 : 39

(B) 39 : 1

(C) 1 : 40

(D) 40 : 1

52. Which of the following statements is *correct* about a regression model ?

(A) Residual sum of squares reduces with every new term added in the model.

(B) Residual sum of squares reduces with new term added in the model provided the response variable is dependent on the new term.

(C) Residual sum of squares increases with the new term added in the model.

(D) Residual sum of squares increases with the new term added in the model provided the response variable is correlated with the new term.

53. Let (Ω, F, P) be a probability space. Let $\{A_n\}$ be a sequence of events such that $P(A_n) = 1$ for each $n \geq 1$.

Then $P\left(\bigcap_{n=1}^{\infty} A_n\right) =$

(A) Zero

(B) One

(C) Infinity

(D) Not defined

54. Let X be a zero-mean unit variance Gaussian random variable. Define the random variable Y as :

$$Y = \begin{cases} X & \text{if} \quad |X| \leq \alpha \\ -X & \text{if} \quad |X| > \alpha \end{cases}$$

α is some positive real number. What is the distribution of X + Y ?

(A) Gaussian (0, 1)

(B) Gaussian (0, 2)

(C) Not Gaussian, but having a distribution which is discontinuous at origin

(D) Not Gaussian, but a continuous distribution

55. Let $\{X_n\}$ be a sequence of independent random variables. Define the σ-field :

$$F = \bigcap_{n=1}^{\infty} \sigma\left(X_n, X_{n+1}, \ldots\ldots\right).$$

Suppose $A \in f_e$. Then, which of the following statements is more appropriate ?

(A) $P(A) = 0$

(B) $P(A) = 1$

(C) $P(A) = P(A^C)$

(D) $P(A) = 0$ or $P(A) = 1$

56. If X is a positive random variable with probability density function $f(x)$, then X^{-1} has probability density function :

(A) $1/f(x)$

(B) $f(1/x)$

(C) $1/x^2 \, f(1/x)$

(D) $1/x \, f(1/x)$

57. Let X_1 and X_2 be iid random variables with distribution function $F(x)$. Then, $P(X_1 \leq X_2)$ is :

(A) 1/3

(B) 2/3

(C) 1/2

(D) 3/2

58. Let X be a r.v. with $U(-\theta, \theta)$. The distribution of $Y = X^2$ is :

(A) $U(0, \theta)$

(B) $U(0, \theta^2)$

(C) $f(y) = \frac{y^{-1/2}}{2\theta}$; $0 < y < \theta$

(D) $f(y) = (2\theta)\ y^{-1/2}$; $0 < y < \theta^2$

59. If X is $F(m, n)$ (F-distribution with m and n degrees of freedom) and Y is $F(n, m)$, given that $P[X \geq c] = a$ then $P\left[Y \geq \frac{1}{c}\right]$ is :

(A) a

(B) 1

(C) $\frac{a}{2}$

(D) $1 - a$

60. If X is distributed as Binomial (n, p), $0 < p < 1$, then :

(A) $-X$ is distributed $B(-n, p)$

(B) $n - X$ is distributed $B(n, 1 - p)$

(C) $X - k$ is distributed $B(n - k, p)$, when k is constant

(D) $\frac{1}{X}$ is distributed as $B\left(n, \frac{1}{p}\right)$

61. Suppose X and Y are independent r.v.'s such that :

$$f(x) = \frac{e^{-x} x^{\alpha - 1}}{\Gamma\alpha}; \; x > 0, \; \alpha > 0$$

$$f(y) = \frac{e^{-y} y^{\beta - 1}}{\Gamma\beta}; \; y > 0, \; \beta > 0$$

The distribution of $\frac{X}{X + Y}$ is :

(A) Beta (α, β)

(B) Gamma $(\alpha + \beta, 1)$

(C) Beta $(\alpha + \beta, \alpha - \beta)$

(D) Gamma $(\alpha\beta, 1)$

62. Let $X_1, X_2, \dots\dots\dots X_n$ be iid r.v.'s from an exponential distribution with mean θ. Then the distribution of X_1 given $T = \sum_{1}^{n} X_i$

(A) $\frac{(n-2)(t - x_1)^{n-3}}{t^{n-2}}; \; 0 < x_1 < t$

(B) $\frac{n(t - x_1)^{n-1}}{t^n}; \; 0 < x_1 < t$

(C) $\frac{(n-1)(t - x_1)^{n-2}}{t^{n-1}}; \; 0 < x_1 < t$

(D) $\frac{n \cdot (t - x_1)^{n-1}}{\theta^n, t^n}; \; 0 < x_1 < t$

63. Let $X_1, X_2, \dots\dots\dots X_n$ be a random sample from Poisson distribution with parameter λ. Then covariance between $\bar{X}$ and $(X_1 - X_2)$ is :

(A) 1/2

(B) 1

(C) –1

(D) 0

64. Based on a random sample of size n from $N(\mu, \sigma^2)$, where μ is known and variance is unknown, sufficient statistic for μ is :

(A) $\sum X_i$

(B) $\sum X_i^2, \sum X_i$

(C) $\bar{X}_n$

(D) $\sum X_i^2$

65. Let $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ be a random sample of size n from Poisson distribution with mean λ. The log likelihood function $L(\lambda \mid x_1, x_2, \ldots\ldots x_n)$ is :

(A) constant in λ

(B) monotonic function of λ

(C) concave function of λ

(D) convex function of λ

66. Let X be a Bernoulli r.v. with parameter θ. Suppose we wish to test $H_0 : \theta = 1/3$ against $H_1 : \theta = 2/3$, based on random sample of size 1. Then :

(A) there exists infinitely many most powerful tests of size 0.05

(B) there exists unique non-randomized most powerful test of size 0.05

(C) there exists unique randomized most powerful test of size 0.05

(D) the most powerful test rejects H_0, if $X = 0$ is observed

67. Let $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ be a random sample of size n from $U(0, \theta)$. Define $T_1 = X_{(n)}$ and $T_2 = \bar{X}_n$. Then :

(A) T_1 is UMVUE and T_2 is not unbiased estimator for θ

(B) T_2 is UMVUE for $\theta/2$

(C) $\left(\frac{n+1}{n}\right)T_1 - 2T_2$ and $\left(\frac{n+1}{n}\right)T_1$ are correlated

(D) $\left(\frac{n+1}{n}\right)T_1 - 2T_2$ and $\left(\frac{n+1}{n}\right)T_1$ are uncorrelated

68. Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ be a random sample observed from exponential distribution with location parameter θ. For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$, the UMP test rejects H_0, if :

(A) $\bar{X} > C$

(B) $X_{(1)} > C$

(C) $X_{(n)} > C$

(D) $\Sigma (X_i - X_{(1)}) > C$

where C is to be chosen suitably.

69. If $r_{12.3}$ is the correlation coefficient between the variables X_1 and X_2 after eliminating the linear effect of X_3, then which of the following are *correct* ?

(1) $-1 \le r_{12.3} \le 1$

(2) $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23} \le 1$

(3) $r_{12.3}^2 = b_{12.3}\, b_{21.3}$, where b's are partial regression coefficients

(A) (1) and (2) only

(B) (1) and (3) only

(C) (2) and (3) only

(D) (1), (2) and (3)

70. The manager of a cyber cafe says that number of customers visiting on week days followed a Binomial distribution. Which one of the following techniques can be used to test the hypothesis at a given level of significance ?

(A) Test of significance of mean

(B) Test of significance of difference of means

(C) Chi-square test as a test of goodness-of-fit

(D) Correlation analysis

71. In a normal population $N(\mu, \sigma^2)$ with $\sigma^2 = 4$, in order to test the null hypothesis $\mu = \mu_0$ against $H_1 : \mu = \mu_1$, where $\mu_1 > \mu_0$ based on a random sample of size n, the value of K such that $\bar{X} > K$ provides a critical region of size $\alpha = 0.05$ is :

(A) $\mu_0 + 1.645/\sqrt{n}$

(B) $\mu_0 + 3.290/\sqrt{n}$

(C) $\mu_0 + 1.96/\sqrt{n}$

(D) $\mu_0 + 3.92/\sqrt{n}$

72. Consider the 2×2 contingency table on two attributes A and B :

	A_1	A_2
B_1	10	20
B_2	30	40

What is the value of χ^2 for testing the independence of the attributes A and B ?

(A) 0.645

(B) 0.794

(C) 0.812

(D) 0.853

73. The following is an arrangement of men (M) and women (W), lined up to purchase tickets for a rock concert :

MWMWMMMWMWMMMWWMMMMWW

MWMMMWMMMWWW

What is the expected value of runs for testing the hypothesis that the arrangement is random ?

(A) 14.92

(B) 15.88

(C) 16.76

(D) 18.20

74. In an ANOVA for an experimental design involving 3 treatments and 10 observations per treatment, if SSE = 399.6, then the estimated variance of error term is :

(A) 133.2

(B) 87.8

(C) 66.6

(D) 14.8

75. If orders are placed with the size determined by the economic order quantity, the re-order costs component is :

(A) equal to the holding cost component

(B) greater than the holding cost component

(C) less than the holding cost component

(D) either greater or less than the holding cost component

76. Gantt chart is used to solve the :

(A) Job sequencing problems

(B) Inventory problems

(C) Replacement problems

(D) All of the above

77. In usual notations of queueing systems, which of the following relationships is *not true* ?

(A) $W_s = W_q + \frac{1}{\mu}$

(B) $L_s = \lambda W_s$

(C) $L_q = \lambda W_q$

(D) $L_s = L_q + 1/\lambda$

78. For any primal problem and its dual :

(A) optimal value of objective function is same

(B) primal will have an optimal solution iff dual does

(C) both primal and dual can not be infeasible

(D) optimal solution does not exist

79. The payoff value for which each player in a game always selects the same strategy is known as :

(A) saddle point

(B) equilibrium point

(C) both (A) and (B)

(D) none of the above

80. The number of non-negative variables in a basic feasible solution to a $m \times n$ transportation problem is :

(A) $m + n - 1$

(B) mn

(C) $m + n$

(D) $m + n + 1$

81. If ρ_{wsy} denotes the intra-class correlation coefficient between pairs of units that are in the same systematic sample and if $\rho_{wsy} = 0$, then :

(A) Systematic sampling is as efficient as SRSWOR

(B) Systematic sampling is more efficient than SRSWOR

(C) Systematic sampling is as efficient as SRSWR but less efficient than SRSWOR

(D) Systematic sampling is more efficient than SRSWR

82. For an SRSWOR (N, n), the probability that a specified unit is included in the sample is :

(A) $\frac{1}{N}$

(B) $\frac{n}{N}$

(C) $\frac{1}{\left({}^{N}C_{n}\right)}$

(D) $\frac{1}{N(N-1)}$

83. In a 2^5 factorial design in block of 8 plots each, total number of interaction confounded with blocks is :

(A) 2

(B) 7

(C) 3

(D) 4

84. Identify the treatments x_1, x_2, x_3 and x_4 from blocks 1, 2, 3, 4 respectively so that the design is BIBD :

Block 1 : A, B, C, x_1

Block 2 : A, x_2, C, E

Block 3 : A, B, D, x_3

Block 4 : A, x_4, D, E

Block 5 : B, C, D, E

(A) x_1 = B, x_2 = E, x_3 = C, x_4 = D

(B) x_1 = D, x_2 = B, x_3 = E, x_4 = C

(C) x_1 = C, x_2 = D, x_3 = B, x_4 = E

(D) x_1 = E, x_2 = C, x_3 = D, x_4 = B

ROUGH WORK

ROUGH WORK