Roll No. | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilators

1. ______________________________
2. ______________________________

1517

Question Booklet Series **X**

PAPER–II

Question Booklet No.

(Identical with OMR Answer Sheet Number)

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 1 Hour 15 Minutes *Maximum Marks: 100*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be get replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of fifty (50) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

[Please Turn Over]

MATHEMATICAL SCIENCES

PAPER II

1. If f is a real valued function which is continuous and satisfies

$$f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}, \forall\ x,\ y \in \mathbb{R},$$

$f(0) = 1$ and $f'(0) = 2$, then $f(5)$ is equal to

(A) –9

(B) 11

(C) 10

(D) 6

2. $\lim_{n\to\infty} \frac{1}{n}\{(2n+1)(2n+2)\ ....\ (2n+n)\}^{\frac{1}{n}}$ is equal to

(A) $\frac{4e}{27}$

(B) $\frac{27e}{4}$

(C) $\frac{27}{4e}$

(D) $\frac{4}{27e}$

3. Let $p(z) = a_0 + a_1 z + \ldots + a_n z^n$ and
$q(z) = b_1 z + b_2 z^2 + \ldots + b_n z^n$
be complex polynomials. If a_0, b_1 are non-zero complex numbers, then the residue of $\frac{p(z)}{q(z)}$ at '0' is equal to

(A) $\frac{a_0}{b_1}$

(B) $\frac{b_1}{a_0}$

(C) $\frac{a_1}{b_1}$

(D) $\frac{a_0}{a_1}$

4. Let A be a real 3×4 matrix of rank 2. Then the rank of $A^t A$, where A^t denotes the transpose of A, is

(A) exactly 2

(B) exactly 3

(C) exactly 4

(D) at most 2 but not necessarily 2

5. If A is a 5×5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A, each with algebraic multiplicity 2, then the determinant of A is

(A) 0

(B) 24

(C) 120

(D) 180

6. Let V be the vector space of twice differentiable functions f on $\mathbb{R}$ satisfying $f'' - 2f' + f = 0$. Define $T : V \to \mathbb{R}^2$ by $T(f) = (f'(0),\ f(0))$. Then T is

(A) one – one and onto

(B) one – one but not onto

(C) onto but not one – one

(D) neither one – one nor onto

7. The probability that a teacher will give a surprize test during any class meeting is 3/5. If a student is absent on two days, then the probability that he will miss at least one test is

(A) 21/25

(B) 4/25

(C) 2/25

(D) None of the above

8. Suppose that the variables $x_1 \geq 0$ and $x_2 \geq 0$ satisfy the constraints $x_1 + x_2 \geq 3$ and $x_1 + 2x_2 \geq 4$ and let $Z = 5x_1 + 7x_2$. Which of the following is true?

(A) The maximum value of Z is 21 and it does not have any finite minimum.

(B) The minimum value of Z is 17 and it does not have any finite maximum.

(C) The maximum value of Z is 21 and its minimum value is 17.

(D) Neither has a finite maximum nor a finite minimum.

9. The period of the function $\left|\sin^3 \frac{x}{2}\right| + \left|\sin^5 \frac{x}{5}\right|$ is

(A) 2π

(B) 10π

(C) $\sqrt{2\pi}$

(D) 5π

10. Consider the power series $\sum_{n=1}^{\infty} z^{n!}$. The radius of convergence of this series is

(A) 0

(B) ∞

(C) 1

(D) a real number greater than 1

11. Which of the following is true?

(A) $\sum \frac{1}{n}$ and $\sum \frac{1}{n \log n}$ both converge

(B) $\sum \frac{1}{n}$ diverges but $\sum \frac{1}{n \log n}$ converges

(C) $\sum \frac{1}{n}$ and $\sum \frac{1}{n \log n}$ both diverge

(D) $\sum \frac{1}{n}$ converges but $\sum \frac{1}{n \log n}$ diverges

12. The radius of convergence of the series $\sum \frac{n^n}{n!} z^n$ is

(A) e

(B) $\frac{1}{e}$

(C) 1

(D) 0

13. Let $f(Z) = \sum_{n=0}^{10} Z^n$, $Z \in \mathbb{C}$. If $\gamma : |Z - i| = 2$, then $\oint_{\gamma} \frac{f(Z)\,dz}{(Z-i)^{10}}$ is equal to

(A) $2\pi i(1 + 10i)$

(B) $-30\pi i$

(C) $2\pi i$

(D) 0

14. The co-efficient of $\frac{1}{z}$ in the Laurent series of $\frac{\sin 2z}{z^2}$ is

(A) 0

(B) 2

(C) –1

(D) 1

15. The number of subgroups of $\mathbb{Z}/48\mathbb{Z}$ is

(A) 4

(B) 8

(C) 10

(D) 12

16. Let G be a group and $a, b \in G$ such that order of a is 5 and $aba^{-1} = b^2$. Then the order of b is

(A) 30

(B) 31

(C) 32

(D) 33

17. Suppose I is the group of integers and $H = \{3x : x \in I\}$ is a normal subgroup of I. Then the elements of I/H are

(A) $\{H, H+1, H+2\}$

(B) $\{H+1, H+2, H+3\}$

(C) $\{H, H+1, H+3\}$

(D) $\{H, H+1, H+4\}$

18. In the set of integers I, define $a \oplus b = a + b + 1$, $a \odot b = a + b + ab$ with $a, b \in I$. Then $(I, \oplus, \odot)$ is a ring such that it is

(A) commutative

(B) an integral domain

(C) a field

(D) None of the above

19. Let $S : \mathbb{R}^3 \to \mathbb{R}^4$ and $T : \mathbb{R}^4 \to \mathbb{R}^3$ be linear transformations such that ToS is the identity map of $\mathbb{R}^3$. Then

(A) SoT is identity map of $\mathbb{R}^4$

(B) SoT is one-one but not onto

(C) SoT is onto but not one-one

(D) SoT is neither one-one nor onto

20. Let $M_3(\mathbb{R})$ be the vector space of all 3×3 real matrices. Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Which one of the following is not a subspace of $M_3(\mathbb{R})$?

(A) $\{X \in M_3(\mathbb{R}) : XA = AX\}$

(B) $\{X \in M_3(\mathbb{R}) : X + A = A + X\}$

(C) $\{X \in M_3(\mathbb{R}) : \text{trace}\,(AX) = 0\}$

(D) $\{X \in M_3(\mathbb{R}) : \det\,(AX) = 0\}$

21. Let A and B be two $n \times n$ matrices such that $BA^2 + B^2 = I - BA^3$, where I is the $n \times n$ identity matrix. Which one of the following is always true?

(A) A is non-singular

(B) B is non-singular

(C) $A + B$ is non-singular

(D) BA is non-singular

22. Which of the following is a subspace of $\mathbb{R}^3$?

(A) $\{(x,y,z) \in \mathbb{R}^3 : xyz = 0\}$

(B) $\{(x,y,z) \in \mathbb{R}^3 : x - y = 3\}$

(C) $\{(x,y,z) \in \mathbb{R}^3 : x + y + z = 1\}$

(D) $\{(x,y,z) \in \mathbb{R}^3 : x + y = 0\}$

23. Let W be the Wronskian of two linearly independent solutions of the ODE:

$$2\frac{d^2 y}{dt^2} + \frac{dy}{dt} + t^2 y = 0, \ t \in \mathbb{R}.$$

Then for all t, $\exists$ a constant $c \in \mathbb{R}$ such that $W(t)$ is

(A) ce^{-t}

(B) $ce^{-t/2}$

(C) ce^{2t}

(D) ce^{-2t}

24. The nature of $\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial y^2}$ is

(A) elliptic

(B) parabolic

(C) hyperbolic

(D) None of the above

25. The initial value problem $\frac{dy}{dx} = 3y^{2/3},\ y(0) = 0$ in an interval around $x = 0$ has

(A) no solution

(B) unique solution

(C) more than one solution

(D) infinite number of linearly independent solutions

26. It is necessary to find cumulative frequencies in order to draw a/an

(A) Histogram

(B) Frequency polygon

(C) Ogive curve

(D) Column chart

27. If one regression coefficient of the two regression lines is greater than unity, the other will be

(A) greater than 1

(B) 1

(C) less than 1

(D) 0·5

28. If X and Y are independent gamma variates with parameters μ and ν respectively, then the ratio $X/(X + Y)$ follows

(A) Beta distribution of type I with μ and ν parameters

(B) Beta distribution of type II with μ and ν parameters

(C) Gamma distribution with parameters μ and ν

(D) None of the above

29. If $X \sim N(0, 1)$, then the pdf of $Y = |X|$ *is*

(A) $g_Y(y) = e^{-y^2/2},\ y \geq 0$

(B) $g_Y(y) = \sqrt{2/\pi}\ e^{-y^2/2},\ y \geq 0$

(C) $g_Y(y) = \sqrt{2/\pi}\ e^{-y/2},\ y \geq 0$

(D) None of the above

30. If X and Y are independent Poisson variates, then the conditional distribution of X given $X + Y$, is

(A) normal

(B) binomial

(C) hypergeometric

(D) Poisson

31. Let E and F be two independent events with $P(E/F) + P(F/E) = 1,\ P(E \cap F) = \frac{2}{9}$ and $P(F) < P(E)$. Then $P(E)$ equals

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

32. Let the random variables X and Y have the joint pmf $p(x, y)$, then its distribution function is

(A) $F(x,y) = \sum_{i=1}^{i\,:\,x_i \le x} \sum_{j=1}^{j\,:\,y_j \ge y} p(x_i, y_j)$

(B) $F(x,y) = \sum_{i=1}^{i\,:\,x_i > x} \sum_{j=1}^{j\,:\,y_j \le y} p(x_i, y_j)$

(C) $F(x,y) = \sum_{i=1}^{i\,:\,x_i \ge x} \sum_{j=1}^{j\,:\,y_j \ge y} p(x_i, y_j)$

(D) $F(x,y) = \sum_{i=1}^{i\,:\,x_i \le x} \sum_{j=1}^{j\,:\,y_j \le y} p(x_i, y_j)$

33. The standard chi-squared test for a 2 by 2 contingency table is valid only if

(A) all the expected frequencies are greater than five

(B) both variables are continuous

(C) at least one variable is from a normal distribution

(D) None of the above

34. Let X be a random variable such that $E|X| < \infty$. Then $E|X - c|$ is minimized if we choose c is equal to

(A) the mean of the distribution.

(B) the median of the distribution.

(C) the quartile deviation of the distribution.

(D) the standard deviation of the distribution.

35. The test statistic for testing the significance of an observed partial correlation coefficient is

(A) $\dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

(B) $\dfrac{r_{12\cdot34\,...\,(k+2)}}{\left(1-r^2_{12\cdot34\,...\,(k+2)}\right)^{1/2}} \sqrt{n-k-2}$

(C) $\dfrac{r_{12\cdot34\,...\,(k+1)}}{\left(1-r^2_{12\cdot34\,...\,(k+1)}\right)^{1/2}} \sqrt{n-k-1}$

(D) $\dfrac{r_{12\cdot34\,...\,(k+2)}}{\left(1-r^2_{12\cdot34\,...\,(k+2)}\right)} \sqrt{n-k-1}$

36. An experiment is conducted under the following circumstances:

(a) when there are pairs of observations on two things being compared.

(b) for any pair, each of the two observations is made under similar extraneous conditions.

(c) different pairs are observed under different conditions.

In such a situation which test can be used?

(A) Paired t-test

(B) Sign test

(C) Median test

(D) Independent t-test

37. Homogeneity of several variances can be tested by

(A) Bartlett's test

(B) Fisher's exact test

(C) F-test

(D) t-test

38. Let $X_{1,\ldots,}X_n$ be a random sample of size n drawn from $R(0, \theta)$. Define $T_1 = X_{(n)}$, $T_2 = X_{(1)} + X_{(n)}$. Then which one is correct?

(A) T_1 is consistent but not T_2 .

(B) T_2 is consistent but not T_1 .

(C) Both are consistent.

(D) None of the above

39. Asymptotic distribution of U statistic is

(A) $N\left(\frac{mn}{2}, \frac{mn(m+n-1)}{12}\right)$

(B) $N\left(\frac{mn}{2}, \frac{mn(m+n+1}{12}\right)$

(C) $N\left(\frac{mn}{2}, \frac{mn(m-n-1)}{12}\right)$

(D) $N\left(\frac{mn}{2}, \frac{mn(m-n+1)}{12}\right)$

40. The statistic t for testing the hypothesis $\rho = 0$ based on a sample of size n from a bivariate population has degrees of freedom

(A) n

(B) $n - 1$

(C) $n - 2$

(D) $n - 3$

41. If in a linear programming problem the number of variables in primal is n, and the number of constraints in its dual is m, then

(A) $m = n$

(B) $m \geq n$

(C) $m \leq n$

(D) $m + n - 1 \geq 0$

42. Game is a situation where

(A) players have same objectives.

(B) players have conflicting objectives.

(C) players have no objectives.

(D) None of the above

43. The demand for an item is deterministic and constant over the time and it is equal to 2400 unit per year. The per unit cost of the item is ₹ 50 while the cost of placing an order is ₹ 5. The inventory carrying cost is 20% of the cost of inventory per annum and the cost of shortage is ₹ 12 per unit per year. When the stock outs are permitted the optimal ordering quantity is

(A) 33 units

(B) 66 units

(C) 181 units

(D) 88 units

44. When the equipment starts deteriorating with respect to time, its maintenance cost gradually starts/remains

(A) decreasing

(B) constant

(C) increasing

(D) zero

45. Under the proportional allocation, the size of the sample from each stratum depends on

(A) total sample size

(B) size of the stratum

(C) population size

(D) All of the above

46. A population is perfectly homogeneous in respect of a characteristic. What size of sample would you prefer?

(A) A large sample

(B) A small sample

(C) A single sample

(D) No item

47. Snowball sampling is a

(A) Random sampling

(B) Cluster sampling

(C) Non-random sampling

(D) None of the above

48. In the ANOVA, treatment refers to

(A) experimental units

(B) different levels of a factor

(C) a factor

(D) None of the above

49. The Standard Error (S.E.) of any treatment mean is given by for RBD

(A) S.E. $\left(\bar{t}_i\right) = s_E,\ i = 1, 2, \ldots, t$

(B) S.E. $\left(\bar{t}_i\right) = s_E / r,\ i = 1, 2, \ldots, t$

(C) S.E. $\left(\bar{t}_i\right) = s_E / r^{3/2},\ i = 1, 2, \ldots, t$

(D) S.E. $\left(\bar{t}_i\right) = s_E / \sqrt{r},\ i = 1, 2, \ldots, t$

where s_E stands for sum of squares due to error and r the no. of replication

50. For a factorial experiment with three factors N, P and K, each at two levels, the key block of a replicate is given below:

(1)	*pk*	*nk*	*np*

The confounded effect is

(A) *pk*

(B) *np*

(C) *nk*

(D) *npk*

ROUGH WORK

ROUGH WORK

ROUGH WORK