Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

A

Paper-II

MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

APR - 30217

Time Allowed : 1¼ Hours] **[Maximum Marks : 100**

Number of Pages in this Booklet : **28** Number of Questions in this Booklet : **84**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. (a) This paper consists of **Eighty Four (84)** multiple choice questions, each question carrying **Two (2)** marks.
   (b) There are *three* sections, Section-I, II, III in this paper.
   (c) Students should attempt all questions from Sections I and II *or* Sections I and III.
   (d) Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/ questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (iii) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.

   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. (a) या प्रश्नपत्रिकेत एकूण **चौऱ्यांशी (84)** बहुपर्यायी प्रश्न दिलेले आहेत, प्रत्येक प्रश्नाला **दोन (2)** गुण आहेत.
   (b) या प्रश्नपत्रिकेत खण्ड-I, II, III असे तीन खण्ड आहेत.
   (c) विद्यार्थ्यांनी खण्ड-I आणि II किंवा खण्ड I आणि III यांचे सगळे प्रश्न सोडावे.
   (d) खाली दिलेल्या प्रश्नाचे चार पर्याय किंवा उत्तर दिलेले आहेत. प्रश्नाचे बहुपर्यायी उत्तरांमधून केवळ एक 'बरोबर' आहे.
3. परीक्षा सुरू झाल्यानंतर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (i) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (ii) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (iii) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.

   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science

Paper II

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Section I

1. The function $f(x) = x + \frac{1}{2}\cos x$, $x \in \mathbf{R}$ is :

(A) non-periodic

(B) periodic with period 2π

(C) periodic with period π

(D) periodic with period 4π

2. Lim sup of the sequence $a_n = \frac{1}{n} + \sin\frac{n\pi}{3}$ is :

(A) $1 + \frac{\sqrt{3}}{2}$

(B) $1 - \frac{\sqrt{3}}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{\sqrt{2}}$

3. If $f(z) = z + 1$, then $\frac{1}{f(2 + 3i)}$ is :

(A) $\frac{1 + i}{6}$

(B) $\frac{1 - i}{6}$

(C) $\frac{2 - 3i}{13}$

(D) $3 + 3i$

4. $\int\limits_{|z| = 2} \frac{(z + 1)dz}{z}$ is :

(A) $-2\pi i$

(B) 2π

(C) $2\pi i$

(D) $4\pi i$

5. Let A be $n \times n$ real matrix. Then $\det((\det A)\,A) =$

(A) det A

(B) $(\det A)^2$

(C) $(\det A)^n$

(D) $(\det A)^{n+1}$

6. If $A = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$, then AA^t is :

(A) orthogonal

(B) symmetric

(C) skew-symmetric

(D) not defined

7. The following system of equations :

$$x + 2y + 2z = 5$$
$$2x + 2y + 3z = 5$$
$$3x + 4y + 5z = 9$$

(A) has a unique solution

(B) has no solution

(C) has two solutions

(D) has infinitely many solutions

8. If A is orthogonal matrix, then :

(A) $\det A = \pm 1$

(B) $\det A = 1$

(C) $\det A = 0$

(D) $\det A \neq 1$

9. Let $X_1, X_2, \ldots\ldots, X_n$ be independent rvs with exponential distribution having mean one. Define :

$Z = \text{Max}(X_1, X_2, \ldots\ldots, X_n)$ and

$W = X_1 + \frac{X_2}{2} + \frac{X_3}{3} + \ldots\ldots + \frac{X_n}{n}$.

Then :

(A) MGF of $Z = \prod_{i=1}^{n} \frac{1}{i-t}$

(B) MGF of $W = \prod_{i=1}^{n} \frac{i}{i-t}$

(C) MGF of Z and W are the same

(D) MGF of Z and W are different

10. Let X_1 and X_2 be independent rvs with $U(0, \theta_i)$; $i = 1, 2$ respectively.

Let $Z_1 = \text{Min}(X_1, X_2)$ and

$$Z_2 = \begin{cases} 0 \; ; & Z_1 = X_1 \\ 1 \; ; & Z_1 = X_2 \end{cases}$$

Which of the following statements is *correct* ?

(A) $P[Z_2 = 1] = \dfrac{\theta_1}{2\theta_2}$; if $\theta_1 < \theta_2$

(B) $f(Z_1) = \dfrac{\theta_1 + \theta_2}{\theta_1\theta_2} - \dfrac{2Z_1}{\theta_1\theta_2}$;

$0 < Z_1 < \theta_2$ and $\theta_1 < \theta_2$

(C) $f(Z_1) = \dfrac{\theta_1 + \theta_2}{\theta_1\theta_2} - \dfrac{2Z_1}{\theta_1\theta_2}$;

$0 < Z < \theta_1$ and $\theta_1 > \theta_2$

(D) Cannot find the probability mass function of Z_2 if $\theta_1 > \theta_2$

11. If $P(A \cup B) = 0.7$ and $P(A \cup B^C) = 0.9$, then P(A) is given by :

(A) 0.2

(B) 0.4

(C) 0.6

(D) 0.8

12. A sample space consists of five simple events E_1, E_2, E_3, E_4 and E_5. If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$ and $P(E_4) = 2P(E_5)$, then probability of E_4 and E_5 is :

(A) (0.30, 0.20)

(B) (.20, .30)

(C) (0.10, 0.30)

(D) (0.20, 0.10)

13. If $P(D|F)$ and $P(E|F)$ and $P(D|\bar{F}) > P(E|\bar{F})$, then the relation between P(D) and P(E) is :

(A) $P(D) > P(E)$

(B) $P(D) < P(E)$

(C) $P(D) = P(E)$

(D) Cannot be determined

14. For a set of m equations in n variables $(n > m)$, a solution obtained by setting $(n - m)$ variables equal to zero and solving for remaining m equations in m variables is called a/an :

(A) basic solution

(B) feasible solution

(C) basic feasible solution

(D) optimum solution

15. The set $B = \left\{(x_1, x_2) : x_1^2 + x_2^2 \le 4\right\}$ is a :

(A) convex set

(B) concave set

(C) unbounded set

(D) concave and convex set

16. The problem Max $Z = 3x_1 + 2x_2$ subject to :

$$x_1 + x_2 \le 1$$

$$2x_1 + 2x_2 \ge 4$$

$$x_1, x_2 \ge 0$$

has :

(A) No solution

(B) Optimum solution

(C) Feasible solution

(D) Feasible but not optimal solution

Section II

17. If $f(x) = x|x|$, $x \in [-1, 1]$, then :
(A) f is not continuous
(B) f is continuous but not differentiable
(C) f is differentiable but not continuously differentiable
(D) f is c^1 but not c^2

18. The Taylor series for $\cos z$ around 0 is :
(A) $1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \ldots\ldots..$
(B) $1 + \frac{z^2}{2!} + \frac{z^4}{4!} - \ldots\ldots..$
(C) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots\ldots..$
(D) $z + \frac{z^3}{3!} + \frac{z^5}{5!} + \ldots\ldots..$

19. Minimal polynomial of the matrix :

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 4 & -2 \\ 0 & 3 & -1 \end{pmatrix}$$

is :
(A) $(\lambda - 1)(\lambda - 2)^2$
(B) $(\lambda - 1)^2 (\lambda - 2)$
(C) $(\lambda - 1)(\lambda - 2)$
(D) $(\lambda - 1)^2$

20. If A is a 3×3 real matrix such that $A^3 = 0$ and $A^2 \neq 0$. Then the Jordan canonical form of A is :

(A) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

21. Let I be the set of integers. The points of discontinuity of $\tan x$ are :

(A) $n\pi$, $n \in$ I

(B) $2n\pi$, $n \in$ I

(C) $\frac{(2n+1)\pi}{2}$, $n \in$ I

(D) $\frac{n\pi}{2}$, $n \in$ I

22. The function $f(x) = x^2$, $x \in \mathbf{R}$ is :

(A) Lipschitz but not locally Lipschitz

(B) Locally Lipschitz but not Lipschitz

(C) Neither Lipschitz nor locally Lipschitz

(D) Lipschitz as well as locally Lipschitz

23. Consider sequences of functions :

$$f_n(x) = \frac{nx}{1 + n^2x^2}, \ |x| \le 1$$

$$g_n(x) = \frac{x}{1 + nx^2}, \ |x| \le 1$$

(A) Sequence f_n is uniformly convergent but sequence g_n is not uniformly convergent

(B) Sequence f_n is not uniformly convergent but sequence g_n is uniformly convergent

(C) Neither f_n nor g_n converges uniformly

(D) Both f_n and g_n converge uniformly

24. The maximum value of the function $y(x) = x(x - 1)^2$, $0 \le x \le 2$ is :

(A) 0

(B) 4

(C) 2

(D) 4/27

25. Let f be a real valued function defined on $[a, b]$. Which of the following statements is *false* ?
(A) If f is continuous on $[a, b]$, then it is Riemann integrable on $[a, b]$
(B) If f is Riemann integrable, then $|f|$ is Riemann integrable
(C) If $|f|$ is Riemann integrable, then f is Riemann integrable
(D) If f is Riemann integrable, then f^2 is Riemann integrable

26. The radius of convergence for the series :

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{.........}$$

is :
(A) 0
(B) 1
(C) e
(D) ∞

27. The complex function $f(z) = x^2 + y^2 + 1$ has derivative :
(A) everywhere
(B) nowhere
(C) only at $z = 0$
(D) only on the points on x-axis and y-axis.

28. The function $-e^x \cos y + 1$:
(A) is not harmonic
(B) is harmonic and its harmonic conjugate is $-e^x \sin y + c$, c constant.
(C) is harmonic and its harmonic conjugate is itself
(D) is harmonic and its harmonic conjugate is $-e^x \sin y + y + c$, c constant

29. The complex numbers z_1, z_2, z_3, z_4 are either collinear or they lie on a circle if their cross ratio :

$$\frac{z_1 - z_3}{z_2 - z_3} \cdot \frac{z_2 - z_4}{z_1 - z_4}$$

is :

(A) 5

(B) $5i$

(C) $2i$

(D) $1 + i$

30. Let D be a region (non-empty, open, connected) in **C**. An analytic function on D :

(A) has continuous derivative on D, but the derivative may not be analytic on D.

(B) may not have continuous derivative on D.

(C) has derivatives of all orders on D.

(D) may have derivative of some order as a discontinuous function on D.

31. Which of the following is an essential singularity of :

$$f(z) = \frac{z-3}{z-1} + e^{1/(z-2)} \ ?$$

(A) $z = 1$

(B) $z = 0$

(C) $z = 3$

(D) $z = 2$

32. Let D be a non-empty connected open subset of **C** and let $f : D \to \mathbf{C}$ be an analytic function. Which of the following statements is not equivalent to each of the remaining three statements ?

(A) $f \equiv 0$

(B) f has infinitely many zeros in D

(C) There is a complex number a in D such that $f^{(n)}(a) = 0$ for all $n \geq 0$

(D) $\{z \in D \mid f(z) = 0\}$ has a limit point in D

33. In which of the following groups every subgroup is a normal subgroup ?

(A) The group of invertible upper triangular real 2 × 2 matrices $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ $(a, d \neq 0)$.

(B) The permutation group s_3 (on a three elements set).

(C) The group of all non-zero $n \times n$ matrices over real numbers w.r.t. multiplication such that all the entries of the matrix are equal.

(D) The group of all bijective linear transformations from $\mathbf{R}^n$ to $\mathbf{R}^n$.

34. Consider the following three groups :

$G_1 = <\mathbf{C}^*, \bullet>$ — the multiplicative group of non-zero complex numbers.

$G_2 = <\mathbf{R}, +>$ — the additive group of real numbers.

$G_3 = <\mathbf{R}^*_+, \bullet>$ — the multiplicative group of positive reals.

Which of the following is *true* ?

(A) $G_2 \cong G_3$

(B) $G_1 \cong G_3$

(C) $G_1 \cong G_2$

(D) G_1 is not cyclic but G_2 and G_3 are cyclic

35. Which of the following is *true* ?

(A) The ring of polynomials over a ring is an integral domain.

(B) The ring of $n \times n$ — matrices over a field is an integral domain.

(C) If in the ring $M_n(F)$ of $n \times n$ — matrices over a field F, A is a matrix such that $AB \neq 0$ for every $B(\neq 0) \in M_n(F)$, then A is invertible.

(D) If in the ring of polynomials over a ring R we have a polynomial $f(x)$ such that $f(\alpha) = 0$ for every $\alpha \in R$ then $f \equiv 0$ polynomial.

36. The group of automorphisms of the cyclic group of order 7 is isomorphic to :

(A) $(\mathbf{Z}_6, +)$

(B) $(\mathbf{Z}_7, +)$

(C) $(\mathbf{Z}_5, +)$

(D) $(\mathbf{Z}_8, +)$

37. Let G be a finite group of order n. Then G is isomorphic to :

(A) S_n

(B) a quotient group of S_n

(C) a subgroup of S_n

(D) a normal subgroup of S_n

38. Let F be a field with 64 elements. Then which of the following statements is *true* ?

(A) The multiplicative group of non-zero elements of F is cyclic.

(B) F is a vector space over $\mathbf{Z}_3$.

(C) The characteristic of F is 4

(D) F is a direct product of two non-trivial fields.

39. If a real matrix A has the characteristic polynomial as $(x-2)^2$ $(x-1)$ $(x-3)$ and minimum polynomial as $(x-2)(x-1)(x-3)$, then which of the following statements is *not true* ?

(A) Trace A = 8

(B) Det A = 12

(C) A is diagonalizable over **R**

(D) A is not diagonalizable over **R**

40. If the characteristic polynomial of matrix A is $(x-1)^2$ $(x-2)^2$ and the minimum polynomial is $(x-1)^2$ $(x-2)$, then the Jordan canonical form of A is :

(A) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

41. Let :

$S = \{(x_1, x_2, x_3) \in \mathbf{R}^3 \mid x_1 + x_2 + x_3 = 0\}$

$T = \{(x_1, x_2, x_3) \in \mathbf{R}^3 \mid x_1 x_2 + x_3 = 0\}$

Then which of the following statements is *correct* ?

(A) S is a vectorspace of dimension 2 and T is a vectorspace of dimension 2.

(B) S is a vectorspace of dimension 2 but T is not a vectorspace.

(C) T is a vectorspace of dimension 2 but S is not a vectorspace.

(D) Neither S nor T is a vectorspace.

42. Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be defined as :

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x + y \\ 2x + y - z \end{pmatrix}$$

Then the matrix representation of T with respect to standard basis is :

(A) $\begin{pmatrix} -1 & 2 \\ 1 & 1 \\ 0 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} -1 & 1 & 0 \\ 2 & 1 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 0 \end{pmatrix}$

43. Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ have characteristic polynomial $(x - 5)^3 (x - 6)$. If T is diagonalizable, then :

(A) dim ker (T − 5I) = 1

(B) dim ker (T − 5I) = 3

(C) minimal polynomial of T is $(x - 5)^3 (x - 6)$

(D) minimal polynomial of T is $(x - 5)^2 (x - 6)$

44. Let u and v be vectors in an inner product space such that $||u + v|| = 8$, $||u - v|| = 6$ and $||u|| = 7$. Then $||v|| =$

(A) $\sqrt{51}$

(B) 1

(C) $\sqrt{2}$

(D) 2

45. The partial differential equation :

$u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0$

(A) is hyperbolic on $\mathbf{R}^2$

(B) is parabolic on $\mathbf{R}^2$

(C) is elliptic on $\mathbf{R}^2$

(D) is hyperbolic on the set $S = \{(x, y) \in \mathbf{R}^2 \mid x > 0\}$

46. Consider the following statements :

I : If ϕ_1, ϕ_2 are linearly independent functions on an interval I, they are linearly independent on any interval J contained inside I.

II : If ϕ_1, ϕ_2 are linearly independent solutions of $L(y) = 0$ on an interval I, they are linearly independent on any interval J contained inside I.

Then :

(A) Both I and II are true

(B) Both I and II are false

(C) Only I is true

(D) Only II is true

47. The solution of the differential equation :

$$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

is :

(A) $x + e^{\frac{x}{y}} = c$

(B) $x + ye^{\frac{x}{y}} = c$

(C) $y + e^{\frac{x}{y}} = c$

(D) $y + xe^{\frac{x}{y}} = c$

48. The function $f(x, y) = y^2$ satisfies a Lipschitz condition on :

(A) $\{(x, y) \mid |x| < \infty, |y| < \infty\}$

(B) $\{(x, y) \mid |x| \le a, |y| < \infty, (a > 0)\}$

(C) $\{(x, y) \mid -7 \le x \le 10, |y| < \infty\}$

(D) $\{(x, y) \mid |x| \le a, |y| \le b, (a, b > 0)\}$

49. The partial differential equation obtained by eliminating arbitrary constants a and b from :

$$z = a \cdot e^{bx} \sin by$$

is :

(A) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial y} = 0$

(B) $\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$

(C) $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} = 0$

(D) $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} = 0$

50. The complete integral of the partial differential equation :

$$xpq + yq^2 - 1 = 0$$

is :

(A) $z^2 = (ax + y)^2 + by$

(B) $(z + b) = 4(ax + y)^2$

(C) $(z + b)^2 = 4(ax + y)$

(D) $z^2 = (ax + y)^2 + bx$

Section III

51. Let the regression lines of Y on X and X on Y be $Y = aX + b$ and $X = cY + d$ respectively. Consider the following statements :

(1) The ratio of the variances of X and Y is c/a.

(2) The correlation coefficient between X and Y is $\sqrt{ac}$.

(3) The values of $\bar{X}$ and $\bar{Y}$ are $\frac{cb + d}{1 - ac}$ and $\frac{ad + b}{1 - ac}$ respectively.

Which of the above statements are *correct* ?

(A) 1 and 2 only

(B) 1 and 3 only

(C) 2 and 3 only

(D) All are correct

52. If $Y = aX + 3$ and $X = 2Y + 6$ are the regression lines of Y on X and X on Y respectively, then which one of the following is *correct* ?

(A) $0.5 \leq a \leq 1$

(B) $0 \leq a \leq 0.5$

(C) $-0.5 \leq a \leq 0$

(D) $a \geq 1$

53. Let $X_n = X$ and $Y_n = -X$, where $X \sim N(0, 1)$. Let $Y \sim N(0, 1)$. Then, which of the following is more appropriate ?

(A) $X_n \xrightarrow{d} X, X_n + Y_n \xrightarrow{d} X + Y$

(B) $X_n \xrightarrow{d} X, Y_n \xrightarrow{d} Y$

(C) $Y_n \xrightarrow{d} Y, X_n + Y_n \xrightarrow{d} X + Y$

(D) $X_n \xrightarrow{d} X$, $Y_n \xrightarrow{d} Y$, $X_n + Y_n \xrightarrow{d} X + Y$

54. Let $\{X_n\}$ be a sequence of iid random variables with $E(X_1) \neq 0$, and var $(X_1) < \infty$. Then, which of the following statements is *true* ?

(A) $\frac{1}{n} \sum_{i=1}^{n} j X_j \xrightarrow{p} E(X_1)$

(B) $\frac{1}{n(n+1)} \sum_{j=1}^{n} j X_j \xrightarrow{p} E(X_1)$

(C) $\frac{1}{n+1} \sum_{j=1}^{n} j X_j \xrightarrow{p} 0$

(D) $\frac{1}{n(n+1)} \sum_{j=1}^{n} X_j \xrightarrow{p} E(X_1)$

55. Let X be a non-constant positive random variable with finite mean. Then, which of the following is *true* ?

(A) $E(\log X) < \log E(X)$

(B) $[E(X)]^{-1} > E(X^{-1})$

(C) $\log E(X) < E(\log X)$

(D) $|E(X)| \geq E(|X|)$

56. Let X be a random variable with probability density function $f_X(x)$. Let $Y = X^2$ then, the probability density function of Y is :

(A) $\left[f_X\left(\sqrt{x}\right) + f_X\left(-\sqrt{x}\right)\right]/2$, $x > 0$

(B) $\left[f_X\left(\sqrt{x}\right) + f_X\left(-\sqrt{x}\right)\right]/2\sqrt{x}$, $x > 0$

(C) $\left[f_X(x)\right]^2$, $x > 0$

(D) $2f_X\left(\sqrt{x}\right) - 1$, $x > 0$

57. Let X_1 and X_2 be two independent random variables such that $X_1 + X_2$ is a degenerate random variable. Then :

(A) X_1 is a degenerate random variable, but X_2 is not

(B) X_2 is a degenerate random variable, but X_1 is not

(C) Both X_1 and X_2 are degenerate

(D) Not possible to conclude whether X_1 and X_2 are degenerate

58. If the r.v. X has $N(\mu, \sigma^2)$ then the distribution of $Z = \left(\frac{X - \mu}{\sigma}\right)^2$ is :

(A) Normal (0, 1)

(B) Chi-square with 1 df

(C) Gamma (3/2, 1/2)

(D) Half-normal

59. Let X_1, X_2 be distributed as Poisson (λ) then, the distribution of $2X_1 + X_2$ is :

(A) Poisson (3λ)

(B) Poisson (2λ)

(C) Poisson (λ)

(D) None of the above

60. Let X_1, X_2 have the following pdf :

$$f(x \mid \lambda) = \frac{\lambda}{(1 + x)^{\lambda + 1}} ; x > 0$$

The distribution Z = (log 1 + X_1) + log (1 + X_2) is :

(A) Gamma (2, λ)

(B) Gamma (1, 2λ)

(C) Gamma (1, λ)

(D) Gamma (2, log λ)

61. Let $X_1, X_2, ..., X_m$ be iid rvs with $B(n, p)$. The distribution $X_{(m)} = \text{Max } X_i$ is :

(A) $\left(\sum_{i=0}^{x} \binom{n}{i} p^i q^{n-i}\right)^m - \left(\sum_{i=0}^{x-1} \binom{n}{i} p^i q^{n-i}\right)^m$

(B) $m\left[\sum_{i=0}^{x} \binom{n}{i} p^i q^{n-i}\right]^{m-1}$

(C) $m\left[\sum_{i=0}^{x-1} \binom{n}{i} p^i q^{n-i}\right]^{m-1}$

(D) $m \cdot \left[\sum_{i=0}^{x-2} \binom{n}{i} p^i q^{n-i}\right]^{m-2}$

62. Suppose X_1, X_2, ..., X_n has the following pdf :

$f(x \mid \theta) = \theta x^{\theta - 1}$; $0 < x < 1$, $\theta > 0$

The distribution of $\sum_{i=1}^{n} -\log X_i$ is :

(A) Same as $f(x \mid \theta)$

(B) Gamma $(n, 1/\theta)$

(C) Gamma (n, θ)

(D) Gamma $(1, \theta)$

63. Let X_1, X_2, ..., X_n be a random sample from normal distribution with mean θ and variance σ^2, where σ^2 is known. The confidence coefficient of the confidence interval $\left(\bar{X} \pm K\sigma/\sqrt{n}\right)$, where $K > 0$ is :

(A) an increasing function of K

(B) an increasing function of σ

(C) decreasing function of n

(D) constant for all values of K

64. Let X_1, X_2, ..., X_n be a random sample from one-parameter exponential family of a distributions, which has monotone likelihood ratio in statistic $T(\underset{\sim}{x})$. Suppose $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$ is to be tested. Then MP test based on $T(\underset{\sim}{x})$ for testing H_0 against H_1 is :

(A) not UMP

(B) UMP for all $\theta_1 > \theta_0$

(C) UMP for all $\theta_1 \neq \theta_0$

(D) UMP for all $\theta_1 < \theta_0$

65. Let X_1, X_2, ..., X_n be a random sample from normal distribution with mean θ and known variance σ^2. Then UMVUE of θ^2 is :

(A) $\bar{X}^2$

(B) $\bar{X}^2 - 1/n$

(C) $\bar{X}^2 + 1/n$

(D) $\bar{X}^2 - \dfrac{\sigma^2}{n}$

66. The random variable X has geometric distribution with parameter 'p'. A random sample of size n observations contains all the observations greater than or equal to an integer 'r'. Then MLE of p is :

(A) r/n

(B) greater than r/n

(C) less than or equal to $1/(r + 1)$

(D) greater than or equal to $1/(r + 1)$

67. Let X be a r.v. with first kind beta distribution with parameters (α, β). Then based on a single observation on X, Most Powerful test for testing $H_0 : (\alpha, \beta) = (1, 1)$ against $H_1 = (\alpha, \beta) = (3, 1)$ has critical region of the form :

(A) $\{x : x < c_1 \text{ or } x > c_2\}, c_1 < c_2$

(B) $\{x : x < c\}$

(C) $\{x : x > c\}$

(D) $\{x : c_1 < x < c_2\}$

where c, c_1 and c_2 are suitable constants.

68. Based on a random sample $X_1, X_2, ..., X_n$ of size n from double exponential distribution with location parameter θ and scale parameter 1, sufficient statistic for θ is :

(A) $\sum_{i=1}^{n} X_i$

(B) sample median

(C) the entire sample $X_1, X_2, ..., X_n$

(D) $\sum_{i=1}^{n} |X_i|$

69. For testing $H_0 : \sigma = \sigma_0$ in a normal population $N(0, \sigma^2)$, the critical region is obtained as $\sum X_i^2 < K$. For which alternative hypothesis does this provide a UMP test ?

(A) $\sigma > \sigma_0$

(B) $\sigma < \sigma_0$

(C) $\sigma \neq \sigma_0$

(D) $\sigma = \sigma_1$

70. Consider a single observation from the population :

$f(x; \theta) = \theta e^{-\theta x}$, $0 < x < \infty$.

Let $X \geq 1$ be the critical region for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$. What are the values of type 1 and type 2 errors respectively ?

(A) $\frac{e-1}{e}, \frac{1}{e^2}$

(B) $\frac{1}{e}, \frac{e-1}{e^2}$

(C) $\frac{e-1}{e^2}, \frac{1}{e}$

(D) $\frac{1}{e^2}, \frac{e-1}{e}$

71. In usual notations, if a quantity $\phi(T, \theta)$ exists, whose distribution is independent of θ, then which of the following statements are *correct* ?

(1) $\phi(T, \theta)$ is called a confidence interval.

(2) $\phi(T, \theta)$ is a pivotal quantity.

(3) $\phi(T, \theta)$ can be used to define $(1 - \alpha)$ 100% confidence interval for θ.

Select the *correct* answer using the choice given below :

(A) (1) and (2) only

(B) (1) and (3) only

(C) (2) and (3) only

(D) All are correct

72. In the general linear model $Y = X\beta + \varepsilon$, the unbiased estimator of the variance of the error term when $\hat{\varepsilon} = Y - X\hat{\beta}$ is :

(A) $\frac{\hat{\varepsilon}\hat{\varepsilon}'}{n - K}$

(B) $\frac{\hat{\varepsilon}'\hat{\varepsilon}}{n - K}$

(C) $\frac{\hat{\varepsilon}'\hat{\varepsilon}}{n}$

(D) $\frac{\hat{\varepsilon}\hat{\varepsilon}'}{K}$

73. If F_X is any continuous distribution function, and D_n^+ is the Kolmogorov-Smirnov test Statistics, then $\forall Z \geq 0$, the limiting distribution of $V = 4n D_n^{+2}$ as $n \to \infty$ is :

(A) Chi-square distribution with 2 df

(B) Chi-square distribution with n df

(C) Chi-square distribution with $(n - 1)$ df

(D) Chi-square distribution with $2n$ df

74. The first of two samples consists of 23 pairs and gives a correlation of 0.5 while the second of 28 pairs has a correlation of 0.8. Which of the following is *true* ?

(A) The difference is not significant.

(B) The difference is significant at 1% level.

(C) The difference is significant at 5% level.

(D) The difference is significant at 10% level.

75. The average number of units in M/M/1 : (∞, FICFS) model is equal to :

(A) ρ

(B) $\rho/(1 - \rho)$

(C) $1 - \rho$

(D) $\frac{1 - \rho}{\rho}$

76. There are 6 jobs each of which must go through the two machines A and B in the order of AB. The processing times (in hrs) are given as :

Jobs	Time for A	Time for B
1	4	6
2	8	3
3	3	7
4	6	2
5	7	8
6	5	4

Write down the sequence of jobs that minimizes the total elapsed time required to complete the job.

(A) 4—3—2—1—6—5

(B) 3—1—5—6—2—4

(C) 4—2—3—1—6—5

(D) 1—2—4—3—6—5

77. If the half yearly demand of an item is 1600 units, inventory carrying charges 25% per annum, the unit cost is Rs. 6.00 and the cost of per order is Rs. 150, then the economic order quantity will be :

(A) $400\sqrt{2}$

(B) 800

(C) $800\sqrt{2}$

(D) 400

78. If dual has an unbounded solution, then primal has :

(A) Unbounded solution

(B) Feasible solution

(C) No feasible solution

(D) Optimal solution

79. Let there be m-origins, ith origin possessing a_i units of a certain item, whereas there are n-destinations with jth destination requiring b_j units in a transportation problem, then the number of basic variables are at the most :

(A) $2(m + n)$

(B) $m + n - 2$

(C) $m + n$

(D) $m + n - 1$

80. Consider the coin matching game involving two players A and B. Each player selects either a head (H) or a tail (T). If the coin match, then player A wins Re. 1 from player B, otherwise player B wins Re. 1 from player A. Then the optimum strategies for the players and the value of the game is :

(A) (1, 0), (0, 1), V = 1

(B) (1, 0), (0, 1), V = 0

(C) $\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)$, V = 0

(D) $\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)$, V = 1

81. Suppose the variable Y is uniformly distributed over $(b, b + h)$. The range is divided into L strata of equal sizes. A simple random sample of size n/L is selected from each stratum. If V and V_1 denote the variance of estimator (mean) for a simple random sample of size n and a stratified random sample respectively, then :

(A) $V_1 = V$

(B) $V_1 = VL^2$

(C) $V_1 = VL$

(D) $\frac{V_1}{V} = \frac{1}{L^2}$

82. Which of the following would generally require the largest sample size ?

(A) Simple random sampling

(B) Stratified random sampling

(C) Cluster sampling

(D) Systematic sampling

83. Layout of a block design with 5 blocks and 4 treatments A, B, C, D is given below :

Block	1	A, B, C
	2	A, B, D
	3	A, C, D
	4	B, C, D
	5	A, B, C, D

Which one of the following statements is *true* ?

(A) Every treatment occurs same number of times.

(B) Every block contains same number of plots.

(C) Every pair of treatment occurs three times.

(D) Any two blocks contains same number of treatments common.

84. The degrees of freedom for the error sum of squares in a Latin square design with V rows, V columns and V treatments with two missing observation is :

(A) $V^2 - 3$

(B) $(V - 1)(V - 2) - 2$

(C) $(V - 1)(V - 2)$

(D) $(V - 1)^2 - 2$

ROUGH WORK

ROUGH WORK