Roll No. | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilators

1. ______________________

2. ______________________

1515

Question Booklet Series **A**

PAPER–III

Question Booklet No.

Subject Code : 15

OMR Sheet No. ....

(To be filled by the candidate)

MATHEMATICAL SCIENCES

Time : 2 Hours 30 Minutes *Maximum Marks: 150*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet and the OMR Sheet Number should be entered on this Question Booklet.
3. This paper consists of seventy-five (75) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

[Please Turn Over]

1. The Laplace transform of a function $f(t)$ defined for $t \geq 0$ is defined by

$$F(p) = \int_0^\infty e^{-pt} f(t)dt.$$

If $f(t) = e^{t^2}$, then

(A) $F(p) = e^{p^2}$

(B) $F(p) = e^{-p^2}$

(C) $F(p) = e^{p}$

(D) $F(p)$ does not exist

2. Laplace transform of a function $f(t)$ is denoted by $\mathcal{L}[f(t), p] = \int_0^\infty f(t)e^{-pt}dt$.

Then $\mathcal{L}\left[\frac{sin\omega t}{t}, p\right]^0$ is given by

(A) $tan\, p/\omega$

(B) $cot\, p/\omega$

(C) $cos\, p/\omega$

(D) $cot^{-1} p/\omega$

3. Fourier transform of a function $f(t)$ is denoted by $\mathfrak{F}[f(t), \xi] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{i\xi t}dt$.

Then $\mathfrak{F}\left[e^{-a|t|}, \xi\right]$ is $(a > 0)$

(A) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2+\xi^2}$

(B) $\sqrt{\frac{2}{\pi}} \frac{\xi}{a^2+\xi^2}$

(C) $\sqrt{\frac{2}{\pi}} \frac{1}{a^2+\xi^2}$

(D) does not exist

4. The velocity field for a fluid motion is given by $\left(\frac{3x_1^2-r^2}{r^5}, \frac{3x_1x_2}{r^5}, \frac{3x_1x_3}{r^5}\right)$ where $r^2 = (x_1^2 + x_2^2 + x_3^2)$, the motion is

(A) rotational

(B) irrotational

(C) cyclic

(D) None of (A), (B), (C)

5. The complex potential for a two-dimensional irrotational motion is given by $\omega = m \log Z$ where $Z = re^{i\theta}$. Then streamlines for the motion are

(A) straight lines issuing from the origin.

(B) concentric circles about the origin.

(C) confocal ellipses.

(D) None of (A), (B) and (C)

6. Using Euler's method the value of $y(0.02)$ for the differential equation $\frac{dy}{dx} + y = 0$ with $y(0) = 1$ taking $n = 0.01$ is

(A) 0.9801

(B) 0.9800

(C) 0.9802

(D) 0.9804

7. Let $\left\{0, \frac{1}{2}, 1\right\}$ be three distinct points on $[0,1]$. Let $p(x)$ be an unique polynomial of suitable degree in $[0,1]$ such that $p(0) = 0$, $p\left(\frac{1}{2}\right) = 0$, $p(1) = 1$. Then $p\left(\frac{1}{4}\right)$ is equal to

(A) $-\frac{1}{8}$

(B) $-\frac{1}{2}$

(C) $\frac{2}{5}$

(D) 1

8. The midpoint integration rule

$$\int_0^\pi f(x)dx \approx \pi f\left(\pi/2\right)$$ is exact if

(A) $f(x)$ is a trigonometric function of x.

(B) $f(x)$ is a linear function of x.

(C) $f(x)$ is an exponential function of x.

(D) $f(x)$ is a quadratic function of x.

9. The rate of convergences of secant method, Newton-Raphson and bisection method are

(A) 1, 2, 2
(B) 1.6, 2, 1
(C) 2, 2, 1.6
(D) 1, 2, 1.6

10. The extremal of the functional
$I[y(x)] = \int_0^1 (y^2 + y'^2)dx$
satisfying $y(0) = 0, y(1) = 1$ is

(A) $\frac{coshx}{cosh1}$
(B) $\frac{sinhx}{sinh1}$
(C) $\frac{cosh1}{coshx}$
(D) $\frac{sinh1}{sinhx}$

11. The extremal of the functional

$J[y] = \int_1^2 \frac{(1+y'^2)^{\frac{1}{2}}}{x} dx \quad \left(y' = \frac{dy}{dx}\right)$
with $y(1) = 0, y(2) = 1$ is

(A) $(y-2)^2 + x^2 = 5$
(B) $(y-1)^2 + (x-1)^2 = 1$
(C) $y - x = -1$
(D) $3y = x^2 - 1$

12. The solution of $y'' - y + x = 0$ optimizes $I[y] \equiv \int_0^1 F(x, y, y')dx$ if $F(x, y, y')$ is

(A) $2xy - y^2 - y'^2$
(B) $2xy + y^2 - y'^2$
(C) $2xy + y^2 + y'^2$
(D) $-2xy - x^2 - y'^2$

13. In rotational motion of a symmetric rigid body (e.g. sphere) the angular velocity $\vec{\omega}$ and the angular momentum $\vec{\Omega}$ are

(A) always same in magnitude and direction.
(B) always same in magnitude but are different in directions.
(C) in general, different in magnitude but have same direction.
(D) always different in magnitudes and directions.

14. The Hamilton's equation of motion for an isotropic oscillator moving in 2D with force constant λ in terms of cartesian coordinates as generalized coordinates is

(A) $\dot{x} = p_x,\ \dot{y} = p_y,\ \dot{p}_x = \lambda x,\ \dot{p}_y = \lambda y$
(B) $\dot{x} = p_x, \dot{y} = -p_y, \dot{p}_x = -\lambda x, \dot{p}_y = -\lambda y$
(C) $\dot{x} = p_x,\ \dot{y} = p_y,\ \dot{p}_x = -\lambda x,\ \dot{p}_y = -\lambda y$
(D) $\dot{x} = -p_x, \dot{y} = p_y, \dot{p}_x = -\lambda x, \dot{p}_y = -\lambda y$

15. A two dimensional surface S⊂$\mathbb{R}^3$ is called a surface of positive, zero and negative curvature if and only if the

(A) mean curvature $H > 0, H = 0, H < 0$ in some neighbourhood of some points within S respectively.
(B) Gaussian curvature $K > 0, K = 0, K < 0$ in some neighbourhood of some point within S respectively.
(C) Gaussian curvature $K > 0, K = 0, K < 0$ at all points of the surface respectively.
(D) mean curvature $H > 0, H = 0, H < 0$ at some points within the surface.

16. The mean curvature H and the Gaussian curvature K of the two dimensional surface $Z = f(x, y)$ embedded in $\mathbb{R}^3$ are

(A) $H = \nabla^2 f,\ K$= determinant of Hessian of f.
(B) H=Determinant of Hessian of f, $K = \nabla^2 f$.
(C) $H = \nabla^2 f, K$=Hessian of f.
(D) H= Hessian of $f, K = \nabla^2 f$.

17. Integral equation corresponding to the initial value problem

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0, 0 \leq x \leq 1,$
$y(0) = 1, \frac{dy}{dx}(0) = 0$ is

(A) $y(x) + \int_0^x t\, y(t)dt = 1$
(B) $y(x) - \int_0^x t\, y(t)dt = 1$
(C) $y(x) + \int_0^x t\, y(t)dt = 0$
(D) $y(x) - \int_0^x t\, y(t)dt = 0$

18. Solution of the integral equation

$\phi(x) = 1 + \int_0^x \phi(t)dt$ is given by

(A) e^{-x}

(B) e^{x^2}

(C) $e^{x/2}$

(D) e^x

19. The solution of differential equation $x^2 \frac{\partial^2 Z}{\partial x^2} - y^2 \frac{\partial^2 Z}{\partial y^2} - y\frac{\partial Z}{\partial y} + x\frac{\partial Z}{\partial x} = 0$ is given by following forms for $x > 0, y > 0$.

(A) $Z = \phi(\log(xy)) + \psi\left(\log\frac{x}{y}\right)$

(B) $Z = g_1(x+y) + g_2(x-y)$

(C) $Z = F_1(x^2+y^2) + F_2(x^2-y^2)$

(D) None of (A), (B) and (C)

20. The maximum value of $u(x,y,z)$ in a region D bounded by a surface S satisfying

$\nabla^2 u = 0$ in D,

$u = f$ on S,

where f is a known function, is attained

(A) in region $D - S$

(B) anywhere in $D \cup S$

(C) nowhere in $D \cup S$

(D) on the boundary S

21. Green's function $G(x,t)$ corresponding to the boundary value problem

$\frac{d^2y}{dx^2} = f(x),\ 0 \le x \le 1,$

$y(0) = y(1) = 0.$

is given by

(A) $G(x,t) = \begin{cases} x(1-t) & 0 \le x < t \\ t(1-x) & t < x \le 1 \end{cases}$

(B) $G(x,t) = \begin{cases} x(2-t) & 0 \le x < t \\ t(2-x) & t < x \le 1 \end{cases}$

(C) $G(x,t) = \begin{cases} x(1+t) & 0 \le x < t \\ t(1+x) & t < x \le 1 \end{cases}$

(D) $G(x,t) = \begin{cases} x(2+t) & 0 \le x < t \\ t(2+x) & t < x \le 1 \end{cases}$

22. For the differential equation

$z^2 \frac{d^2\omega}{dz^2} + z\frac{d\omega}{dz} + (z^2 - \alpha^2)\omega = 0,$

$z \in \mathbb{C}, \alpha$ is a constant, point at infinity is a

(A) regular singular point.

(B) critical point.

(C) irregular singular point.

(D) ordinary point.

23. If the strain components are given by

$e_{xx} = e_{yy} = e_{zz} = e_{xy} = 0,\ e_{zx} = \frac{\partial\varphi}{\partial y}$ and

$e_{yz} = -\frac{\partial\varphi}{\partial x}$ where φ is a function of x, y only, then φ satisfies

(A) $\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2} =$ constant

(B) $\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial y} =$ constant

(C) $\frac{\partial\varphi}{\partial x} - \frac{\partial\varphi}{\partial y} =$ constant

(D) $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} =$ constant

24. Find which of the following statements is true: Under homogeneous strain

(A) straight lines remain straight lines.

(B) parallel straight are transformed to nonparallel straight lines.

(C) a sphere is transformed into a sphere.

(D) a straight line is transformed into a curved line.

25. $\nabla^r y_k$ is same as, where $\Delta = E+h$, $\nabla = E - h$

(A) $\nabla^r y_{k+r}$

(B) $\nabla^r y_{k-r}$

(C) $\nabla^r y_{kr}$

(D) $\nabla^{kr} y_k$

26. Let f be an entire function such that f has a zero at the origin of multiplicity 7. Then

(A) there exist $\delta(>0)$ and $\varepsilon(>0)$ such that for $|a|<\varepsilon$, the equation $f(z)=a$ has at least one multiple root in $0<|z|<\delta$.

(B) there exist $\delta(>0)$ and $\varepsilon(>0)$ such that for $|a|<\varepsilon$ the equation $f(z)=a$ has seven simple roots in $0<|z|<\delta$.

(C) there exist $\delta(>0)$ and $\varepsilon(>0)$ such that for $|a|<\varepsilon$, the equation $f(z)=a$ has a root of multiplicity seven in $0<|z|<\delta$.

(D) there exist $\delta(>0)$ and $\varepsilon(>0)$ such that for $|a|<\varepsilon$, the equation $f(z)=a$ has no root in $0<|z|<\delta$.

27. Let f be a holomorphic function defined on $\mathbb{C}\backslash\{0\}$ such that $f\left(\frac{1}{n}\right)=0$ for all positive integer n. Then

(A) $f(0)=0$

(B) $f(z)=0$ for all z

(C) f is bounded in any neighbourhood of 0,

(D) $f(z)$ may assume all complex values except possibly one.

28. Suppose that f is an entire function and $|f(z)|<1+|z|^{0.5}$ for all z. Then

(A) $f^{(1)}(0)\neq 0$.

(B) $f^{(1)}(z)=0$ for all z.

(C) $f^{(1)}(z)\neq 0$ for some z.

(D) $f^{(1)}(z)=0$ only in $|z|<1$.

29. The value of the integral $\int_C \frac{\sin^2 z\, dz}{\left(z-\frac{\pi}{6}\right)^3}$, C is the circle $|z|=1$ is

(A) $2\pi i$

(B) πi

(C) $3\pi i$

(D) $\frac{\pi i}{2}$

30. The bilinear transformation, which maps the points $z=1,i,-1$ onto the points $\omega=i,0,-i$ is

(A) $\frac{1-z}{1+z}$

(B) $\frac{1+z}{1-z}$

(C) $\frac{1+iz}{1-iz}$

(D) $\frac{1-iz}{1+iz}$

31. Let $g(x,y)=(x^2-y^2,2xy),\quad (x,y)\in\mathbb{R}^2$. Then

(A) the Jacobian of g' at each point on $\mathbb{R}^2$ is non-zero.

(B) each point $(x,y)\neq(0,0)$ has a neighbourhood throughout which g is one-one.

(C) g is one-one on some neighbourhood of (0,0).

(D) the range of the function g is a proper subset of $\mathbb{R}^2$.

32. Let $f:[0,1]\to\mathbb{R}$ be defined by $f(x)=x\cos\frac{\pi}{2x}$ for $0<x\leq 1$ and $f(0)=0$. Then which of the following is not true?

(A) f is Lebesgue integrable.

(B) f is Riemann integrable.

(C) f is of bounded variation.

(D) there exists a sequence of functions $f_n:[0,1]\to\mathbb{R}$ such that $f_n\to f$ uniformly.

33. Let $g(x)=\begin{cases}1, & \text{if } x \text{ is rational}\\ -1, & \text{if } x \text{ is irrational}\end{cases}$, $x\in[0,1]$

Then

(A) g is Riemann integrable over $[0,1]$.

(B) there is a closed subinterval $[\alpha,\beta]$ of $[0,1]$ such that g is Riemann integrable over $[\alpha,\beta]$

(C) g is Lebesgue integrable over $[0,1]$

(D) The Lebesgue measure of the sets of points at which g is continuous is positive.

34. If $y_n = \int_1^2 \frac{e^{-nt}}{t} dt$ then

(A) $\{y_n\}$ converges to 0.
(B) $\{y_n\}$ converges to 1.
(C) $\{y_n\}$ is not convergent.
(D) $\{y_n\}$ is divergent to $+\infty$.

35. Choose which is not true.

(A) The space l^p is a Banach space for $1 \le p \le \infty$.
(B) $l^q \subset l^p, 1 \le p \le q < \infty$
(C) $c_{00} \subset l^\infty$
(D) c and c_0 are Banach spaces.

36. If n pigeons are assigned to m pigeon holes and $m < n$, then at least one pigeon hole contains

(A) at least three pigeons.
(B) at least four pigeons.
(C) at least two pigeons.
(D) None of the above

37. Let $f: X \to Y$ be a continuous function, where X, Y are two topological spaces such that X has a dense connected subset. Then

(A) $f(X)$ has only one component.
(B) $f(X)$ has at least two components.
(C) $f(X)$ has at most countable number of components.
(D) $f(X)$ has uncountable number of components.

38. Let X be a normed linear space over $K(= \mathbb{R} \text{ or } \mathbb{C}), x_0 \in X\setminus\{0\}$ and $V = \{Cx_0 : C \in K\}$. Then for any non-zero functional f on V,

(A) f is an isomorphism.
(B) dimension of $ker f$ is same as that of V.
(C) $Ker\ f \ne \{0\}$.
(D) f is not an epimorphism.

39. Suppose $f: [0,1] \to \mathbb{R}$ is a continuous function and $g: [0,1] \to \mathbb{R}$ is a function which agrees with f almost everywhere on $[0,1]$. Then

(A) g is necessarily continuous over $[0,1]$.
(B) g is lebesgue measurable.
(C) $\{x \in [0,1]: f(x) \ne g(x)\}$ is necessarily a countable set.
(D) If $\{x \in [0,1]: f(x) \ne g(x)\}$ is a singleton, then g is surely continuous over $[0,1]$.

40. Let $C^1[0,1]$ stand for the real normed linear space of all real valued smooth functions over $[0,1]$ equipped with the supremum norm. Further let for each $n \in \mathbb{N}, X_n \in C^1[0,1]$ be defined by

$$X_n(t) = \sqrt{t^2 + \frac{1}{n}} \ (t \in [0,1]) .\ \text{Then}$$

(A) $\{X_n\}$ is a Cauchy sequence in $C^1[0,1]$.
(B) there exists a subsequence of $\{X_n\}$, which converges to a point in $C^1[0,1]$.
(C) $\{X_n\}$ can not converge to any point in the space $C^1[0,1]$.
(D) $\{X_n\}$ is a convergent sequence in $C^1[0,1]$.

41. The linear Diophantine equation $3x + 2y = 6$ in two variables x, y

(A) has only 2 solutions.
(B) has more than 2 but finitely many solutions.
(C) has all its solutions of the form $x = 6 + 2n, y = -6 - 3n$, where n is any integer.
(D) has solutions for which none of the above is true.

42. Let M be a Lebesgue measurable set in $\mathbb{R}$ and $\psi: M \to \psi(M) (\le \mathbb{R})$ a homeomorphism onto $\psi(M)$. Then

(A) $\psi(M)$ is Lebesgue measurable.
(B) For suitable choice of M and ψ, $\psi(M)$ is a non(Lebesgue) measurable set.
(C) $\psi(M)$ is a Borel Set.
(D) If $\mu(M) = 0$, then $\mu^*[\psi(M)] = 0$ 'μ' standing Lebesgue measure and μ^* for Lebesgue outer measure.

43. Let $f \in C[x]$ (the polynomial ring over $\mathbb{C}$) be an irreducible polynomial in the ring. Then

(A) f is necessarily of degree 1.

(B) f may be a non-zero constant polynomial.

(C) f may be a polynomial of degree 3.

(D) f may be a quadratic polynomial of the form $f(z) = az^2 + bz + c\ (a \neq 0)$ over $\mathbb{C}$ with $b^2 - 4ac < 0 (b, c \in \mathbb{R})$.

44. Let $\chi(G)$ be the (vertex) chromatic number of a graph G of order n, then which one is true?

(A) $\sqrt{n} < \chi(G) + \chi(\bar{G}) < n + 1$

(B) $2\sqrt{n} < \chi(G) + \chi(\bar{G}) < n + 1$

(C) $\sqrt{n} < \chi(G) + \chi(\bar{G}) < 2n$

(D) None of the above

45. In a metric space (X, d),

(A) closure of an open ball $B(a, r)(a \in X, r > 0)$ is the corresponding closed ball $\bar{B}(a, r) = \{x \in X : d(x, a) \leq r\}$.

(B) every open set is an F_σ-set.

(C) there exists a closed set A in X, which strictly contains the set $\{x \in X : d(x, A) = 0\}$, where $d(x, A) = \bigcup_{a \in A} d(x, a)$

(D) on a finite set X with $n(> 1)$ elements, at most n different metrics can be defined.

46. The remainder when $1! + 2! + 3! + \cdots + 100!$ is divided by 15 is

(A) 1

(B) 2

(C) 3

(D) 5

47. Let X be a T_1-space with exactly one non-isolated point. Then X is

(A) T_2 but not necessarily T_3.

(B) T_3 but not necessarily Tychonoff.

(C) Tychonoff but not necessarily T_4.

(D) T_4

48. Let X and Y be metrizable topological spaces and $f: X \to Y$ be a map. Then which of the following is true?

(A) If f is a homeomorphism and X is complete, then so is Y.

(B) If f is a homeomorphism and $\{x_n\}$ is a Cauchy sequence in X, then $\{f(x_n)\}$ is also Cauchy in Y.

(C) If f is uniformly continuous and $\{x_n\}$ is a Cauchy sequence in X, then so is $\{f(x_n)\}$ in Y.

(D) If f is uniformly continuous and X is complete, then so is Y.

49. The set $\mathbb{R}$ with lower limit topology

(A) is not separable.

(B) is totally disconnected.

(C) is metrizable.

(D) contains at least one isolated point.

50. Which of the following is true for a topological space X?

(A) Interior of a connected subset of X is connected.

(B) Boundary of a connected subset of X is connected.

(C) X is connected if X has a dense connected subset.

(D) Intersection of two connected subsets of X is connected.

51. Let X be a continuous random variable with probability distribution $F(x)$. Then $F(X)$ is a random variable

(A) with Poisson distribution in $(0, \infty)$.

(B) with exponential distribution in $(0, \infty)$.

(C) which is uniformly distributed over $(0, 1)$.

(D) with Normal distribution in $(-\infty, \infty)$.

52. State which of the following statements in connection with Dynamic Programming (DP) is false:

(A) In DP models, the number of stages is equal to the number of subproblems.
(B) DP problems can be decomposed either additively or multiplicatively.
(C) DP provides specific procedures for optimizing subproblems of each stage.
(D) In any DP model, a reduction in the number of constraints that bind all the stages together can lead to computations savings.

53. The withdrawal of items from certain inventory without refilling in queueing systems can be stated as

(A) the pure birth process.
(B) the pure death process.
(C) the birth-death process.
(D) None of the above

54. The expected waiting time in the system is 10 min and expected waiting time in the queue is 5 min, then the service rate is

(A) $\frac{1}{10}$
(B) $\frac{1}{5}$
(C) 5
(D) 10

55. Consider a gambler who at each play has probability p of winning one rupee and probability $q = 1 - p$ of losing one rupee. Assuming successive plays of the game are independent, the gambler will eventually go broke when playing against a rich adversary if the value of p is

(A) $p = 0.50$
(B) $p = 0.51$
(C) $p = 0.75$
(D) $p = 0.99$

56. For the fixed effect model
$y_{ij} = \mu_i + e_{ij}, i = 1,2, \dots, k; j = 1,2, \dots, n_i$
where μ_i is

(A) mixed effect
(B) fixed effect
(C) random effect
(D) None of the above

57. Suppose X has density
$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0, \theta > 0$. Define Y as follows :

$Y = K$ if $K \leq X < KH, K = 0,1,2 \dots$

Then the distribution of Y is

(A) Normal
(B) Binomial
(C) Poisson
(D) Geometric

58. The regression line of X on Y

(A) minimizes the total of squares of horizontal deviations.
(B) minimizes the total of squares of vertical deviations.
(C) minimizes the total of squares of both horizontal and vertical deviations.
(D) minimizes the total of squares of perpendicular distances of the points from the regression lines.

59. Which of the following is a contrast?

(A) $3T_1 + T_2 - 3T_3 + T_4$
(B) $T_1 + 3T_2 + 3T_3 + T_4$
(C) $-3T_1 - T_2 + T_3$
(D) $T_1 + T_2 + T_3 + T_4$

60. To examine whether two different skin creams, A and B, have different effect on the human body n randomly chosen persons were enrolled in a clinical trial. Then cream A was applied to one of the randomly chosen arms of each person and cream B to the other arm. Then the design is a

(A) CRD
(B) RBD
(C) $BIBD$
(D) LSD

61. A series system consists of n independent components. Let the life time of ith component follows an exponential distribution with parameter $\lambda_i, i = 1,2,\dots,n$. Then the lifetime of the system follows an exponential distribution with parameter

(A) $\prod_{i=1}^{n} \lambda_i$

(B) $\min(\lambda_1, \lambda_n)$

(C) $\max(\lambda_1, \lambda_n)$

(D) $\sum_{i=1}^{n} \lambda_i$

62. The relationship between Average Sample Number (ASN) and Average Amount of Total Inspection (ATI) in sampling inspection plan for attribute with fraction defective p is

(A) $ATI \geq ASN$
(B) $ATI < ASN$
(C) $ATI = p.ASN$
(D) None of the above

63. If we have last census population, migration, births and deaths data for a region in a given period, the population at the time t can be estimated by the formula (using usual notations) as

(A) $\hat{P}_t = P_0 + (B - D) + (I - E)$
(B) $\hat{P}_t = (B - D) + (I - E)$
(C) $\hat{P}_t = P_0\{(B - D) + (I - E)\}$
(D) None of the above

64. Think of different columns of life tables like l_x, ${}_nq_x$, ${}_nL_x$, and T_x. e_{15}^{o} (expectation of life at age 15) is computed as

(A) $\frac{L_{15}}{l_{15}}$

(B) $\frac{T_{15}}{l_{15}}$

(C) $\frac{T_{15}}{L_{15}}$

(D) None of the above

65. Let $\epsilon_t, \epsilon_{th}, \dots$ and ξ be independent variables with zero mean and unit variance. Suppose $U_t = a\xi + \epsilon_t, -\infty < t < \infty$. The process is

(A) stationary
(B) non-stationary
(C) oscillatory
(D) None of the above

66. If the trend line with 1975 as origin is $Y = 20.6 + 1.68X$, then the trend line with 1971 as origin is

(A) $Y = 20.6 + 6.72X$
(B) $Y = 13.88 + 1.68X$
(C) $Y = 34.61 + 1.68X$
(D) None of the above

67. For a population with linear trend, you will prefer

(A) cluster sampling
(B) systematic sampling
(C) stratified sampling
(D) simple random sampling

68. The sampling with replacement, $E(S^2)$ is equal to

(A) $\frac{n-1}{n}\sigma^2$

(B) $\frac{n}{n-1}\sigma^2$

(C) $\frac{N}{N-1}\sigma^2$

(D) $\frac{1}{n}\sigma^2$

69. First canonical correlations will maintain the following:

(A) Higher than any individual correlation between one set and a variable of the other set.
(B) Lower than the largest multiple correlation between a variable of one set and a variable of the other set.
(C) Higher than any individual correlation both belonging to any set.
(D) None of the above

70. Let $\bar{x}$ and S be the sample mean vector and sample variance-covariance matrix for a random sample of size N drawn from $N_p(\mu, \Sigma), \Sigma > 0$. Then a Hotelling T^2 statistic may be constructed as

(A) $(N-1)(\bar{x}-\mu)'S^{-1}(\bar{x}-\mu)$

(B) $N(\bar{x}-\mu)'S^{-1}(\bar{x}-\mu)$

(C) $\frac{1}{N-1}(\bar{x}-\mu)'S^{-1}(\bar{x}-\mu)$

(D) $\frac{1}{N}(\bar{x}-\mu)'S^{-1}(\bar{x}-\mu)$

71. Fisher's Z-transformation is applied to

(A) sample standard deviation

(B) sample coefficient of variation

(C) sample correlation coefficient

(D) sample proportion

72. Large sample standard error of sample kurtosis coefficient is

(A) $\frac{6}{n}$

(B) $\frac{8}{n}$

(C) $\frac{12}{n}$

(D) $\frac{24}{n}$

73. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from a Gamma distribution with pdf $f(x;\theta,p) = \frac{\theta^p}{\Gamma(p)} e^{-\theta x} x^{p-1}, 0 < x < \infty, p > 0$. Which of the following statement is correct?

(A) $\left(\sum_i X_i, \prod_i X_i\right)$ are jointly sufficient for (θ, p).

(B) $\sum_i X_i$ is sufficient for p, $\prod_i X_i$ is sufficient for θ.

(C) Sufficient statistic does not exist.

(D) None of the above

74. Consider the problem of testing $H_0: X \sim \cup(0,1)$ against $H_1: X \sim \cup\left(0, \frac{3}{2}\right)$. Then for some $\alpha \in (0,1)$, the test $\omega = \{X: 1-\alpha < x < 1\}$ is

(A) of size α

(B) of size $< \alpha$

(C) unbiased

(D) None of the above

75. The characteristic function of Laplace distribution with density function $f(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty$ is

(A) $\frac{t}{1+t^2}$

(B) $\frac{1}{1+t^2}$

(C) $\frac{t}{1+t}$

(D) $\frac{1}{1+t}$

ROUGH WORK