Roll No. | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilators

1. ____________________

2. ____________________

1515

Question Booklet Series **A**

PAPER–II **Question Booklet No.**

Subject Code : 15

OMR Sheet No. ....

(To be filled by the candidate)

MATHEMATICAL SCIENCES

Time : 1 Hour 15 Minutes *Maximum Marks: 100*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet and the OMR Sheet Number should be entered on this Question Booklet.
3. This paper consists of fifty (50) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

[Please Turn Over]

1. Let X be a metric space such that the total number of open sets of X is n. Then a possible value of n is

(A) 1012

(B) 1024

(C) 1036

(D) 1089

2. The solution of

$\phi(x) = cosx + \lambda \int_0^{\pi} sinx\, \phi(\xi)d\xi, \lambda \neq \frac{1}{2}$ is

(A) $sinx$

(B) $-sinx$

(C) $cosx$

(D) $-cosx$

3. The condition on p and q such that

$f(x) = \begin{cases} x^p \sin\frac{1}{x^q}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous

(A) $p > q$

(B) $p < q$

(C) $p^2 = q$

(D) for any value of p and q

4. The characteristic root of a Skew Hermitian matrix is

(A) 0

(B) complex number

(C) non zero real

(D) either 0 or a purely imaginary number.

5. Choose which does not hold in a group.

(A) Every element of a finite group is of finite order.

(B) The order of each non-zero element of $\{\mathbb{Z}, +\}$ is finite.

(C) If $0(a) = n, a \in G$, then $a, a^2, a^3, \dots, a^n$ are distinct elements of the cyclic group G.

(D) If $0(a) = n, a \in G$ and $a^m = e$, then n is a divisor of m.

6. Which of the following is false?

(A) For a subset A of a metric space (X, d), the set $\{x \in X: d(x, A) = 0\}$ is a closed set.

(B) If $f: (X, d) \to (Y, d_1)$ is a continuous function and (X, d) is complete, then so is (Y, d_1).

(C) A complete metric space need not be totally bounded.

(D) Closure of a bounded subset in a metric space is bounded.

7. Let A be a 7×7 real matrix and the sum of the entries of each row of A is 1. Then the sum of all the entries of the matrix A^7 is

(A) 7

(B) 49

(C) 7^7

(D) 1

8. Let A be a real symmetric matrix of order n. Then

(A) $I_n + A$ is singular.

(B) $I_n - A$ is singular.

(C) $I_n + A$ is singular and $I_n - A$ is non-singular.

(D) $I_n + A$ and $I_n - A$ are both non-singular.

9. Which of the following sequence of functions (f_n) is uniformly convergent on $[0,1]$?

(A) $f_n(x) = nxe^{-nx^2}$

(B) $f_n(x) = x^{n-1} - x^n$

(C) $f_n(x) = nx(1 - x^2)^n$

(D) $f_n(x) = \frac{nx}{1+n^2x^2}$

10. Which of the following is false?

(A) The zeros of an analytic function are isolated points unless it is identically zero.

(B) The derivative of an analytic function is also analytic.

(C) A bounded entire function must be constant.

(D) A non-constant analytic function maps closed sets into closed sets.

11. Which one is *not* correct?

(A) The characteristic of a field is an even number.

(B) The characteristic of an integral domain is either zero or a prime number.

(C) For every prime number n, the ring $\{\mathbb{Z}_n, \oplus, \odot\}$ is an integral domain.

(D) The set of matrices $\left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ is a field under matrix addition and matrix multiplication.

12. The residue of $f(z) = e^{-1/z}$ at $z = 0$ is

(A) 0

(B) 1

(C) – 1

(D) ½

13. Let A, B be finite sets with $|A| = m$ and $|B| = n$; then the number of non-constant functions from A into B is

(A) $m^n - n$

(B) $n^m - n$

(C) $n^m - m$

(D) $m^n - m$

14. Let G be a group with $|G| = 200$. H is a subgroup of G such that H is not cyclic and H does not contain any element of order 2. Then $|H| =$

(A) 8

(B) 20

(C) 25

(D) 40

15. The series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx)^2}$ is

(A) convergent on [0,1] but not uniformly.

(B) divergent on [0,1].

(C) uniformly convergent on [0,1].

(D) None of the above

16. Find $\lim_{x \to -1} \frac{\cos 2 - \cos 2x}{x^2 - |x|}$

(A) $\sin 2$

(B) $-\sin 2$

(C) $2 \sin 2$

(D) $-2 \sin 2$

17. Which one of the following is true?

(A) If A is a real square matrix such that $(I + A)^{-1}(I - A)$ is skew symmetric then A is orthogonal.

(B) If A is real orthogonal matrix of order $n \times n$ with det $A = -1$, then $A + I_n$ is a non-singular matrix.

(C) There exists a real skew-symmetric matrix of rank one.

(D) If A and B are $n \times n$ equivalent matrices, then A^2 and B^2 are also equivalent.

18. Let x, y be linearly independent vectors in R^2 and $T: R^2 \to R^2$ be a linear transformation such that $Ty = \alpha x$ and $Tx = 0$, then with respect to some basis in R^2, T is of the form

(A) $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}, a > 0$

(B) $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, a, b > 0; a \neq b$

(C) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

19. Tick which one is not correct.

(A) The set of all linear combinations of a finite subset S of a vector space V over F is the smallest subspace of V containing S.

(B) If S, T are two finite subsets of a vector space V with SCT, then $L(S) \subset L(T)$.

(C) Let $V \in \mathbb{R}^n$, $S = \{\alpha, \beta\}$ and $T = \{\alpha, \beta, \alpha + \beta\}$, $\alpha, \beta \in \mathbb{R}^n$, then $L(S) \neq L(T)$.

(D) If $u = (1,1,1)$, $v = (1,0,1)$, then the set $\{u, v\}$ is linearly independent.

20. Let V be a vector space of all real 3×3 matrices and let A be a diagonal matrix with eigen values 1, 2, 3. Suppose that a linear transformation $T: V \to V$ is defined by $T(X) = \frac{1}{2}(AX + XA)$. Then determinant of T is

(A) $\frac{1125}{2}$
(B) $\frac{675}{2}$
(C) $\frac{625}{2}$
(D) $\frac{1225}{2}$

21. The Series $\sum \frac{sin\, nx}{n}$

(A) converges uniformly on [5, 2π – 5].
(B) converges uniformly on [10, 2π – 10].
(C) converges uniformly on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.
(D) does not converge uniformly.

22. Which one of the following is false?

(A) If $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $|f'|$ is bounded, then f is uniformly continuous.
(B) Let $p(x)$ be a real polynomial such that all the roots of $p(x) = 0$ are real and distinct. The all roots of $p'(x) = 0$ are real.
(C) Every continuous function $f: [10,20] \to [10,20]$ has a fixed point.
(D) Let $f: [0, \infty) \to [0, \infty)$ be a function such that $f(x + y) = f(x) + f(y) \forall x, y \geq 0$, then $f(x) = ax$, for some constant a.

23. Let $f: \mathbb{R} \to \mathbb{R}$ be a two times continuously differentiable function that statisfies the differential equation $f^{(2)} + f^{(1)} = f$ for all $x \in [0,1]$. If $f(0) = f(1) = 0$, then

(A) there exists $(\alpha, \beta) \subset (0,1)$ such that $f(x) > 0$ in (α, β).
(B) there exists $(\alpha, \beta) \subset (0,1)$ such that $f(x) < 0$ in (α, β).
(C) $f(x) = 0$ in $[0,1]$.
(D) such function does not exist.

24. The solution of $z = pq$, where z is a complex number, is

(A) $(x + ay - c)^2 = 4az$
(B) $(x + az - c)^2 = 4ay$
(C) $(y + ax - c)^2 = 4az$
(D) $(z + ax - c)^2 = 4ay$

25. Let f and g be two entire functions such that $|f(z)| \leq |g(z)|$. Then

(A) f must be a constant.
(B) f and g assume real values only.
(C) f and g does not assume any real value.
(D) $f(z) = cg(z)$ for some constant c.

26. If $y^{\propto}$ is an integrating factor of the differential equation $2xy\, dx - (3x^2 - y^2)dx = 0$, then the value of $\propto$ is

(A) 1
(B) – 1
(C) – 4
(D) 4

27. The solution of

$$x^2(y - z)\frac{\partial z}{\partial x} + y^2(z - x)\frac{\partial z}{\partial y} = z^2(x - y)$$

has the form

(A) $\phi(x + y + z, xyz) = 0$
(B) $\phi(xyz, xy + yz + zx) = 0$
(C) $\phi\left(xyz, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$
(D) $\phi\left(x + y + z, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

28. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$, $x \in D, a_i, i = 1,2$ are constants then

$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ is

(A) only constant in D
(B) zero at some points of D
(C) never equal to zero in D
(D) identically equal to zero in D

29. The solution of the differential equation $\frac{d^2y}{dx^2} - y = 1$ which vanishes at $x = 0$ and tends to a finite limit as $x \to -\infty$, is

(A) $y = e^{-x} + 1$
(B) $2y = e^{-x} + 1$
(C) $y = e^{x} - 1$
(D) $y = e^{-x} - 1$

30. For a given nonlinear differential equation of first order (a) singular solution contains no arbitrary constant, (b) singular solution can be obtained from complete primitive.

(A) (a) and (b) are both true.
(B) (a) is false but (b) is true.
(C) (a) and (b) are both false.
(D) (a) is true but (b) is false.

31. Given $y = x$ a solution of the differential equation $(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, then the other solution is

(A) $y = x^2 - 1$
(B) $y = x^2$
(C) $y = x^3$
(D) None of the above

32. The equation
$U_{xx} + 4U_{xy} + (x^2 + y^2)U_{yy} = \sin(xy)$ is elliptic if

(A) $\frac{x^2}{4} + y^2 = 1$
(B) $\frac{x^2}{4} + y^2 < 1$
(C) $\frac{x^2}{4} - y^2 = 1$
(D) $\frac{x^2}{4} + y^2 > 4$

33. The inequality $2x_1 + 5x_2 \geq 8$ with $x_1, x_2 \geq 0$, is converted into the equality $2x_1 + 5x_2 - x_3 = 8$ with $x_1, x_2, x_3 \geq 0$. The variable x_3 is called

(A) a slack variable.
(B) a surplus variable.
(C) an artificial variable.
(D) None of the above

34. The optimal value of the objective function in the LPP

Minimize $z = 2x_1 + 5x_2$
subject to

$$x_1 + x_2 \leq 2$$
$$x_1 - x_2 \geq 1$$

with $x_1, x_2 \geq 0$, is

(A) $\frac{7}{2}$
(B) $\frac{11}{2}$
(C) 1
(D) 2

35. In an $M/G/1$ queueing model the number of servers is

(A) M
(B) G
(C) 1
(D) infinity

36. If the joint pdf of X and Y, $f(x, y) = 3 - x - y$ for $0 \leq x \leq 1$, $0 \leq y \leq 1$, the marginal distribution of Y is

(A) $f_Y(y) = \frac{5}{2} - y$
(B) $f_Y(y) = y - \frac{5}{2}$
(C) $f_Y(y) = 3 - y$
(D) $f_Y(y) = 3$

37. Quartile deviation or semi inter-quartile deviation is given by the formula

(A) $\frac{Q_3 + Q_1}{2}$
(B) $Q_3 - Q_1$
(C) $\frac{Q_3 - Q_1}{2}$
(D) $\frac{Q_3 - Q_1}{4}$

38. A histogram can be drawn for the distribution with unequal class intervals by considering

(A) class frequency.
(B) height of bars proportional to class intervals.
(C) height of bars proportional to frequency density.
(D) None of the above

39. In the $M/M/\infty$ queueing model, M stands for

(A) Modulus

(B) moment

(C) memoryless

(D) matching

40. Let X and Y be two independent random variables and let $Z = aX + bY$ where a and b are non-zero constants.

Then $Var(Z)$ is

(A) $a\,Var\,(X) + b\,Var(Y)$

(B) $a^2\,Var(X) + b^2\,Var(Y)$

(C) $\frac{a}{b}\,Var(X) + \frac{b}{a}\,Var(Y)$

(D) $\frac{b}{a}\,Var(X) + \frac{a}{b}\,Var(Y)$

41. The value of the constant c for which the function

$$f(x) = \begin{cases} c\,x\,e^{-2x} & \text{for } x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

is a probability density function, is

(A) 1

(B) 2

(C) 3

(D) 4

42. For two events X and Y find which of the following statements is false.

(A) $P(Y|X) = \frac{P(X\cap Y)}{P(X)}$ provided $P(X) > 0$

(B) $P(X|Y) = \frac{P(X\cap Y)}{P(Y)}$ provided $P(Y) > 0$

(C) $P(X|Y)P(Y|X) = 1$

(D) $P(X)P(Y) \geq 0$

43. In a 3^3 factorial experiment with factor P, Q and R each at 3 levels, the interaction P^2QR^2 is same as the interaction

(A) PQR

(B) PQ^2R

(C) PQ^2R^2

(D) P^2QR

44. In the analysis of data of a randomised block design with b block and v treatments, the error degrees of freedom is

(A) $b(v-1)$

(B) $v(b-1)$

(C) $(b-1)(v-1)$

(D) None of the above

45. The finite population correction (f.p.c.) in sample survey is

(A) $\frac{n}{N}$

(B) $\frac{N}{n}$

(C) $1 - \frac{n}{N}$

(D) $1 - \frac{N}{n}$

46. If the universe from which the sample is to be obtained is non-homogeneous, which of the following sampling techniques will be most appropriate?

(A) Stratified Random Sampling

(B) Simple Random Sampling

(C) Cluster Sampling

(D) Systematic Random Sampling

47. If n_1 and n_2 in Mann-Whitney test are large, the variable v is distributed with mean

(A) $\frac{n_1+n_2}{2}$

(B) $\frac{n_1-n_2}{2}$

(C) $\frac{n_1 n_2}{2}$

(D) $n_1 n_2$

48. Analysis of variance utilises

(A) F-test

(B) x^2-test

(C) Z-test

(D) t-test

49. The ratio of the likelihood function under H_0 and under the entire parametric space is called

(A) probability ratio

(B) sequential probability ratio

(C) likelihood ratio

(D) None of the above

50. If the variance of an estimator attains the Crammer-Rao lower bound, the estimator is

(A) most efficient

(B) sufficient

(C) consistent

(D) admissible

Rough Work

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