Test Booklet No.
प्रश्नपत्रिका क्र.

M

Paper-III
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

   (Name) ....

2. (Signature) ....

   (Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

AUG - 30315

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **75**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **75** objective type questions. Each question will carry *two* marks. *All* questions of Paper-III will be compulsory, covering entire syllabus (including all electives, without options).
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   - (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   - (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/ questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   - (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.

   Example : where (C) is the correct response.

   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 75 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. या प्रश्नपत्रिकेतील **सर्व** प्रश्न सोडविणे अनिवार्य आहे. सदरचे प्रश्न हे या विषयाच्या संपूर्ण अभ्यासक्रमावर आधारित आहेत.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   - (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   - (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   - (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.

   उदा. : जर (C) हे योग्य उत्तर असेल तर.

   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science

Paper III

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Note : This paper contains **Seventy Five (75)** multiple choice questions, each carrying **Two (2)** marks. Attempt *All* questions.

1. Let a function $f = [0, 1] \to \mathbf{R}$ be defined by :

$$f(x) = \begin{cases} 1, \text{ when } x \text{ is rational} \\ 0, \text{ when } x \text{ is irrational} \end{cases}$$

Then :

(A) f is Riemann integrable on [0, 1]

(B) f is continuous on [0, 1]

(C) f is not integrable on [0, 1]

(D) f is of bounded variation on [0, 1]

2. Consider the subset :

$$A = \{x \in Q^+ : 2 \leq x^2 \leq 5\}$$

of the space **R** with usual metric.

Then :

(A) A is closed and bounded

(B) A is neither open nor closed

(C) A is compact

(D) A is connected

3. The rate of the convergence of the Newton-Raphson method used for solving algebraic equations is :

(A) 1

(B) 2

(C) 4

(D) 3

4. Let Δ denote the forward difference operator. Then the value of Δ^2 (cos x) is :

(A) $-4 \sin^2 \frac{h}{2} \cos(x+h)$

(B) $4 \sin^2 h \cos(x+h)$

(C) $3 \cos^2 \frac{h}{2} \cos(x+h)$

(D) $-3 \sin^2 h \cos(x+h)$

5. Given the QPP :

Minimize

$$Z = x_1^2 - x_1 x_2 + 2x_2^2 - x_1 - x_2$$

Subejct to $2x_1 + x_2 \leq 1$

and $x_1 \geq 0$; $x_2 \geq 0$

Using Wolfe's method, we observe that the problem has a/an :

(A) Unique optimum solution

(B) Unbounded solution

(C) Infinite optimum solution

(D) Infeasible solution

6. The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n!+1}$ converges to :

(A) $\frac{\pi}{2}$

(B) 4π

(C) $\frac{\pi}{4}$

(D) 2π

Or

A sequence of sets $\{A_n, n \geq 1\}$ converges iff :

(A) $\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k \subseteq \bigcup_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$

(B) $\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k \subseteq \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$

(C) $\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k \subseteq \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$

(D) $\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k \subseteq \bigcap_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$

7. Let X be a complete metric space and $T : X \to X$ satisfies :

$$d(T(x), T(y)) < d(x, y)$$

for all $x, y \in X$, $x \neq y$. Then :

(A) T has at most one fixed point

(B) T has at least one fixed point

(C) T has exactly one fixed point

(D) T has two fixed points

Or

Let $\{X_n, n \geq 1\}$ be a sequence of random variables with p.m.f. :

$$P(X_n = 1) = \frac{1}{n}, P(X_n = 0) = 1 - \frac{1}{n}.$$

Then :

(A) $X_n \xrightarrow{\phi m} 0$

(B) $X_n \xrightarrow{P} c \neq 0$

(C) $X_n \xrightarrow{P} 1$

(D) X_n does not converge to any X in probability

8. The series $\sum_{n=2}^{\infty} \frac{\sin nx}{n \log n}$ is :

(A) absolutely convergent

(B) not convergent pointwise on $[0, 2\pi]$

(C) uniformly convergent on $[0, 2\pi]$

(D) pointwise convergent but not uniformly convergent on $[0, 2\pi]$

Or

Which of the following statements is *not* true ?

(A) If $0 \le X_n \uparrow X$, then $EX_n \uparrow EX$

(B) If $X_n \ge 0$, then :

$$E\left\{\sum_{k=1}^{\infty} X_k\right\} = \sum_{k=1}^{\infty} EX_k$$

(C) If $Y \le X_n$ and Y is integrable random variable, then $\underline{\lim}\ X_n$ may not be finite

(D) If $Y \le X_n$ and Y is integrable random variable, then $\overline{\lim}\ X_n$ may not be finite

9. Let $\mathbf{R}_u$ and $\mathbf{R}_d$ denote the spaces of reals with usual metric and the discrete metric, respectively. Then the mapping :

$$f : \mathbf{R}_d \to \mathbf{R}_u$$

defined by $f(x) = [x]$, for all x in $\mathbf{R}$ ($[x]$ denote the integral part of x) is :

(A) monotonically decreasing

(B) not continuous

(C) continuous

(D) homeomorphism

Or

If $|X_n| \leq Y$ a.s., Y is integrable, then

$$X_n \xrightarrow{P} X \Rightarrow EX_n \to EX.$$

This theorem is known as :

(A) Monotone convergence theorem

(B) Fatou's Lemma

(C) Dominated convergence theorem

(D) Poisson weak law of large numbers

10. For the point i in **C**, the corresponding point of the unit sphere S in $\mathbf{R}^3$ under the stereographic projection is :

(A) (1, 1, 1)

(B) (1, 0, 0)

(C) (0, 1, 0)

(D) (0, 0, 1)

Or

Let $\{X_n, n \geq 1\}$ be a sequence of i.i.d. Bernoulli random variables with $P(X_n = 0) = \frac{1}{3}$.

If $S_n = X_1 + X_2 + X_3 + \dots\dots + X_n$, $n \geq 1$.

Then we have :

(A) $\frac{S_n}{n} \xrightarrow{L} \frac{1}{3}$

(B) $\frac{S_n}{n} \xrightarrow{am} \frac{1}{3}$

(C) $\frac{S_n}{n} \xrightarrow{a.s.} \frac{2}{3}$

(D) $\frac{S_n}{n} \xrightarrow{a.s.} 0$

11. Let $\gamma : [0, 2\pi] \to \mathbf{C}$ be defined by $\gamma(t) = e^{int}$, where n is some integer. Then the value of the integral $\int_{\gamma} \frac{1}{z}\, dz$ is :

(A) $2\, ni$

(B) $2\, \pi$

(C) $2\, \pi n$

(D) $2\, \pi in$

Or

Which of the following statements is *wrong* ?

(A) Theoretical basis of Central Limit theorem was first introduced by Laplace

(B) Suppose $\{A_n, h \geq 1\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$, then $P(\overline{\lim}\ A_n) = 0$

(C) If $X \leq Y \Rightarrow E^{\mathbf{B}}(X) \leq E^{\mathbf{B}}(Y)$ where $E^{\mathbf{B}}(\cdot)$ denote conditional expectation

(D) If $X_n \xrightarrow{a.s.} X$ iff as $n \to \infty$

$$P\left[\bigcup_{k=n}^{\infty} \{w| \ X_k(w) - X(w)| < 1/r\}\right] \to 0$$

$\forall\, r$, an integer

12. Let f be analytic in the disk $B(a; R)$ and $Z \in B(a; R)$. Then :

(A) $f(z)$ has a power series representation

(B) $f(z)$ cannot have a power series representation

(C) $f(z)$ is always a real number

(D) $f(z)$ is always a purely imaginary number

Or

If $X_1, X_2, ... X_n$ be independent and identically distributed random variables having uniform distribution on $[\theta_1, \theta_2]$, then the joint pdf of the order statics $X_{(1)}, X_{(2)}, ... X_{(n)}$ will be given by one of the following over the range $\theta_1 < X_{(1)} < X_{(2)} < ...... < X_{(n)} < \theta_2$ and zero otherwise :

(A) $n!/(\theta_2 - \theta_1)^n$

(B) $n!(\theta_2 - \theta_1)^n$

(C) $\dfrac{n!}{\theta_1 \theta_2^n}$

(D) $n!\,\theta_1^n\,\theta_2$

13. Let f be analytic in the disk B(a; R) and suppose that γ is a closed rectifiable curve in B(a; R). Then the value of the integral $\int_{\gamma} f(z)\, dz$ is :

(A) $2\pi i$

(B) 1

(C) -1

(D) 0

Or

Let X_1, X_2 be a random variable from N(0, 1) and Y_1, Y_2 be another random sample from N(1, 1). X's and Y's are independent rvs. The distribution of :

$$z = \frac{(X_1 + X_2)/\sqrt{2}}{\sqrt{[(Y_1 - 1)^2 + (Y_2 - 1)^2]/2}}$$

(A) F(1, 1)

(B) X^2 with 3df

(C) N(1, 2)

(D) t with 2df

14. Which of the following ideals is a maximal ideal of $\mathbf{Z}[x]$?

(A) (2)

(B) (x)

(C) (3, x)

(D) (x^2 + 1)

Or

Let X be a random variable such that variance of X is $\frac{1}{2}$. Then an upper bound for $P[|X - EX| > 1]$ as given by the Chebyshev's inequality is :

(A) $\frac{1}{4}$

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

15. Which of the following positive integers has the property that every group having that order is a simple group ?

(A) 10

(B) 17

(C) 21

(D) 6

Or

Let X be a rv with B(n, p). Then the distribution of $n - X$ is :

(A) B ($n - 1$, p)

(B) B (n, $1 - p$)

(C) B ($n - 1$, q), $q = 1 - p$

(D) B (n, p)

16. The number of similarity classes of 6×6 matrices over **C** with minimal polynomial $(x - 1)(x - 2)^2$ and characteristic polynomial $(x - 1)^2 (x - 2)^4$ is :

(A) 1

(B) 2

(C) 3

(D) 4

Or

Let X be a rv with the following df :

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ \frac{1}{4} & ; \quad 0 \le x < 1 \\ \frac{1}{2} + \frac{1}{2}[1 - e^{-(x-1)}] & ; \quad x \ge 1 \end{cases}$$

Then EX is :

(A) $\frac{1}{4}$

(B) $\frac{5}{4}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

17. Which of the following quadratic forms is positive definite ?

(A) $x^2 + y^2 + 2yz + z^2$

(B) $x^2 - y^2 + 2z^2$

(C) $x^2 + y^2 - yz + z^2$

(D) $x^2 - y^2 - z^2$

Or

Let X be a rv with $U(-\theta, \theta),\ \theta > 0$, then the distribution of $Y = |X|$ is :

(A) $f(y) = \frac{1}{2\theta};\ 0 < y < 2\theta$

(B) $f(y) = \frac{1}{3\theta};\ -\theta < y < 2\theta$

(C) $f(y) = \frac{1}{\theta};\ 0 < y < \theta$

(D) $f(y) = \frac{1}{3\theta};\ 0 < y < 3\theta$

18. Let $\{f_n\}$, be a sequence of monotonically increasing real functions on [0, 1], converges pointwise to the function $f \equiv 0$.

(A) Then $\{f_n\}$ need not be uniformly convergent to f

(B) Then $\{f_n\}$ converges uniformly to f

(C) If the functions f_n are non-negative, then f_n must be continuous for sufficiently large n

(D) If the function f_n are non-negative, then f_n must be constant for sufficiently large n

Or

Consider one parameter exponential family $\{f(x, \theta); \theta \in \Theta \subset R_1\}$ with probability function :

$$f(x, \theta) = \exp[u(\theta)\, k(x) + v(\theta) + w(x)]$$

When this family has monotone likelihood ratio in $k(x)$?

(A) $u(\theta)$ is decreasing function of θ

(B) $u(\theta)$ is non-decreasing function of θ

(C) $v(\theta)$ is decreasing function of θ

(D) $v(\theta)$ is non-decreasing function of θ

19. Which of the following statements is *true* ?

(A) Any uniformly continuous function on $[a, b]$ is of bounded variation on $[a, b]$

(B) If $f : \mathbf{R} \to \mathbf{R}$ is continuously differentiable function, then it is of bounded variation on $[-n, n]$ for any $n \in \mathbf{N}$

(C) Any bounded function on $[a, b]$ is of bounded variation on $[a, b]$

(D) Difference of any two monotonically increasing functions on $[a, b]$ is always of bounded variation on $[a, b]$

Or

Which of the following does *not* belong to exponential family of distributions ?

(A) $f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0$

(B) $f(x, \theta) = e^{-(x-\theta)}, x > \theta$

(C) $f(x, \theta) = \frac{5x^4}{\theta} e^{-x^5/\theta}, x > 0$

(D) $f(x, \theta) = \frac{e^{-x/\theta} x^2}{2\theta^3}, x > 0$

20. Let f be a real function define on $\mathbf{R}$ as :

$$f(t) = \begin{cases} \frac{p+\sqrt{2}}{q+\sqrt{2}} - \frac{p}{q} & , \text{ if } t = \frac{p}{q}, p, q \in \mathbf{Z}, (p, q) = 1 \\ 0 & , \text{ if } t \text{ is irrational} \end{cases}$$

Then :

(A) f is not continuous at $t = 1$

(B) f is continuous at all rationals

(C) f is not continuous at all irrationals

(D) f is continuous at all irrational

Or

For testing simple null against simple alternative hypothesis which of the following statements is most appropriate ?

(A) UMP level α test exists

(B) UMPU level α test exists

(C) UMP invariant test exists

(D) Most powerful level α test exists

21. Let $f : \mathbf{R}^3 \to \mathbf{R}$ be a continuous function. If

$$D = \{(x, y, z) \in \mathbf{R}^3 | \ x^2 + y^2 + z^2 \leq 4\}$$

Then $f(D)$:

(A) need not be an interval in $\mathbf{R}$

(B) is always an interval of the form (a, b)

(C) is union of two disjoint closed sets in $\mathbf{R}$

(D) is always an interval of the form $[a, b]$

Or

A size α test is said to unbiased if :

(A) It has maximum power in the class of all size α tests

(B) Size and power are equal

(C) Power is smaller than size

(D) Size of the test does not exceed its power

22. Let N, H, G be groups. Which of the following is *true* ?

(A) If $N \Delta G$, then G/N is isomorphic to a subgroup of G

(B) If $H \Delta N$ and $N \Delta G$, then $H \Delta G$

(C) If $N \Delta G$ and H is a subgroup of G, then $NH = HN$

(D) If $N \Delta G$, then G/N is isomorphic to a subgroup of G

Or

Let X be a binomial random variable with parameters n and p. Consider the prior distribution $\pi(p) = 1, 0 < p < 1$. Then the Bayes estimator under squared error loss function is :

(A) $\delta(X) = X / n$

(B) $\delta(X) = \dfrac{X + n}{n}$

(C) $\delta(X) = \dfrac{X + 1}{n + 2}$

(D) $\delta(X) = \dfrac{X + 1}{n + 1}$

23. Which of the following is *not* a Noetherian ring ?

(A) $\mathbf{Z}$

(B) $\mathbf{Z}[x]$

(C) $\mathbf{Q}[x_1, x_2, \ldots, x_n, \ldots]$

(D) $\mathbf{Q}(x_1, x_2, \ldots, x_n, \ldots)$

Or

Let X have the hypergeometric distribution with probability mass function :

$$P(X = x) = \frac{\binom{M}{x}\binom{N-m}{n-x}}{\binom{N}{n}},$$

$$x = 0, 1, 2, \ldots\ldots, M$$

The UMP level α test for $H_0 : M \le M_0$ against $H_1 : M > M_0$ is given by :

(A) Reject H_0 if $M_0 - X > k$ such that $\sup P_{H_0}(M_0 - X > k) \le \alpha$

(B) Reject H_0 if $N - X > k$ such that $\sup_{H_0} P(N - X > k) \le \alpha$

(C) Reject H_0 if $X > k$ such that $\sup_{H_0} P(X > k) \le \alpha$

(D) Reject H_0 if $N - M - x > k$ such that $\sup_{H_0} P(N - M - X > k) \le \alpha$

24. $\mathbf{Q}(\sqrt{2})$ and $\mathbf{Q}(\sqrt{3})$ are isomorphic as :

(A) **Q**-vector spaces but not as fields

(B) abelian groups but not as **Q**-vector spaces

(C) fields

(D) commutative rings

Or

Which of the following distribution is *not* an exact sampling distribution ?

(A) Beta distribution

(B) Chi-square distribution

(C) *t*-distribution

(D) F-distribution

25. A field extension L/K is a Galois extension and the Galois group is cyclic of order 10. Then the number of intermediate fields F other than L and K is :

(A) 10

(B) 4

(C) 2

(D) 5

Or

In likelihood ratio test, under some regularity conditions on $f(x, 0)$, the rv $-2 \log \lambda(x)$ (where $\lambda(x)$ is a likelihood ratio) is asymptotically distributed as :

(A) Normal

(B) Exponential

(C) Chi-square

(D) F-distribution

26. Let f be a real function on [0, 1] and

$$\{(x, f(x)) : x \in [0, 1]\}$$

be its graph. Then :

(A) f is continuous implies its graph is an open set in $\mathbf{R}^2$

(B) f is continuous implies its graph is a compact set in $\mathbf{R}^2$

(C) f has at least one fixed point

(D) f is continuous implies it is an open map

Or

Let X_n converges in probability to X, then :

(A) X_n converges almost sum to X

(B) X_n converges in distribution to X

(C) $\frac{X_n}{k}$ converges almost sum to $\frac{X}{k}$, $k \neq 0$

(D) $\frac{X_n}{Y_n}$ converges in distribution to $\frac{X_n}{C}$ if Y_n converges in law to X if $P[Y_n = 0] = 1$

27. For any $f \in C[a, b]$, define :

$$\|f\|_p = (\int_a^b |f(x)|^p \, dx)^{1/p}, \quad p \geq 1$$

and $\|f\|_\infty = \sup \{|f(x)| : x \in [a, b]\}$.

Then :

(A) $(C[a, b], \| \|_\infty)$ is a separable metric space

(B) $(C[a, b], \| \|_1)$ is a Hilbert space

(C) $(C[a, b], \| \|_\infty)$ is not a Banach space

(D) $(C[a, b], \| \|_1)$ is a Banach space

Or

Let $X_1, X_2, \ldots\ldots, X_n$ be i.i.d. random variable following $U(0, \theta)$. Suppose $H_n(x) = P(X_{(n)} \leq x)$. Then $H_n(x)$ converges uniformly to :

(A) a degenerate distribution at θ

(B) a degenerate distribution at 0

(C) a degenerate distribution at 1

(D) a degenerate distribution at $\frac{\theta}{2}$

28. The space $L^P[0, 1]$, $p \geq 1$ is :

(A) not a complete metric space

(B) a compact metric space

(C) a Hilbert space for $p = 1$

(D) a separable metric space

Or

Let $X_1, X_2, \ldots, X_n$ be iid rvs with Cauchy distribution as :

$$f(x) = \frac{1}{\pi(1+x^2)}; \; -\infty < x < \infty$$

Then by using CLT the distribution of $\sum_{i=1}^{n} Y_i$, where $Y_i = \frac{X_i}{1+X_1^2}$ is :

(A) AN (0, 1)

(B) $AN\left(0, \frac{8}{n}\right)$

(C) Does not exist

(D) $AN\left(0, \frac{n}{8}\right)$

29. Let :

$$f_n(x) = \begin{cases} 0 & \text{if } 0 \leq x < \frac{1}{2} - \frac{1}{n} \\ n(x - \frac{1}{2}) + 1 & \text{if } \frac{1}{2} - \frac{1}{n} \leq x \leq \frac{1}{2} \\ 1 - n(x - \frac{1}{2}) & \text{if } \frac{1}{2} < x \leq \frac{1}{2} + \frac{1}{n} \\ 0 & \text{if } \frac{1}{2} + \frac{1}{n} < x \leq 1 \end{cases}$$

for $n = 2, 3, \ldots\ldots$

Then the sequence $\{f_n\}_{n=2}^{\infty}$ is :

(A) not a Cauchy sequence in $(C[0, 1], \|\,\|_\infty)$

(B) not convergent in $(C[0, 1], \|\,\|_1)$

(C) not a Cauchy sequence in $(C[0, 1], \|\,\|_1)$

(D) not convergent in $(C[0, 1], \|\,\|_\infty)$

Or

A multivariate method for investigating the relationship between two sets of variables is :

(A) Discriminant Analysis

(B) MANOVA

(C) Multiple Logistic Regression

(D) Canonical Correlation

30. Let $\mathbf{R}_l$ denotes the space of reals where the topology is generated by all intervals of the form $[a, b)$ and $\mathbf{R}_k$ denotes space of reals where the topology is generated by all open intervals (a, b) and all sets of the form $(a, b) - \mathrm{K}$, where

$$\mathrm{K} = \left\{\frac{1}{n} \mid n \in \mathbf{N}\right\}.$$

Then which of the following statements is *correct* ?

(A) The topology of $\mathbf{R}_l$ is finer than the topology of $\mathbf{R}_k$

(B) The topology of $\mathbf{R}_k$ is finer than the topology of $\mathbf{R}_l$

(C) The topology of $\mathbf{R}_l$ is coarser than the topology of $\mathbf{R}_k$

(D) The topologies of $\mathbf{R}_l$ and $\mathbf{R}_k$ are not comparable

Or

.... occurs when variables in the data are highly correlated.

(A) Heteroscedasticity

(B) Heterogeneity

(C) Estimation bias

(D) Multicollinearity

31. Let $\mathbf{R}$ denotes the set of real numbers in its usual topology and $\mathbf{R}_l$ denotes the same set in the topology generated by all intervals of the form $[a, b)$. Let $f : \mathbf{R} \to \mathbf{R}_l$ be defined by $f(x) = x$ for every real number x. Then which of the following statements is *true* ?

(A) f is not continuous

(B) f is continuous

(C) f is homeomorphism

(D) f^{-1} is not continuous

Or

Principal Component Analysis is a multivariate method that :

(A) reduces dimension of data

(B) reduces heterogeneity of data

(C) reduces multicollinearity of data

(D) reduces skewness of data

32. $\mathbf{R}^W$ denotes the infinite product of $\mathbf{R}$ with itself. Suppose $\mathbf{R}^W$ is given the box topology. Then which of the following is *true* ?

(A) $\mathbf{R}^W$ is connected

(B) $\mathbf{R}^W$ is not connected

(C) $\mathbf{R}^W$ is compact

(D) $\mathbf{R}^W$ is connected and compact

Or

Multiple correlation coefficient :

(A) must be between –1 and +1

(B) must be between 0 and 1

(C) must be non-negative

(D) can take any value

33. Consider the following four statements :

(I) Every separable topological space is second countable

(II) Every second countable space is separable

(III) Every separable metric space is second countable

(IV) Every first countable space is second countable

Then which of the following statements is *correct* ?

(A) All are correct

(B) Only (I) and (II) are correct

(C) Only (II) and (III) are correct

(D) Only (III) and (IV) are correct

Or

Hotelling's T^2 test is a generalization of :

(A) Chi-square test

(B) F-test

(C) T-test

(D) Likelihood ratio test

34. Which of the following statements is *not* true ?

(A) A tree is a bipartite graph

(B) K_5, the complete graph on 5 vertices is a planar graph

(C) Every connected graph contains a spanning tree

(D) The number of vertices of odd degree in a graph is always even

Or

Consider the linear model :

$$y_1 = \theta_1 + \theta_3 + \epsilon_1$$

$$y_2 = \theta_2 + \theta_3 + \epsilon_2$$

where ϵ_1 and ϵ_2 are errors. The linear combination :

$$\lambda_1\theta_1 + \lambda_2\theta_2 + \lambda_3\theta_3$$

is estimable if and only if :

(A) $\lambda_1 = \lambda_2 + \lambda_3$

(B) $\lambda_3 = \lambda_1 + \lambda_2$

(C) $\lambda_2 = \lambda_3 - \lambda_1$

(D) $\lambda_3 = \lambda_2 - \lambda_1$

35. The number of distinct trees on 4 vertices is :

(A) 16

(B) 14

(C) 15

(D) 12

Or

Consider the simple linear regression model :

$y_i = \theta_0 + \theta_1 x_i + \in_i, \quad i = 1, 2$

where $\in_i \sim \text{NID}(0, \sigma^2)$, $x_1 = -1$ and $x_2 = 1$.

The best linear unbiased estimators of θ_0 and θ_1 respectively are :

(A) $\left[\bar{y}, \frac{y_2 - y_1}{2}\right]$

(B) $\left[\frac{y_2 - y_1}{2}, \bar{y}\right]$

(C) $\left[y_1 - \bar{y}, y_2 - \bar{y}\right]$

(D) $\left[\bar{y}, \bar{y}\right]$

36. A debating team consists of three boys and two girls. Then the number of ways they can sit in a row is :

(A) 100

(B) 120

(C) 125

(D) 110

Or

Consider the model $\underline{y} = X\underline{\beta} + \underline{\in}$. A linear function of observations belongs to the error space if and only if its coefficient vector is :

(A) orthogonal to the rows of matrix X

(B) orthogonal to the columns of matrix X

(C) parallel to the rows of matrix X

(D) parallel to the columns of matrix X

37. Suppose that $n + 1$ objects are put into n boxes. Then which one of the following statements is *true* ?

(A) No box contains more than one object

(B) Exactly one box contains two objects

(C) At most one box contains two or more objects

(D) At least one box contains two or more of the objects

Or

The *k*th order polynomial model in one variable is :

$$y = \beta_0 + \beta_1 X + \beta_2 X^2 + ... + \beta_k X^k + \in,$$

where $\in \sim NID(0, \sigma^2)$. If we set $x_j = X^j, j = 1, 2, ..., k$, then the model is :

(A) simple linear regression model

(B) non-linear regression model

(C) logistic regression model

(D) multiple linear regression model

38. The solution of differential equation

$$(\sec x . \tan x . \tan y - e^x)\, dx + (\sec x . \sec^2 y)\, dy = 0$$

is :

(A) $\tan y = e^x + c$

(B) $\tan y + e^x = c$

(C) $\tan x \,.\, \tan y = e^x + c$

(D) $\tan y \,.\, \sec x = e^x + c$

Or

In simple linear regression model, if the data contain repeated observations on Y at the same value of X, then the maximum value of coefficient of determination (R^2) is :

(A) greater than 1

(B) equal to 1

(C) equal to 0

(D) less than 1

39. The orthogonal trajectories of the Parabola $6ay^2 = (x - 3)$, where *a* is variable parameter, is given by equation :

(A) $(x + 3)^2 + y^2 = c^2$

(B) $(x - 3)^2 + \frac{y^2}{2} = c^2$

(C) $\frac{(x - 3)^2}{2} + y^2 = c^2$

(D) $(x - 3)^2 - y^2 = c^2$

Or

For a sampling design (U, S, P), let, for $i = 1, 2, \ldots\ldots, N$, y_i denotes y-value of ith element of the population

$$T_i = \begin{cases} 1 & \text{if } i\text{th element of the population belongs to the sample} \\ 0 & \text{otherwise} \end{cases}$$

and

π_i denotes the inclusion probability of ith element of the population. Then the Horvitz-Thompson estimator for population mean is given by :

(A) $\bar{Y}_{HT} = \frac{1}{N} \sum_{i=1}^{N} \frac{y_i}{\pi_i}$

(B) $\bar{Y}_{HT} = \frac{1}{N} \sum_{i=1}^{N} \frac{y_i T_i}{\pi_i}$

(C) $\bar{Y}_{HT} = \frac{1}{n} \sum_{i=1}^{N} \frac{y_i T_i}{\pi_i}$

(D) $\bar{Y}_{HT} = \frac{1}{n} \sum_{i=1}^{N} y_i T_i$

40. The solution of differential equation :

$$\frac{d^2 y}{dx^2} + 4y = \sin^2 x$$

is :

(A) $y(x) = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{8} - \frac{x \sin 2x}{8}$

(B) $y(x) = c_1 e^{2x} + c_2 e^{-2x} + \frac{1 - x \sin 2x}{8}$

(C) $y(x) = c_1 \cos 2x + c_2 \sin 2x + 1 - x \sin 2x$

(D) $y(x) = c_1 e^{2x} + c_2 e^{-2x} + 1 - x \sin 2x$

Or

Based on the random sample of size n = 100 taken by using SRSWOR it is observed that sample mean $\bar{Y} = 150$ and standard error $SE(\bar{Y}) = 8.1$ then the 95% confidence interval for population mean is :

(A) (134.124, 165.876)

(B) (145.950, 154.050)

(C) (125.700, 174.300)

(D) (141.900, 158.100)

41. The set of linearly independent solutions of differential equation :

$$\frac{d^4 y}{dx^4} - \frac{d^2 y}{dx^2} = 0$$

is :

(A) $\{1, x, e^x, e^{-x}\}$

(B) $\{1, x, e^x, x.e^x\}$

(C) $\{1, x, e^{-x}, x.e^{-x}\}$

(D) $\{1, x, e^{-x}, x e^x\}$

Or

Which of the following statements is *wrong* ?

(A) For a fixed effective size (FES) design with n-draws, row sum of ith row of double inclusion probability matrix is $(n-1)\,\pi_i$

(B) PPSWR is not a FES design

(C) Under ordered SRSWR design with N = 5 and n = 2, $P(\nu(s) = 2) = 4/5$, where $\nu(s)$ denotes the number of distinct elements in the sample

(D) For FES design, $P(\nu(s) = \text{constant}) = 1$

42. The number of quadratic non-residues modulo 23 is :

(A) 10

(B) 22

(C) 11

(D) 2

Or

Under SRSWOR sampling design, the bias of the regression estimator of population mean $\overline{Y}_{Reg}$ is given by :

(A) $-\text{cov}\,(\hat{\beta}, \overline{X})$

(B) $\text{cov}\,(\hat{\beta}, \overline{X})$

(C) $-\text{cov}\,(\hat{\beta}, \overline{X})$

(D) $\dfrac{-\text{cov}\,(\hat{\beta}, \overline{X})}{s_{xy}}$

43. Which of the following arithmetic functions is totally multiplicative ?

(A) $f(n) = \sigma(n)$

(B) $f(n) = \tau(n)$

(C) $f(n) = \mu(n)$

(D) $f(n) = n^3$

Or

Let $\overline{\overline{Y}}$ be the mean of cluster means of clusters selected under cluster sampling then variance of $\overline{\overline{Y}}$ is given by :

(A) $V(\overline{\overline{Y}}) = \frac{1}{n} s_b^2$

(B) $V(\overline{\overline{Y}}) = \left(\frac{1}{n} - \frac{1}{N}\right) s_w^2$

(C) $V(\overline{\overline{Y}}) = \left(\frac{1}{n} - \frac{1}{N}\right) s_b^2 s_w^2$

(D) $V(\overline{\overline{Y}}) = \left(\frac{1}{n} - \frac{1}{N}\right) s_b^2$

44. The number of solutions of the congruence $x^2 \equiv 1 \pmod{260}$ is :

(A) 8

(B) 2

(C) 4

(D) 10

Or

Auxiliary variable's information is useful in :

(*i*) ratio estimation

(*ii*) stratum formation in stratified sampling

(*iii*) cluster sampling

(*iv*) PPS sampling

Which of the above statements is/are *correct* ?

(A) only (*i*), (*ii*) and (*iv*)

(B) only (*i*) and (*iv*)

(C) only (*iii*) and (*iv*)

(D) only (*i*)

45. Which of the following linear diophantine equations is *not* solvable ?

(A) $2x + 3y = 7$

(B) $4x + 3y = 19$

(C) $8x - 3y = 36$

(D) $21x + 15y = 62$

Or

The coefficients of orthogonal contrast $\sum_{i=1}^{3} c_i d_i$ are :

(A) $c_1 = 1,\ c_2 = 1,\ c_3 = 1,\ d_1 = -1,\ d_2 = -1,\ d_3 = -1$

(B) $c_1 = 1,\ c_2 = -1,\ c_3 = 1,\ d_1 = -1,\ d_2 = 1,\ d_3 = -1$

(C) $c_1 = 1,\ c_2 = 2,\ c_3 = 0,\ d_1 = -1,\ d_2 = 0,\ d_3 = 1$

(D) $c_1 = -2,\ c_2 = 1,\ c_3 = 1,\ d_1 = 0,\ d_2 = -1,\ d_3 = 1$

46. A particle of mass *m* is moving along the trajectory :

$$x = a(\phi - \sin\phi),\ y = a(1 + \cos\phi)$$

where $\phi = \phi(t)$, in a gravitational field of uniform acceleration *g* along downward *y* direction. Then $2ma^2\dot{\phi}^2 \sin^2\frac{\phi}{2}$ will represent :

(A) Kinetic energy

(B) Potential energy

(C) Lagrangian

(D) Hamiltonian

for the motion. (Choose the *correct* answer)

Or

Multiple comparison of treatment means given by Dunnett is used for :

(A) comparison of one particular treatment with other treatment means

(B) comparison of any two treatment means

(C) comparison of any three treatment means

(D) comparison of several treatment means simultaneously

47. A dynamical system consists of three particles m_1, m_2 and m_3 in motion in space. The distances between m_1 and m_2, m_1 and m_3 remain constant throughout motion. The motion will be governed by n Lagrange's equation. Then :

(A) $n = 7$

(B) $n = 6$

(C) $n = 5$

(D) $n = 9$

Or

In BIBD, there are a treatments, b blocks, each block contains k treatments and each treatment occurs r times and that there are $ar = bk$ total observations. The number of times each pair of treatments appears in the same block is :

(A) $\frac{r(k-1)}{(a-1)}$

(B) $\frac{k(r-1)}{(a-1)}$

(C) $\frac{a(k-1)}{(r-1)}$

(D) $\frac{b(r-1)}{(k-1)}$

48. Displacement of a rigid body in space is described by a (3×3) matrix A. Then :

(A) A is a real, Orthogonal matrix

(B) A is a real, Singular matrix

(C) A is complex Hermitian matrix

(D) A is a non-Hermitian matrix

Or

In 2^k factorial design with $k = 2$, the main effects A and B, interaction effect AB. The interaction effect AB is given by :

(A) $[ab + (1) - a - b]/2$

(B) $[ab + a - b - (1)]/2$

(C) $[ab + b - a - (1)]/2$

(D) $[a + b - ab - (1)]/2$

49. A bead is sliding along a wire rotating with uniform angular velocity ω. If the motion is force free Lagrange's equations will imply :

(A) $\frac{1}{r}\frac{d^2r}{dt^2} = \omega$

(B) $\frac{1}{r}\frac{d^2r}{dt^2} = \omega^2$

(C) $\frac{1}{r}\frac{d^2r}{dt^2} = \frac{1}{\omega}$

(D) $\frac{1}{r}\frac{d^2r}{dt^2} = \frac{1}{\omega^2}$

Or

When the interaction effect AB is confounded in 2^2 factorial design :

(A) the block effect and the main effect A are identical

(B) the block effect and the interaction effect AB are identical

(C) the block effect and the main effect B are identical

(D) the block effect and the interaction effect AB are not identical

50. Hamiltonian of a dynamical system represents total energy of the system, then :

(A) System is time dependent and conservative

(B) System is time independent and conservative

(C) The potential is velocity dependent

(D) System is time independent and non-conservative

Or

In 2^{3-1} fractional factorial design with treatments A, B and C, the estimate of the main effect A is :

(A) $[abc + b - a - c]/2$

(B) $[abc + a - b - c]/2$

(C) $[abc + c - a - b]/2$

(D) $[abc - a - b - c]/2$

51. The velocity components of two-dimensional steady fluid flow are known to be :

$$u = y \text{ and } v = x$$

Which of the following statements *does not* describe the fluid motion properly ?

(A) The stream lines are rectangular hyperbolas

(B) The fluid motion is irrotational and steam lines are rectangular hyperbolas

(C) The fluid motion is rotational

(D) The fluid motion is irrotational

Or

Secular trend in a time series can be measured by :

(A) two methods

(B) three methods

(C) four methods

(D) five methods

52. A curve C : $\bar{r} = \bar{r}(s)$ is a plane curve if and only if :

(A) $\bar{r}' \times \bar{r}'' = 0$

(B) $(\bar{r}' \times \bar{r}'') . \bar{r}''' = 0$

(C) $\bar{r}' \times \bar{r}'' \times \bar{r}''' = 0$

(D) $(\bar{r}' \times \bar{r}'') . \bar{r}''' = \text{constant}$

Or

Seasonal variations can occur in a time series within a period of :

(A) nine years

(B) one year

(C) three years

(D) four years

53. If ϕ and Ψ denote velocity potential and stream function of a two-dimensional motion of an incompressible fluid, then :

(A) Both ϕ and Ψ are harmonic functions

(B) Only ϕ is a harmonic function

(C) Only Ψ is a harmonic function

(D) Neither ϕ is a harmonic function nor Ψ

Or

The most frequently used mathematical model of a time series is the :

(A) multiplicative model

(B) exponential model

(C) mixed model

(D) additive model

54. If $\overline{v}$ denotes velocity field of an incompressible fluid in motion, then the mass conservation will not imply the following :

(A) $\nabla \cdot \overline{v} = 0$

(B) $\frac{\partial \rho}{\partial t} + \nabla \cdot \overline{v} = 0$

(C) $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \overline{v}) = 0$

(D) $\nabla \times \overline{v} = 0$

Or

When the trend is of exponential type, the moving averages should be computed by using the :

(A) weighted mean

(B) harmonic mean

(C) arithmetic mean

(D) geometric mean

55. The equations of motion for two-dimensional fluid motion due to a point source do not lead to the following :

(A) Stream lines are concentric circles

(B) Flow lines are straight lines diverging from the location of the source

(C) Stream lines and flow lines do not intersect

(D) Stream lines and flow lines are orthogonal

Or

Consider a two state Markov chain $\{X_n : n \geq 0\}$ with state space $S = \{0, 1\}$ and stationary transition probability matrix :

$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$$

Let $\pi(0) = P(X_0 = 0)$ and $\pi(1) = P(X_0 = 1)$, then to have $P(X_n = 0) = \pi(0)$ and $P(X_n = 1) = \pi(1)$ the values of $\pi(0)$ and $\pi(1)$ are :

(A) $\pi(0) = p, \ \ \pi(1) = q$

(B) $\pi(0) = 1 - p, \ \ \pi(1) = q$

(C) $\pi(0) = \dfrac{q}{p+q}, \ \ \pi(1) = \dfrac{p}{p+q}$

(D) $\pi(0) = \dfrac{p}{p+q}, \ \ \pi(1) = \dfrac{q}{p+q}$

56. Which of the following curves drawn on a right circular cylinder will *not* be a geodesic ?

(A) Ellipse

(B) Straight line

(C) Circle

(D) Helix

Or

Consider a Markov chain $\{X_n, n \geq 0\}$. Let

$p_{ij}^{(n)} = P(X_{m/n} = j | \ X_m = i)$.

The following relation holds :

(A) $p_{ij}^{(n)} = \sum_{k \in s} p_{ik}^{(r)} \ p_{kj}^{(n-r)}$ for all r such that $1 \leq r < n$

(B) $p_{ij}^{(n)} = \sum_{k \in s} p_{ik}^{(r)} \ p_{kj}^{(n-r)}$ for some $r < n$

(C) $p_{ij}^{(n)} = \sum_{k \in s} p_{ik}^{r} \ p_{kj}^{n-r}$ for some $r < n$

(D) $p_{ij}^{(n)} = \sum_{k \in s} p_{ik}^{r} \ p_{kj}^{n-r}$ for all $1 \leq r < n$

57. The first and second fundamental forms on a two-dimensional surface are :

I : $du^2 + (u^2 + c^2)\, dv^2$

II : $\dfrac{-2c}{\sqrt{u^2 + c^2}}\, du dv$

Which of the following statements does follow ?

(A) Parametric curves are not orthogonal

(B) Asymptotic lines are not orthogonal

(C) Parametric curves are conjugate directions

(D) Parametric curves are asymptotic lines

Or

Let :

$$f_{ij}^{(n)} = \mathrm{P}\,(\mathrm{X}_{m+1} \neq j, \mathrm{X}_{m+2} \neq j, \ldots\ldots,$$
$$\mathrm{X}_{m+n-1} \neq j, \mathrm{X}_{m+n} = j \mid \mathrm{X}_m = i)$$

and $\lim\limits_{n\to\infty} f_{ij}^{(n)} = f_{ij}$.

Suppose i is a recurrent state and $i \to j$. Which of the following is *true* ?

(A) j is transient and $f_{ij} < 1$

(B) j is null recurrent

(C) j is transient and $f_{ji} = 1$

(D) j is recurrent and $f_{ij} = f_{ji} = 1$

58. C is a curve on the surface S with first fundamental form :

I : $dz^2 + a^2 d\theta^2$, a-constant.

K and τ denote curvature and torsion of C.

If C is a non-geodesic curve on S, then :

(A) K and τ in general will be position dependent

(B) K and τ are both constants

(C) K may be constant and $\tau = 0$

(D) K/τ will be a constant

Or

Consider a Markov chain having transition matrix :

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 & 0 \\ 0 & 1/5 & 2/5 & 1/5 & 0 & 1/5 \\ 0 & 0 & 0 & 1/6 & 1/3 & 1/2 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1/4 & 0 & 3/4 \end{bmatrix}$$

and state space S = {0, 1, 2, 3, 4, 5}. Which of the following statements is *correct* ?

(A) 2 is transient state

(B) 0 is transient state

(C) 5 is transient state

(D) 3 is transient state

59. The parametric curves on the two-dimensional surface S with first fundamental form :

I : $g_{11}(dx^1)^2 + 2g_{12}(dx^1)(dx^2) + g_{22}(dx^2)^2$

will be non-orthogonal if and only if :

(A) $g_{11} \neq 0$

(B) $g_{12} \neq 0$

(C) $g_{22} \neq 0$

(D) $g_{11} g_{22} - (g_{12})^2 \neq 0$

Or

Let $\{N(t), t \geq 0\}$ denotes Poisson process. If N(*t*) = 1, then the arrival time for the first event has distribution :

(A) Exponential with mean *t*

(B) Uniform (0, *t*)

(C) Gamma $(1, \frac{t}{2})$

(D) Poisson with mean 1

60. If the function F does not contain the variables x and y explicitly, then the extremal is :

(A) Parabola

(B) Straight line

(C) Ellipse

(D) Circle

Or

What is the denominator in the General Fertility Rate (GFR) ?

(A) All married women

(B) Married women in age group 15-49

(C) All women in age group 15-49

(D) All women

61. The number of degrees of freedom of a simple pendulum with a variable length is :

(A) 1

(B) 6

(C) 3

(D) 2

Or

In the demographic study of population, a country with low birth rate and low death rate is in the following phase :

(A) First

(B) Second

(C) Third

(D) Fourth

62. The extremal of the functional :

$$I[y(x)] = \int_{-1}^{1} (12xy - y'^2)\, dx$$

satisfying $y(-1) = 1$, $y(0) = 0$ is :

(A) $x = y^2$

(B) $y^3 + x = 0$

(C) $y = x^2$

(D) $y + x^3 = 0$

Or

New Zealand has 23% of its population less than 15, 12% over 65, and the remaining 65% between 15 and 65.

The young dependency ratio for New Zealand is :

(A) 53.8

(B) 16.9

(C) 18.5

(D) 35.4

63. The variational problem of extremizing the functional :

$$I[y(x)] = \int_{1}^{3} y(2x - y)\, dx$$

$$y(1) = 0, \quad y(0) = 0$$

has :

(A) a unique solution

(B) exactly two solutions

(C) no solution

(D) Infinite number of solutions

Or

Which of the following statements is *not correct* ?

(A) Control charts are a proven technique for improving productivity

(B) Control charts are effective in defect prevention

(C) Control charts never prevent unnecessary process adjustment

(D) Control charts provide information about process capability

64. The resolvent kernel of the Volterra integral equation having kernel :

$$K(n, t) = e^{(x-t)}$$

is :

(A) $e^{(x+t)(1+\lambda)}$

(B) $e^{(x-t)(1-\lambda)}$

(C) $e^{(x-t)(1+\lambda)}$

(D) $e^{(x-t)(1+\lambda)^2}$

Or

The use of warning limits used in control charts increases :

(A) proportion of defectives

(B) process capability

(C) risk of false alarms

(D) process variability

65. The eigenvalue λ of Fredholm integral equation,

$$y(x) = \lambda \int_0^1 (x^2 + y(t))\, dt$$

is :

(A) $\lambda = 4$

(B) $\lambda = 2$

(C) $\lambda = -2$

(D) $\lambda = -4$

Or

The probability of false alarm for $\bar{X}$-chart with 3-sigma control limits is :

(A) 0.0027

(B) 0.00027

(C) 0.002

(D) 0.027

66. The initial value problem corresponding to the integral :

$$y(x) = 1 + \int_0^x y(t)\, dt$$

is :

(A) $y' + y = 0, \quad y(0) = 0$

(B) $y' - y = 0, \quad y(0) = 1$

(C) $y' - y = 0, \quad y(0) = 0$

(D) $y' + y = 0, \quad y(0) = 1$

Or

If $C_P = \dfrac{USL - LSL}{6\sigma}$, where

USL : Upper specification limit

LSL : Lower specification limit

σ : Process standard deviation

Then the probability of non-conformance (p), when process mean,

$\mu = \dfrac{USL + LSL}{2}$ is given by :

(A) $p = 2\Phi(C_P)$

(B) $p = 2\Phi(-3\, C_P)$

(C) $p = 2\Phi(-C_P)$

(D) $p = 2\Phi(6\, C_P)$

where ϕ denotes the distribution function of standard normal distribution.

67. For the homogeneous Fredholm integral equation :

$$\phi(x) = \lambda \int_0^1 e^{x+t}\, \phi(t)\, dt$$

a non-trivial solution exists when λ has the value :

(A) $\lambda = \dfrac{2}{e-1}$

(B) $\lambda = \dfrac{1}{e^2+1}$

(C) $\lambda = \dfrac{1}{e+1}$

(D) $\lambda = \dfrac{2}{e^2-1}$

Or

Which of the following statements is *not correct* ?

(A) Sampling inspection by variables provides better quality protection than by attributes

(B) Sampling inspection by variables require less inspection than by attributes

(C) Errors of measurements are bettter surfaced in sampling inspection by variables than by attributes

(D) Sampling plan (N = 10,000, n = 500, C = 2) provides less protection to the vendor that the sampling plan (N = 10,000, n = 500, C = 0)

68. Let * be the usual convolution product on $L^1(\mathbf{R})$. For any two differentiable functions :

$$f, g \in L^1(\mathbf{R}), \ \frac{d}{dx}((f * g)(x)).$$

(A) $= f(x) * \frac{d}{dx} g(x) + g(x) * \frac{d}{dx} f(x)$

(B) need not be equal to $f(x) * \frac{d}{dx} g(x)$

(C) $= f(x) * \frac{d}{dx} g(x) = g(x) * \frac{d}{dx} f(x)$

(D) need not be equal to $g(x) * \frac{d}{dx} f(x)$

Or

Let ϕ be the coherent structure of n associated components with component reliabilities $p_1, p_2, ..., p_n$ then :

(A) $P[\phi(\underline{X}) = 1] = \prod_{i=1}^{n} p_i$

(B) $P[\phi(\underline{X}) = 1] \leq \prod_{i=1}^{n} p_i$

(C) $\prod_{i=1}^{n} p_i \leq P[\phi(\underline{X}) = 1] \leq \coprod_{i=1}^{n} p_i$

(D) $P[\phi(\underline{X}) = 1] = \coprod_{i=1}^{n} p_i$

69. Let $L[-\pi, \pi]$ be the class of all 2π-periodic Lebesgue integrable functions. Which of the following statements is *true* ?

(A) $f * g$ need not be equal to $g * f$

(B) $(f * g)^\wedge(n) = \hat{f}(n) . \overline{\hat{g}(n)} + \overline{\hat{f}(n)} . \hat{g}(n), \ \forall n \in \mathbf{Z}$

(C) $\|f\|_1 \|g\|_1 \leq \|f * g\|_1$

(D) $(f * g)^\wedge(n) = \hat{f}(n) . \hat{g}(n), \ \forall n \in \mathbf{Z}$

Or

For a probabilistic discrete inventory model with instantaneous demand and no setup cost, the optimum stock level '*z*' can be obtained by (Here C_1–inventory carrying cost and C_2–shortage cost) :

(A) $\sum_{d=0}^{z} p(d) \leq \frac{c_2}{c_1 + c_2} \leq \sum_{d=0}^{z-1} p(d)$

(B) $\sum_{d=0}^{z} p(d) \leq \frac{c_2}{c_1 + c_2} \geq \sum_{d=0}^{z-1} p(d)$

(C) $\sum_{d=0}^{z} p(d) \geq \frac{c_2}{c_1 + c_2} \leq \sum_{d=0}^{z-1} p(d)$

(D) $\sum_{d=0}^{z} p(d) \geq \frac{c_2}{c_1 + c_2} \geq \sum_{d=0}^{z-1} p(d)$

70. Let L be the Laplace transform. Then $L(\sqrt{t})$:

(A) $= \sqrt{\frac{\pi}{s}}, s > 0$

(B) $\frac{\sqrt{\pi}}{2}$

(C) $\frac{\sqrt{\pi}}{2s^{3/2}}, \ s > 0$

(D) $\frac{\pi}{s}, \ s > 0$

Or

The probability distribution of monthly sale of a Paragon shoe is as follows :

Monthly Sale	Probability
0	0.01
100	0.06
200	0.25
300	0.35
400	0.20
500	0.03
600	0.10

The cost of carrying inventory is Rs. 20 per unit per month and the cost of unit shortage is Rs. 80 per month. Then optimum stock level which minimizes the total expected cost is given by :

(A) 100

(B) 300

(C) 400

(D) 200

71. The series :

$$\sum_{n=1}^{\infty} \frac{1}{n^p (\log n)^q}$$

is convergent for :

(A) $p > 0$ and $q > 0$

(B) $p = 1$ and $q = 1$

(C) $p \geq 1$ and $q > 1$

(D) $p \geq 1$ and $q \geq 1$

Or

In $(M/G/1):(\infty/GD)$ queueing model, if

λ is average customer arrival rate

σ is Std. deviation

ρ is Traffic intensity

then average queue length is given by :

(A) $\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)}$

(B) $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)^2}$

(C) $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$

(D) $\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)^2}$

72. Let f be a real function on (a, b). If

$f''(x) \geq 0, \quad \forall\ x \in (a, b)$.

Then which of the following is *true* ?

(A) $f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) \leq \frac{\sum_{i=1}^{n} f(x_i)}{n}$,

$x_i \in (a, b), \forall\ i = 1, 2, ..., n$

(B) $f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) \geq \frac{\sum_{i=1}^{n} f(x_i)}{n}$,

$x_i \in (a, b), \forall\ i = 1, 2, ..., n$

(C) $f\left(\frac{\sum_{i=1}^{n} (x_i)}{n}\right) > \frac{\sum_{i=1}^{n} f(x_i)}{n}$,

$x_i \in (a, b), \forall\ i = 1, 2, ..., n$

(D) $f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \frac{1}{n}\left(\sum_{i=1}^{n} f(x_i)\right)$,

$x_i \in (a, b), \forall\ i = 1, 2, ..., n$

Or

In $(M / D / 1) : (GD / \infty / \infty)$ queue system where the service time is constant and $\rho = \frac{\lambda}{\mu}$. The Pollaczek-Khintchine formula for expected number of customers in the system reduces to :

(A) $L_S = \rho + \frac{\rho^2}{2(1-\rho)}$

(B) $L_S = \rho - \frac{\rho^2}{2(1-\rho)}$

(C) $L_S = \rho - \frac{\rho^2}{(1-\rho)^2}$

(D) $L_S = \rho + \frac{\rho^2}{(1-\rho)^2}$

73. Let **Z** be the set of integers and Q be the set of rationals $(\mathbf{Z} \subset Q)$. If m is the Lebesgue measure, then :

(A) $m(\mathbf{Z}) < m(Q)$

(B) $m(\mathbf{Z}) = m(Q)$

(C) $m(Z) > m(Q)$

(D) $0 < m(Z) \leq m(Q) < \infty$

Or

In dynamic programming, determine the values of x_1, x_2 and x_3 so as to :

Maximize : $Z = x_1 \cdot x_2 \cdot x_3$

Subject to : $x_1 + x_2 + x_3 = 20$

and $x_1, x_2, x_3 \geq 0$

(A) (10/3, 10/3, 10/3)

(B) (20/3, 20/3, 20/3)

(C) (20, 20, 20)

(D) (30, 30, 30)

74. If $f : [0, 1] \to R$ defined as :

$$f(x) = \begin{cases} 1, & \text{if } x \in Q \cap [0, 1] \\ 0, & \text{otherwise} \end{cases}$$

then :

(A) f is Riemann integrable function

(B) $(L)\int_0^1 f(x)\,dx = 0$

(C) $(L)\int_0^1 f(x)\,dx = 1$

(D) f is not Lebesgue integrable function

Or

In a non-linear programming problem, which of the following is *not correct* ?

(A) The objective function is concave, if the principal mirrors of bordered Hessian matrix, alternate in sign, beginning with the negative sign

(B) If $f(x)$ is strictly convex, the Kuhn-Tucker conditions are sufficient conditions for an absolute maximum

(C) In case of maximization of NLPP, all constraints must be converted into '$\leq$' type and in the case of minimization NLPP into '$\geq$' type

(D) If the principal minors of bordered Hessian matrix are positive, the objective function is convex

75. Let E_1 and E_2 be any two distinct measurable subsets of **R**. If $m(E_1) = 0$ and $m(E_2) > 0$, then :

(A) every subset of E_2 is measurable

(B) every superset of E_1 is measurable

(C) $E_2 \cap (\mathbf{R} - E_1)$ need not be measurable

(D) $E_2 - E_1$ is measurable

Or

Dynamic programming deals with the :

(A) single stage decision-making problem

(B) time independent decision-making problem

(C) problems which fix the levels of different variables so as to maximize profit or minimize cost

(D) multi-stage decision-making problems

ROUGH WORK

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