Test Booklet No.
प्रश्नपत्रिका क्र. **F**

Paper-II
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

AUG - 30215

Time Allowed : 1¼ Hours] **[Maximum Marks : 100**

Number of Pages in this Booklet : **28** Number of Questions in this Booklet : **50**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **50** objective type questions. Each question will carry *two* marks. *All* questions of Paper-II will be compulsory, covering entire syllabus (including all electives, without options).
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/ questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 50 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. या प्रश्नपत्रिकेतील **सर्व** प्रश्न सोडविणे अनिवार्य आहे. सदरचे प्रश्न हे या विषयाच्या संपूर्ण अभ्यासक्रमावर आधारित आहेत.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

MATHEMATICAL SCIENCE

Paper II

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Note : This Paper contains **Fifty (50)** multiple choice questions, each question carrying **Two (2)** marks. Attempt *All* questions.

1. Let $f : \mathbf{R} \to \mathbf{R}$ and $g : [a, b] \to \mathbf{R}$ be defined as $f(x) = x^2 \ \forall \ x \in \mathbf{R}$ and $g(x) = x^2 \ \forall \ x \in [a, b]$. Then :

   (A) f and g both are uniformly continuous

   (B) f is uniformly continuous and g is continuous

   (C) f is continuous and g is uniformly continuous

   (D) f and g both are continuous but not uniformly continuous

2. Let $\{a_n\}$ be a sequence of positive numbers. Which of the following is *true* ?

   (A) Convergence of $\Sigma \frac{\sqrt{a_n}}{n}$ implies the convergence of $\Sigma \ a_n$

   (B) Convergence of $\Sigma \frac{a_n}{\sqrt{n}}$ implies the convergence of $\Sigma \ a_n$

   (C) Convergence of $\Sigma \ a_n$ implies the convergence of $\Sigma \sqrt{\frac{a_n}{n}}$

   (D) Convergence of $\Sigma \ a_n$ implies the convergence of $\Sigma \frac{\sqrt{a_n}}{n}$

3. If $S_1 = \sqrt{2}$ and $S_{n+1} = \sqrt{2 + S_n}$, $n = 1, 2, 3......$, then $\{S_n\}$ converges to :

(A) 2

(B) $2 + \sqrt{2}$

(C) $\sqrt{2 + \sqrt{2}}$

(D) $\sqrt{2}$

4. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 x^n}{5^n \sqrt{n^5}}$ is :

(A) $(-5, 5]$

(B) $[-5, 5]$

(C) $[-5, 5)$

(D) $(-5, 5)$

5. Let the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^4$ be such that $T(1, 0, 0) = (1, 2, 0, 4)$, $T(0, 1, 0) = (2, 0, -1, -3)$, $T(0, 0, 1) = (0, 0, 0, 0)$. Then :

(A) $T(x, y, z) = (x + 2y, -x, y, 4x + 3y)$

(B) $T(x, y, z) = (x + 2y, 2x, -y, 4x - 3y)$

(C) $T(x, y, z) = (x - 2y, -x, y, 4x + 3y)$

(D) $T(x, y, z) = (x - 2y, x, -y, -4x - 3y)$

6. Dimension of the vector space $\sigma^2 = \{(x, y) \mid x$ and y are complex numbers$\}$ over the field of reals is :

(A) 4

(B) 2

(C) 3

(D) 1

7. Let C(**R**) be the vector space of all continuous real functions over **R**. Then which of the following sets is linearly independent in C(**R**) ?

(A) $\{1, \cos 2x, \sin^2 x\}$

(B) $\{1, e^x, e^{2x}\}$

(C) $\{1, x^2 - 2x + 1, x^2, (x - 2)^2\}$

(D) $\{1, \log(1 + |x|), \log((1 + |x|)^2)\}$

8. Which of the following is a subspace of the vector space C[*a*, *b*] ?

(A) $\{f \in C[a, b] \mid f(a) = 1\}$

(B) $\{f \in C[a, b] \mid \int_a^b f(x)\,dx > 0\}$

(C) $\left\{f \in C[a, b] \mid f\left(\frac{a+b}{2}\right) = 1\right\}$

(D) $\{f \in C[a, b] \mid f(0) = 0\}$

9. If $P(D \mid F) > P(E \mid F)$ and $P(D \mid \bar{F}) > P(E \mid \bar{F})$, then the relation between P(D) and P(E) is :

(A) $P(D) < P(E)$

(B) $P(D) = P(E)$

(C) $P(\bar{D}) < P(E)$

(D) $P(D) > P(E)$

10. Let X be a degenerate random variable such that $P(X = 2) = 1$. Then which of the following statements is *true* ?

(A) $EX = 1$, $V(X) = 2$

(B) $EX = 2$, $V(X) = 1$

(C) $EX = 2$, $V(X) = 0$

(D) $EX = 2$, Variance does not exist

11. Chebyshev's inequality can be applied to the distribution if it is :

(A) Any distribution regardless of its shape

(B) Normal

(C) Exponential

(D) Gamma

12. Let X_1, X_2 be iid rvs with common pmf

$P[X = \pm 1] = \frac{1}{2}$. Let $X_3 = X_1 X_2$.

Then which of the following statements is *correct* ?

(A) X_1, X_2 and X_3 are mutually independent

(B) X_1, X_2 and X_3 are pairwise independent but not mutually independent

(C) $EX_1 = EX_2 = EX_3 = 1$

(D) X_1, X_2 and X_3 are not mutually independent

13. The set of feasible solutions to a linear programming problem is a :

(A) Concave set

(B) Non-empty set

(C) Bounded set

(D) Convex set

14. In graphical solution method of LPP, the redundant constraint is one :

(A) Which forms the boundary of feasible region

(B) Which does not form boundary of feasible region

(C) Which does not optimize the objective function

(D) Which optimizes the objective function

15. In graphical solution method of LPP, if two constraints do not intersect in the positive quadrant of the graph, then :

(A) one of the constraints is redundant

(B) the solution is infeasible

(C) the solution is unbounded

(D) the solution is optimum

16. Which of the following is the optimal solution to the LPP of maximizing

$$z = x_1 + 2x_2$$

subject to

$$x_1 - x_2 \leq 1$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

(A) (4, 3)

(B) (2, 1)

(C) Unbounded solution

(D) Infeasible solution

17. Which of the following functions is a metric ?

(A) $d(x, y) = \text{Min} \{|x|, |y|\}$, $\forall\ x, y \in \mathbf{R}$

(B) $d(x, y) = |x^2 - y^2|$, $\forall\ x, y \in \mathbf{R}$

(C) $d(x, y) = |x^3 - y^3|$, $\forall\ x, y \in \mathbf{C}$

(D) $d(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right|$, $\forall\ x, y \in \mathbf{R}\setminus\{0\}$

Or

Which of the following statements is true, if $y = -3x - 7$ is the line of regression of y on x ?

(A) The y-intercept is 7

(B) The value of y is expected to decrease by 3 when x increases by 4 units

(C) The slope of the line is 3

(D) The value of y is expected to decrease by 3 when x increases by 1 unit

18. The set **R** of reals with usual metric is :

(A) Connected and complete

(B) Compact

(C) Compact and connected

(D) Complete but not connected

Or

The simplest linear regression model is $y = \alpha + \beta X + \in$, where $\in$ is a random variable with mean $\mu_{\in} = 0$ and variance $\sigma_{\in}^{2} = \sigma^2$. Let X* denote a particular value of the independent variable X. Which of the following statements is true regarding the variance of Y when X = X* ?

(A) $\sigma^2_{y|x^*} = \sigma^2$

(B) $\sigma^2_{y|x^*} = \alpha + \beta x^* + \sigma^2_{\in}$

(C) $\sigma^2_{y|x^*} = \beta x^* + \sigma^2$

(D) $\sigma^2_{y|x^*} = \beta^2\sigma^2$

19. Which of the following sets is a perfect set in $\mathbf{R}^2$?

(A) The set of complex numbers

(B) The set of all complex numbers z such that $|z| < 1$

(C) The set of all complex numbers z such that $|z| > 1$

(D) Q × Q, where Q is the set of rational numbers

Or

The lines of regression of Y on X and X on Y are given as X + 2Y – 5 = 0 and 2X + 3Y = 8. Then the mean values of X and Y respectively are :

(A) 1 and 2

(B) 2 and 1

(C) 2 and 5

(D) 2 and 3

20. Consider the set $A = \left\{x \sin \frac{1}{x} = x \in (0, 1)\right\}$ as a subset of **R**. Then :

(A) A is closed

(B) A is compact

(C) A is connected

(D) A is not bounded

Or

Let A and B be any two events in a probability space such that $P(A^c \cap B) = 0.2$, $P(B^c \cap A) = 0.1$, $P(A \cap B) = 0.4$. Which of the following statements is *correct* ?

(A) $P(A) = 0.3$

(B) $P(A^c \cap B^c) = 0.6$

(C) $P(A \cup B) = 0.7$

(D) $P(B) = 0.5$

21. Which of the following functions is *not* of bounded variation on $\left[0, \frac{\pi}{2}\right]$?

(A) $f(x) = \sin 2x$

(B) $f(x) = \sin x + \cos x$

(C) $f(x) = \sin (\cos 2x)$

(D) $f(x) = x^2 \sin \left(\frac{\pi}{x^2}\right)$

Or

If P_1 and P_2 are two probability measures defined on a measurable space $(\Omega, \mathbf{F})$, then which of the following statements *cannot* be true always ?

(A) $P_1(A) = P_2(A)$ for some $A \in \mathbf{F}$

(B) $P_1(A) = 0 \Rightarrow P_2(A) = 0$

(C) Both (A) and (B)

(D) $P_1(A) > P_2(A) \ \forall \ A \in \mathbf{F}$

22. Let u and d be the usual metric and discrete metric on $\mathbf{R}$, respectively. Define $I = (\mathbf{R}, u) \to (\mathbf{R}, d)$ as $I(x) = x$ for all $x \in \mathbf{R}$. Then :

(A) I is bounded

(B) I is not continuous

(C) I is continuous but not uniformly continuous

(D) I is uniformly continuous

Or

Consider a measurable space $(\Omega, \mathbf{F})$ where $\Omega = \{1,2, 3, 4\}$, $A = \{1, 2\}$ and $\mathbf{F} = \{\phi, A, A^c, \Omega\}$ and define functions :

$X(\omega) = 1 \ \forall \ \omega \in \Omega$

$$Y(\omega) = \begin{cases} 0 & \text{if } \omega \in A \\ 1 & \text{if } \omega \notin A \end{cases}$$

$Z(\omega) = \omega \quad \forall \ \omega \in \Omega$

Then which of the following is *true* ?

(A) All the functions X, Y and Z are random variables

(B) Only X and Y are random variables

(C) Only X is random variable

(D) Only Y is random variable

23. Let z and w be any two complex numbers. Then which of the following is *not* true ?

(A) $|zw| = |z| \ |w|$

(B) $\overline{z + w} = \overline{z} + \overline{w}$

(C) $|z| + |w| \leq |z + w|$

(D) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$, $w \neq 0$

Or

Which of the following statements is *not true* ?

(A) $E(X + Y)^2 \geq \frac{1}{2}\{EX^2 + EY^2\}$

(B) If X is integrable, then $|X|$ is also integrable

(C) E(X) may not exist

(D) Moments of even order always exist

24. The number of zeros of $z^7 - 4z^3 + z + 1$ which lie in the interior of the unit circle $|z| = 1$ is :

(A) 7

(B) 1

(C) 3

(D) 0

Or

Characteristic function of X is $\phi(u)$ then characteristic function of $b + cX$ is :

(A) $e^{ib}\phi(-uc)$

(B) $e^{iu}\phi(ub)$

(C) $e^{iub}\phi(uc)$

(D) $1 - e^{iub}\phi(uc)$

25. Let C be the circle $|z| = 1$. Then the value of the integral $\int_C \frac{2z + 3}{z^2 + 2z + 5} dz$ is :

(A) $-1 - 2i$

(B) $-1 + 2i$

(C) 0

(D) $2 + 3i$

Or

A random sample of 15 people is taken from a population in which 40% favour a particular political stand. What is the probability that exactly 6 individuals in the sample favour this political stand ?

(A) 0.5000

(B) 0.4000

(C) 0.2066

(D) 0.1600

26. Let $r : [a, b] \to \mathbf{C}$ be a Lipschitz function. Then :

(A) f is absolutely continuous and of bounded variation on $[a, b]$

(B) f is absolutely continuous and not of bounded variation on $[a, b]$

(C) f is of bounded variation but not absolutely continuous on $[a, b]$

(D) f is neither absolutely continuous nor of bounded variation on $[a, b]$

Or

A business evaluates a proposed venture as follows. It stands to make a profit of Rs. 10,000 with probability 3/20, to make a profit of Rs. 5,000 with probability 9/20, to break even with probability 1/4, and to incur a loss of Rs. 5,000 with probability 3/20. The expected profit is :

(A) Rs. 3,000

(B) Rs. 3,250

(C) Rs. 10,000

(D) Rs. 15,000

27. The value of $\log(2 - 3i)$ is :

(A) $\log\sqrt{13} + i \text{ arc tan } (3/2)$

(B) $\log\sqrt{13} + i[\pi - \text{arc tan } (3/2)]$

(C) $\log\sqrt{13} - i \text{ arc tan } (3/2)$

(D) $\log\sqrt{13} - i[\pi - \text{arc tan } (3/2)]$

Or

One of the effects of flooding a lake is that mercury is leached from the soil, enters the food chain, and contaminates the fish. Suppose that the concentration of mercury in an individual fish follows an aproximate normal distribution with a mean of 0.25 ppm and a standard deviation of 0.08 ppm. Fish are safe to eat if the mercury level is below 0.30 ppm.

What proportion of fish would be safe to eat ?

(A) 23%

(B) 63%

(C) 73%

(D) 27%

28. Suppose G is open and connected in $\mathbf{C}$ and $f: G \to \mathbf{C}$ is differentiable with $f'(z) = 0$ for all z in G. Then :

(A) f strictly increasing

(B) f is not bounded

(C) f is not constant

(D) f is constant

Or

A professional basketball player sinks 80% of his foul shots in the long run. If he gets 100 shots during a season, then the probability that he sinks 75 and 90 shots (inclusive of both) is approximately given by :

(A) $P[-1.25 \leq Z \leq 2.50]$

(B) $P[-1.125 \leq Z \leq 2.625]$

(C) $P[-1.375 \leq Z \leq 2.375]$

(D) $P[-1.375 \leq Z \leq 2.635]$

29. Let G be a group of order 29. Then :

(A) G need not be abelian

(B) G is cyclic

(C) G is abelian but not cyclic

(D) G has at least one proper subgroup

Or

Marks in a chemistry test follow a normal distribution with a mean of 65 and a standard deviation of 12. Approximately what percentage of the students would score below 50 ?

(A) 22%

(B) 11%

(C) 15%

(D) 18%

30. Let G be a finite group and $p \mid O(G)$, where p is a prime. Then :

(A) if $p^k \mid O(G)$ and H is a subgroup of G of order p^k, then H is cyclic

(B) for every integer i such that $p^i \mid O(G)$, G has a subgroup of order p^i

(C) G has a normal subgroup of order p

(D) if k is largest integer such that $p^k \mid O(G)$, then G has a normal subgroup of order p^k

Or

Let $X_1, X_2, ... X_n$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Then which of the following statements is *not correct*?

(A) $(\Sigma X_i, \Sigma X_1{}^2)$ is jointly sufficient for (μ, σ^2)

(B) $\bar{X}$ is sufficient for μ and S^2 is sufficient for σ^2

(C) $(\bar{X}, S^2)$ is jointly sufficient for (μ, σ^2)

(D) if σ^2 is known, then $\bar{X}$ is sufficient for μ and $\Sigma(X_i - \mu_0)^2$ is sufficient for σ^2 if $\mu = \mu_0$ is known

31. The number of all group homomorphisms from the group $\mathbf{Z}_8$ into the group $\mathbf{Z}_{20}$ is :

(A) 8

(B) 1

(C) 4

(D) 0

Or

Let $X_1, X_2, \ldots, X_n$ be a random sample from Poisson distribution with parameter λ. Then $T = \Sigma X_i$ is sufficient for λ. The distribution of X_1 given $T = t$ is :

(A) $\binom{t}{x_1}\left(\frac{1}{n}\right)^{x_1}\left(1-\frac{1}{n}\right)^{t-x_1}$; $x_1 = 0, 1, 2 \ldots\ldots n$

(B) $\binom{n}{x_1}\left(\frac{\lambda}{n}\right)^{x_1}\left(1-\frac{\lambda}{n}\right)^{n-x_1}$; $x_1 = 0, 1, 2, \ldots\ldots n$

(C) $\binom{t}{x_1}\left(\frac{1}{n}\right)^{x_1}\left(1-\frac{1}{n}\right)^{t-x_1}$; $x_1 = 0, 1, 2, \ldots\ldots t$

(D) $\binom{n}{x_1}\left(\frac{1}{\lambda}\right)^{x_1}\left(1-\frac{1}{\lambda}\right)^{n-x_1}$; $x_1 = 0, 1, 2 \ldots\ldots n$

32. Which of the following is *false* ?

(A) Every PID is UFD

(B) Every UFD is Euclidean domain

(C) Every Euclidean domain is UFD

(D) Every Euclidean domain is PID

Or

If X_1, X_2 are distributed as $B(n, p)$, n known and $0 < p < 1$, the number of unbiased estimators of p is :

(A) 2

(B) infinity

(C) 4

(D) 6

33. Which of the following groups has trivial centre ?

(A) A non-abelian simple group

(B) A group of order prime

(C) A group of order p^3 where p is prime

(D) The group of non-singular $n \times n$ matrices over $\mathbf{R}$

Or

Let $X_1, X_2, \ldots\ldots X_n$ be a random sample from $U(\theta, \theta + 1)$. The number of maximum likelihood estimators of θ is :

(A) Infinity

(B) 2

(C) 5

(D) 7

34. The mapping $\phi : \mathbf{Z}_5 \to \mathbf{Z}_5$ satisfying $\phi(2) = 3$ is a group homomorphism. Then :

(A) $\phi(1) = 1$

(B) $\phi(1) = 2$

(C) $\phi(1) = 0$

(D) $\phi(1) = 4$

Or

For a large n (> 30), the confidence interval for mean μ of a $N(\mu, \sigma^2)$ distribution when σ^2 known, is :

(A) $\left[\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma^2}{n}, \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma^2}{n}\right]$

(B) $\left[\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right]$

(C) $\left[\bar{X} - Z_{\frac{\alpha}{2}}\frac{\sigma^2}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}}\frac{\sigma^2}{\sqrt{n}}\right]$

(D) $\left[\bar{X} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right]$

35. Let V be the set of all $n \times n$ skew-symmetric matrices over **R**. Then V is a real vector space of dimension :

(A) $\frac{n(n-1)}{2}$

(B) $\frac{n(n+1)}{2}$

(C) n^2

(D) $n^2 - n$

Or

Let $X_1, X_2, ..., X_n$ be iid rvs from $U(0, \theta)$. Then which of the following statements is *true* ? Let

$$X_{(n)} = \underset{i}{\text{Max}} \; X_i$$

(A) $X_{(n)}$ is consistent estimator for θ

(B) $X_{(n)}$ is unbiased estimator for θ

(C) $X_{(n)}$ is not sufficient for θ

(D) $X_{(n)}$ is not maximum likelihood estimator of θ

36. Which of the following polynomials is the characteristic polynomial of a matrix that is not invertible ?

(A) $x^3 + 2x - x + 1$

(B) $x^3 + x$

(C) $x^4 + x^3 - 2x^2 + 5$

(D) $(x + 1)(x - 2)(x + 2)$

Or

Let $X_1, ..., X_{n_1}$ and $Y_1, ..., Y_{n_2}$ be two independent random samples from two normally distributed populations with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2) respectively. Then the null distribution of the usual test statistic for testing the equality of σ_1^2 and σ_2^2, when μ_1 and μ_2 are unknown, is :

(A) Standard normal

(B) F-distribution with $(n_1 - 1, n_2 - 1)$ degrees of freedom

(C) t with $n_1 + n_2 - 2$ degrees of freedom

(D) square of Chi-square random variable with $n_1 + n_2 - 1$ degrees of freedom

37. Let $P_n(x)$ be the vector space of all polynomials of degree atmost n with real coefficients. Then its dimension is :

(A) $n - 1$

(B) n

(C) $n + 1$

(D) n^2

Or

In 2×2 contingency table to test independence of two attributes, if only one of the cell frequency is less than 5, then :

(A) Pooling of cell frequencies is used to carry out the test

(B) Fisher's correction is used to carry out the test

(C) Yates' correction is used to carry out the test

(D) The test cannot be carried out

38. Which of the following statements is *false* ?

(A) Consider vector space **C** over **C**. Then for any scalars α, β; $\alpha.1 + \beta.i = 0$ if and only if $\alpha = 0$ and $\beta = 0$.

(B) The set $M = \{f \in C[0, 1] \mid f\left(\frac{1}{2}\right) = 0\}$ is a subspace of the real vector space C[0, 1].

(C) Let V(F) be any vector space. Then for any $a \in F$ and for any vector $\overline{x} \in V$; $\alpha\overline{x} = \overline{0}$ if and only if $\alpha = 0$ or $\overline{x} = \overline{0}$

(D) The class of all polynomials with real coefficients is a real vector space.

Or

A test for testing H_0 against H_1 is called level α test if :

(A) size of the test does not exceed α

(B) size of the test is exactly equal to α

(C) the hypothesis of the test is simple hypothesis

(D) the test is unbiased

39. Let u and v be vector spaces of dimensions m and n, respectively. Then dimension of the space Hom(v, u) is :

(A) $m + n$

(B) n^m

(C) m^n

(D) mn

Or

If the form of the distribution from which the sample is drawn, is known completely except the unknown parameter θ, which of the following statements is *true* ?

(A) MLE, the maximum likelihood estimator of θ exists only if support of the random variable is free from θ

(B) MLE of θ exists only if θ is a real valued parameter

(C) MLE of θ exists only if the distribution belongs to the exponential family

(D) MLE of θ always exists

40. The general solution of the PDE

$$(y - z)p + (z - x)q = x - y$$

is :

(A) $\phi(x^2 - y^2 - z^2, x - y - z) = 0$

(B) $\phi(x + y + z, x^2 + y^2 + z^2) = 0$

(C) $\phi(xyz, x^2 + y^2 + z^2) = 0$

(D) $\phi(x + y + z, x^2 + y^2 + z^2) = 0$

Or

A single observation from $N(\mu, 1)$ resulted in 2.33. It is decided to test the hypothesis that $H_0 : \mu = 0$ against $H_1 : \mu > 0$ using this single observation. What will be the p-value in this case ?

(A) 0.05

(B) 0.95

(C) 0.99

(D) 0.01

41. The particular integral of the equation $r - 2s + t = \cos(2x + 3y)$ is :

(A) $\cos(2x + 3y)$

(B) $\sin(2x + 3y)$

(C) $-\cos(2x + 3y)$

(D) $-\sin(2x + 3y)$

Or

In probability sampling, all units get :

(A) always unequal chances to be selected

(B) always an equal chance to be selected

(C) a chance to be included in the sample

(D) always selected using SRSWOR

42. The characteristic curve of the PDE

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

is :

(A) $u = y - x,\ v = y - x^2$

(B) $u = y + x,\ v = y - x^2$

(C) $u = y - x,\ v = y^2 + x$

(D) $u = y + x,\ v = y - x$

Or

Sampling error :

(A) arises because of wrongly reported data

(B) arises due to mistakes made by field workers

(C) increases with increase in sample size

(D) may arise due to non-representativeness of the samples and the inadequacy of sample size

43. The solution of the equation

$$(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$$

is :

(A) $x = \tan^{-1} y - 1 + c\ e^{-\tan^{-1} y}$

(B) $y = \tan^{-1} x - 1 + ce^{-\tan^{-1} x}$

(C) $x = \tan^{-1} y + ce^{-\tan^{-1} y}$

(D) $y = \tan^{-1} x + ce^{-\tan^{-1} x}$

Or

In SRSWOR of size n from a population of size N, the number of distinct samples which will contain a specific population unit is :

(A) $\binom{N-1}{n-1}$

(B) $\binom{N}{n}$

(C) $\binom{N}{n-1}$

(D) $n\,!$

44. If $y_1(x)$ and $y_2(x)$ are solutions of the equation $y'' + 2y' + (1 - x)y = 0$ such that $y_1(0) = 0$, $y'_1(0) = 1$, $y_2(0) = 1$, $y'_2(0) = 1$, then the value of the Wronskian $W(y_1, y_2)$ on $\mathbf{R}$ is :

(A) -1

(B) 0

(C) 1

(D) 2

Or

In stratified sampling with stratum sizes $N_1 = 800$, $N_2 = 300$ and stratum variances $s_1^2 = 144$, $s_2^2 = 400$, under Neyman allocation, the ratio of sample sizes $\frac{n_1}{n_2}$ is given by :

(A) $\frac{8}{5}$

(B) $\frac{8}{3}$

(C) $\frac{3}{4}$

(D) $\frac{3}{8}$

45. Under usual notations, variance of estimator of population mean based on stratified random sample is :

(A) $V(\bar{Y}_{st}) = \sum_{h=1}^{L} \left(\frac{1}{n_h} - \frac{1}{N_h}\right) s_h^2 w_h^2$

(B) $V(\bar{Y}_{st}) = \sum_{h=1}^{L} \left(\frac{1}{n_h} - \frac{1}{N_h}\right) s_h^2 c_h^2$

(C) $V(\bar{Y}_{st}) = \sum_{h=1}^{L} \left(\frac{1}{N_h} - \frac{1}{n_h}\right) s_h^2 w_h^2$

(D) $V(\bar{Y}_{st}) = \sum_{h=1}^{L} \left(\frac{1}{N_h} - \frac{1}{n_h}\right) s_h^2 N_h^2$

Or

If there are n jobs to be performed, one at a time, on each of m machines, the possible sequences would be :

(A) $(n!)^m$

(B) $(m!)^n$

(C) $(n)^m$

(D) $(m)^n$

46. If C is the initial cost of an item, then the discounted value (d) of all future costs associated with the policy of replacing the item after n years is given by :

(A) $D_n = c/(1 + d)^n$

(B) $D_n = c/(1 + d^n)$

(C) $D_n = c/(1 - d)^n$

(D) $D_n = c/(1 - d^n)$

Or

In Latin square design, there are :

(A) Four sources of variability

(B) Three sources of variability

(C) One source of variability

(D) Two sources of variability

47. For M/M/1 queueing system, the expected number of customers in the system are (Here μ service rate and λ arrival rate) :

(A) $L = \frac{\mu}{\mu - \lambda}$

(B) $L = \frac{\lambda - \mu}{\lambda}$

(C) $L = \frac{\lambda}{\mu - \lambda}$

(D) $L = \frac{\lambda}{\mu(\mu - \lambda)}$

Or

What is the error degrees of freedom in Latin square design (assuming there are p treatments) :

(A) $(P - 1)$

(B) $P^2 - 1$

(C) $(P - 2)\ (P - 1)$

(D) $P - 2$

48. Find the value of p, q and r if the primal problem is

Maximize $Z = px_1 + 2x_2$

Subject to

$$x_1 + x_2 \leq q$$
$$x_1 - x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

and the corresponding dual problem is

Minimize $Z^* = 3w_1 + 2\omega_2$

Subject to

$$w_1 + w_2 \geq 4$$
$$w_1 - w_2 \geq r$$
$$w_1, w_2 \geq 0$$

(A) (4, 1, 3)

(B) (3, 4, 2)

(C) (2, 4, 3)

(D) (4, 3, 2)

Or

In 2^2 factorial design, let A and B be two factors each having two levels each with single replication and interaction effect is included. Then error degrees of freedom is :

(A) 1

(B) 2

(C) 3

(D) 0

49. For a transportation problem, choose the statement which is *not* correct :

(A) The $u_i + v_j - C_{ij}$ value of an unoccupied cell indicates the net change in cost of re-allocating one unit through the route involved

(B) Any of $m + n - 1$ number of occupied cells would allow determining whether a given solution is optimum or not

(C) The maximum quantity that can be re-allocated in a closed path is equal to the minimum quantity in the cells bearing negative sign

(D) In time minimization problem, the cost C_{ij} is replaced by the unit time t_{ij}

Or

In BIBD, the number of blocks is :

(A) greater than or equal to number of plots

(B) greater than or equal to number of treatments

(C) less than or equal to number of plots

(D) less than or equal to number of treatments

50. In game theory, which of the following is *not* correct ?

(A) The constant value, if added to each element of the pay off matrix to formulate the given problem as an LPP, should be substracted from the value of the game, determined from the solution to the LPP

(B) Every two person zero sum game cannot be represented by a pair of LPP's with primal dual relationship

(C) Any two person game can be formulated and solved as an LPP

(D) In a $2 \times n$ and $m \times 2$ game, each of the players can mix at most two strategies if the multiple optimum solutions do not exist

Or

In BIBD, if the number of treatments is equal to the number of plots in a block, then BIBD :

(A) reduces to CRD

(B) reduces to RBD

(C) reduces to LSD

(D) reduces to Graeco LSD

ROUGH WORK

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