Test Paper : III

Test Subject : MATHEMATICAL SCIENCE

Test Subject Code : K-2614

Test Booklet Serial No. : ____________________

OMR Sheet No. : ____________________

Roll No. | | | | | | | | |

(Figures as per admission card)

**Name & Signature of Invigilator/s**

Signature : ____________________ Signature : ____________________

Name : ____________________ Name : ____________________

**Paper : III**

**Subject : MATHEMATICAL SCIENCE**

Time : 2 Hours 30 Minutes Maximum Marks : 150

Number of Pages in this Booklet : **16** Number of Questions in this Booklet : **75**

**ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು**

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ಎಪ್ಪತ್ತೈದು ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ. ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ, ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೇ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.

   **ಉದಾಹರಣೆ :** (A) (B) ● (D)

   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಗಳನ್ನು, **ಪತ್ರಿಕೆ III ಪುಸ್ತಿಕೆಯೊಳಗೆ ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಮಾತ್ರವೇ ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿನ ಅಂಡಾಕೃತಿ ಹೊರತುಪಡಿಸಿ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗಿರುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMR ನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**

**Instructions for the Candidates**

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.

   **Example** : (A) (B) ● (D)

   where (C) is the correct response.
5. Your responses to the question of Paper III are to be indicated in the **OMR Sheet kept inside the Booklet**. If you mark at any place other than in the ovals in OMR Answer Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet soon after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**

# MATHEMATICAL SCIENCE

## PAPER – III

**Note :** This paper contains **seventy-five (75)** objective type questions. **Each** question carries **two (2)** marks. **All** questions are **compulsory.**

**1.** $\lim_{n\to\infty} \frac{1}{n}\left(1+\sqrt{2}+\sqrt[3]{3}+\dots+\sqrt[n]{n}\right)$ is equal to

(A) 0

(B) 1

(C) $\infty$

(D) $\frac{1}{2}$

**2.** If a > 0 then $\int_0^\infty \frac{\log x}{x^2+a^2}\,dx$ is equal to

(A) $\frac{\pi \log a}{2a}$

(B) $\frac{\pi \log a}{2}$

(C) $\frac{\pi \log a}{a}$

(D) $\pi \log a$

**3.** Which one of the following statements is false ?

(A) Every sequentially compact metrizable space is compact

(B) Every locally compact Hausdorff space is regular

(C) Every limit point compact space is compact

(D) If X is locally compact Hausdorff space and A is open in X, then A is locally compact

**4.** The sturm-Liouville problem

$$\frac{d^2y}{dx^2}+\lambda y = 0,\ y(0) = 0,\ y(\pi) = 0$$

(A) has a non-trivial solution if $\lambda \le 0$

(B) has a non-trivial solution if $\lambda = n$, n = 1, 2, 3, ....

(C) has no non-trivial solutions if $\lambda = 2n$, n = 1, 2, 3, ....

(D) has non-trivial solutions if $\lambda = n^2$, n = 1, 2, 3, .....

**5.** The solution to the heat equation $u_t = 3u_{xx}$, 0 < x < 2, t > 0, $u_x(0, t) = u_x(2, t) = 0$, u (x, 0) = 3x, is

(A) $u(x, t) = \frac{3}{2}+\frac{12}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} e^{\frac{-3n^2\pi^2 t}{4}}\cos\left(\frac{n\pi x}{2}\right)$

(B) $u(x, t) = \frac{3}{2}+\frac{12}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} e^{\frac{-3n^2\pi^2 t}{4}}\cos\left(\frac{n\pi x}{4}\right)$

(C) $u(x, t) = \frac{3}{2}+\frac{12}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} e^{\frac{-3n^2\pi^2 t}{4}}\sin\left(\frac{n\pi x}{2}\right)$

(D) $u(x, t) = \frac{3}{2}+\frac{12}{4}\sum_{n=1}^{\infty}\frac{1}{n} e^{\frac{-3n^2\pi^2 t}{4}}\cos\left(\frac{n\pi x}{2}\right)$

**6.** The extremal for the functional

$$\int_a^b (x + y^2 + 3y') \, dx \text{ is given by}$$

(A) $y(x) = x$

(B) $y(x) = c$, where c is any real constant $\neq 0$

(C) $y(x) = 0$

(D) $y(x) = e$

**7.** Consider the Fredholm integral equation

$$\phi(x) = f(x) + \lambda \int_0^1 e^{x-y} \phi(y) \, dy \quad .... \ (1) \text{ for a}$$

given real function $f(x)$ $(0 \leq x \leq 1)$. If $\lambda \neq 1$, then which one of the following is the solution of (1) ?

(A) $\phi(x) = f(x) - \dfrac{\lambda}{\lambda - 1} e^x \displaystyle\int_0^1 e^{-y} f(y) \, dy$

(B) $\phi(x) = f(x) - \left(\dfrac{\lambda + 1}{\lambda - 1}\right) e^x \displaystyle\int_0^1 e^{-y} f(y) \, dy$

(C) $\phi(x) = f(x) + \dfrac{\lambda}{\lambda - 1} e^x \displaystyle\int_0^1 e^{-y} \phi(y) \, dy$

(D) $\phi(x) = f(x) + \dfrac{\lambda^2}{\lambda^2 - 1} e^x \displaystyle\int_0^1 e^{-y} f(y) \, dy$

**8.** The value of $y(0.2)$ obtained by Runge-Kutta method of fourth order, given that $\dfrac{dy}{dx} = x + y$, $y(0) = 1$ with increment $h = 0.2$ is

(A) 1.2426 (B) 1.2425

(C) 1.2428 (D) 1.2424

**9.** Consider the homogeneous linear system

$$\frac{dx_1}{dt} = a_{11}(t)x_1 + ... + a_{1n}(t)x_n$$

$$\frac{dx_2}{dt} = a_{21}(t)\, x_1 + ... + a_{2n}(t)x_n$$

$$\begin{matrix} . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{matrix}$$

$$\frac{dx_n}{dt} = a_{n1}(t)x_1 + ... + a_{nn}(t)x_n.$$

Let $t_0$ be any point of [a, b] and let $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ . \\ . \\ . \\ \phi_n \end{pmatrix}$ be a solution of the above system such that $\phi(t_0) = 0$. Then

(A) $\phi(t) \neq 0$ for atleast one value of t in [a, b]

(B) $\phi(t) = 0 \quad \forall \ t \in [a, b]$

(C) $\phi(t) \neq 0$ and $\phi'(t) = 0$ for all values of $t_0 \neq t \in [a, b]$

(D) $\phi(t) \neq 0$ for all $t \in [a, b]$ with $t \neq t_0$

**10.** If A, B, C, D are nonempty sets, then which one of the following statements is true ?

(A) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

(B) $(A \times B) \cup (C \times D) \subset (A \cup C) \times (B \cup D)$

(C) $(A \times B) \cup (C \times D) \supset (A \cup C) \times (B \cup D)$

(D) $(A \times B) \cap (C \times D) = (A \cup C) \times (B \cup D)$

**11.** If A is a countable subset of $\mathbb{R}^2$ with usual topology, then the set $\mathbb{R}^2 - A$ is

(A) compact

(B) bounded

(C) path connected

(D) closed

**12.** Which one of the following statements is true ?

(A) Every Hausdorff space is regular.

(B) Every regular space is normal.

(C) Product of two normal spaces need not be normal.

(D) Every subspace of a normal space is normal.

**13.** Which one of the following statements is false ?

(A) Every closed subspace of a compact space is compact.

(B) Every compact subspace of any topological space is closed.

(C) The image of a compact space under a continuous map is compact.

(D) The product of finitely many compact spaces is compact.

**14.** Which one of the following statements is not true ?

(A) If X is connected, then for every nonempty proper subset A of X, we have Bd $A \neq \phi$.

(B) The union of a collection of connected sets that have a point in common is connected.

(C) The image of a connected space under a continuous map is connected.

(D) Every connected space is path connected.

**15.** Which one of the following statements is not true ?

(A) The subspaces [a, b] and [0, 1] of $\mathbb{R}$ are homeomorphic to each other.

(B) (–1, 1) and $\mathbb{R}$ are homeomorphic to each other.

(C) The mapping $f : [0, 1) \mapsto S'$ defined by $f(t) = (\cos 2\pi t, \sin 2\pi t)$ is a homeomorphism.

(D) Under the usual topologies, $\mathbb{R}$ and $\mathbb{R}^2$ are not homeomorphic to each other.

**16.** Let p and q be two distinct prime numbers. Then the number of integers a, $1 < a < pq$, which are coprime to pq is

(A) $pq - p - q + 1$ (B) $pq - p - q - 1$

(C) $pq - p - q$ (D) $pq - 1$

**17.** Let E be a finite Galois extension of the field of rational numbers $\mathbb{Q}$. Assume $[E : \mathbb{Q}] > 2$. Which one of the following statements is true ?

(A) Every $\mathbb{Q}$ - vector subspace of E is a subfield of E.

(B) Only finitely many $\mathbb{Q}$-vector subspaces of E are subfields of E.

(C) Number of $\mathbb{Q}$-vector subspaces of E which are subfields is infinite.

(D) No $\mathbb{Q}$-vector subspace of E is a subfield.

**18.** Let $\mathbb{Z}$ [ i ] denote the ring of Gaussian integers. Which one of the following statements is true ?

(A) If $\wp$ is a prime ideal of $\mathbb{Z}[i]$, then $\mathbb{Z}[i]/\wp$ is a field.

(B) If $\wp$ is a prime ideal of $\mathbb{Z}[i]$, then $\mathbb{Z}[i]/\wp$ is always a degree 2 extension of its prime filed.

(C) For any prime number '$p$' in $\mathbb{Z}$, the ideal generated by '$p$' in $\mathbb{Z}[i]$ is a prime ideal.

(D) For any non-zero prime ideal $\wp$ of $\mathbb{Z}[i]$, the intersection $\mathbb{Z} \cap \wp$ is a non-zero ideal of $\mathbb{Z}$.

**19.** In an integral domain R, which one of the following holds ?

(A) Given any a, $b \in R - \{0\}$ there is always a $C \in R$ such that $a \cdot c = b$

(B) Non-zero elements can never be a group under multiplication

(C) The equation $x^2 = a$, $a \in R$ always has a solution

(D) For a, b, $c \in R - \{0\}$, if ac = bc then a = b

**20.** The polynomial $f(x) = x^4 + x^3 + x^2 + x + 1$ is

(A) irreducible over ring of integers $\mathbb{Z}$

(B) reducible over the field of real numbers $\mathbb{R}$

(C) irreducible over the field $\mathbb{F}_5$ of five elements

(D) irreducible over any finite field with 25 elements

**21.** The ring $M_2(\mathbb{R})$ of all $2 \times 2$ real matrices is

(A) a non-associative ring

(B) a commutative ring with identity

(C) a non commutative ring without identity

(D) an associative ring

**22.** Let G be a finite group and $Z \subset G$ be its center. Assume $G/Z$ is cyclic. Then

(A) G is an abelian group

(B) G cannot be an abelian group

(C) G is a cyclic group

(D) G cannot be a cyclic group

**23.** In the set of integers, the relation "a divides b" is

(A) an equivalence relation

(B) a transitive relation

(C) a symmetric relation

(D) both transitive and symmetric relation

**24.** The supremum of the set

$A = \{p \in \mathbb{Q}^+ \mid p^2 < 3\}$ in $\mathbb{Q}$

(A) is 3

(B) is $\sqrt{3}$

(C) exists, but not $\sqrt{3}$

(D) does not exist

**25.** Which one of the following improper integrals diverges ?

(A) $\int_0^1 \frac{\log x}{\sqrt[4]{x}} dx$

(B) $\int_0^\infty e^{-x^2} dx$

(C) $\int_0^\infty \frac{7e^{-x} - 1}{\sqrt[3]{1+2x^2}} dx$

(D) $\int_0^{1/2} \left(\log\left(\frac{1}{x}\right)\right) dx$

**26.** Let $\alpha$ be monotonically increasing on [a, b]. Then the function $f : [a, b] \mapsto \mathbb{R}$ is Riemann-stielties in tegrable with respect to $\alpha$ if and only if

(A) f is continuous on [a, b]

(B) f is monotonic on [a, b]

(C) for every $\epsilon > 0$, there exists a partition p such that

$U(p, f, \alpha) - L(p, f, \alpha) < \epsilon$

(D) f is a product of two Riemann-Stielties integrable functions

**27.** Suppose f is twice differentiable on $(0, +\infty)$, $f''$ is bounded on $(0, +\infty)$ and $f(x) \to 0$ as $x \mapsto \infty$. The $\lim_{x \mapsto \infty} f'(x)$

(A) does not exist

(B) is 0

(C) is 1

(D) is $1/2$

**28.** The series $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \sin\left(1/n\right)\right)^a$

(A) Converges for all real values of a

(B) Converges for $a > \frac{1}{3}$

(C) Diverges for all real values of a

(D) Diverges for $a > \frac{1}{3}$

**29.** The sum of the series $\sum_{n=1}^{\infty} \frac{n}{n^4 + n^2 + 1}$ is

(A) 0 (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 1

**30.** Let $\sum_{n=1}^{\infty} x_n$ be a convergent series in which the terms $x_n$ decrease monotonically. Then

(A) $\lim_{n \mapsto \infty} n x_n = 0$

(B) $\lim_{n \mapsto \infty} n x_n = 1$

(C) $\lim_{n \mapsto \infty} n^2 x_n = 2$

(D) $\lim_{n \mapsto \infty} x_n = 1$

**31.** Let A be a $3 \times 3$ matrix with integer entries. Assume that $A^{-1}$ also has integer entries. Then det (A)

(A) may not be an integer

(B) may be any non-zero integer

(C) is always 1

(D) is either 1 or −1

**32.** $End_F$ (V) denotes the set of all F-endomorphisms of V. Which one of the following statements is true ?

(A) $End_F(V)$ has no F-vector space structure

(B) $End_F(V)$ can never be a commutative ring under usual addition and composition of endomorphisms

(C) $End_F(V)$ is always a F-vector space

(D) Non-zero endomorphisms of $End_F(V)$ always form a group under composition of endomorphisms

**33.** Let A be a 2 × 2 real matrix. Then which of the following is true ?

(A) A is invertible if A has a non real, complex eigen value

(B) If A is invertible then there is always a non-zero real eigen value for A.

(C) If A is invertible then the eigen values must be distinct

(D) If A is invertible then the eigen values cannot be distinct

**34.** Let T be an endomorphism of a two dimensional vector space V over the field of rational numbers. Which one of the following is true ?

(A) The matrix of T is always diagonal

(B) There is always a basis of V over the field of real numbers with respect to which the matrix of T is diagonal

(C) There is always a basis of V over the field of complex numbers with respect to which the matrix of T is digonal

(D) There may not be any basis of V with respect to which the matrix of T is diagonal

**35.** Which of the following pairs of matrices over $\mathbb{R}$ are similar ?

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

**36.** Let $F_2[x]$ be the vector space of all polynomials of degree atmost 2 over a field F. Define $T : F_2[x] \mapsto F_2[x]$ by $T(f) = f' = \frac{df}{dx}$. Then the matrix of T with respect to the basis $\{1 + x, x, x^2\}$ is

(A) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**37.** If $A = \begin{bmatrix} 1 & 0 & -i \\ 0 & 2 & -3+4i \\ i & -3-4i & 5 \end{bmatrix}$ then the eigen values of A

(A) are purely imaginary

(B) are real

(C) have multiplicity 2

(D) have absolute value 1

**38.** Let $V = \{(a, b, c, d) \mid a, b, c, d \in \mathbb{R}, a = c$ and $d = a + b\}$
The dimension of V over $\mathbb{R}$ is

(A) 1

(B) 2

(C) 3

(D) 4

**39.** If $a < b < c < d$ are fixed real numbers and w, x, y and z represent any real numbers, which pair of the following matrices can never be similar ?

(A) $\begin{pmatrix} a & x \\ y & b \end{pmatrix}$ and $\begin{pmatrix} c & w \\ z & d \end{pmatrix}$

(B) $\begin{pmatrix} b & x \\ y & a \end{pmatrix}$ and $\begin{pmatrix} b & w \\ z & c \end{pmatrix}$

(C) $\begin{pmatrix} a & x \\ y & b \end{pmatrix}$ and $\begin{pmatrix} a & y \\ x & b \end{pmatrix}$

(D) $\begin{pmatrix} a & x \\ y & d \end{pmatrix}$ and $\begin{pmatrix} b & w \\ z & c \end{pmatrix}$

**40.** Given that $f(z) = u(x, y) + i\,v(x, y)$ is analytic for all $z \in \mathbb{C}$ and $u(x, y) = y + e^x \cos y$. Then f(z) can be

(A) $e^z + iz$

(B) $e^z - iz$

(C) $ze^{iz}$

(D) $e^z + z$

**41.** Suppose $\alpha$ is real and $z = 1 - \cos\alpha + i \sin\alpha$, then $|z|$ is

(A) $\sqrt{2} \sin(\alpha/2)$

(B) $2 \sin(\alpha/2)$

(C) $\sqrt{2} \cos(\alpha/2)$

(D) $2 \cos(\alpha/2)$

**42.** The value of the integral

$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 5$, positively oriented, is

(A) $2\pi i (e^4 - e^2)$

(B) $2\pi i (e^2 - e^4)$

(C) $2\pi i\, e^2$

(D) $2\pi i\, e^4$

**43.** Let $f(z) = z^3 - 1$ and C denote the circle of radius 2 with center as (1, 0), oriented anticlockwise. Then the value of

$$\oint_C \frac{f(z)}{z-1}\,dz \text{ is}$$

(A) $2\pi i$

(B) 1

(C) $4\pi i$

(D) 0

**44.** The roots of the equation $\sin z = \cosh 4$, $z \in \mathbb{C}$ are

(A) $z = n\pi + (-1)^n\left(\pi/2 - 4i\right),\ n \in \mathbb{Z}$

(B) $z = (-1)^n \cdot n\pi + \left(\pi/2 - 4i\right),\ n \in \mathbb{Z}$

(C) $z = n\pi + \pi/2,\ n \in \mathbb{Z}$

(D) $z = n\pi + (-1)^n\,\pi/2,\ n \in \mathbb{Z}$

**45.** Consider the complex valued function $f(z) = (z-1)^2 \cdot e^{\frac{1}{(z-1)^2}}$. Then f(z) has

(A) a removable singularity at 1

(B) a pole at 0 of order 2

(C) a pole at 1 of order 2

(D) an essential singularity at 1

**46.** If f is the probability density function of uniform random variable over [0, 1], U and Y = f(U), then what is the distribution of Y ?

(A) Uniform over [0, 1]

(B) Degenerate at 0

(C) Degenerate at 1

(D) Standard exponential

**47.** If X and Y are independent Poisson (2) random variables, what is the distribution of X given X + Y = $l$ ?

(A) Poisson (2)

(B) Geometric $\left(\frac{1}{2}\right)$

(C) Binomial $\left(l, \frac{1}{4}\right)$

(D) Binomial $\left(l, \frac{1}{2}\right)$

**48.** What does the distribution function of $U^{1/n}$ converge to, U having uniform distribution over [0, 1] ?

(A) Degenerate distribution at 0

(B) Degenerate distribution at 1

(C) Uniform [0, 1] distribution

(D) Beta distribution

**49.** If (X, Y, Z) has tri-variate normal distribution, which one of the following is correct.

(A) X given Y and Z is normal

(B) $e^X + e^Y + e^Z$ is exponential

(C) $X^2+Y^2+Z^2$ has chi-square distribution

(D) $\frac{X}{X+Y+Z}$ is Cauchy

**50.** If X and Y are independent normal random variables, which one of the following is true ?

(A) X + Y and X – Y are dependent random variables

(B) X + Y and X – Y are identically distributed

(C) (X + Y) (X – Y) has chi-square distribution

(D) X + Y and X – Y are independent normal random variables

**51.** If F and G are distribution functions, which of the following is not a distribution function ?

(A) $\frac{F+G}{2}$

(B) FG

(C) $\frac{F+G}{4}$

(D) $\frac{2F+G}{3}$

**52.** Let $\{X_1, ..., X_n\}$ be a random sample from the probability density function

$f(x;\theta) = \frac{1}{2} e^{-|x-\theta|}$, $x \in \mathbb{R}$, $\theta \in \mathbb{R}$ . Which of the following is correct ?

(A) Sample median is the MLE of $\theta$

(B) Sample mean is the MLE of $\theta$

(C) Sample range is the MLE of $\theta$

(D) MLE of $\theta$ does not exist

**53.** Which of the following is not true ?

(A) Student's t-distribution is a sampling distribution

(B) Student's t-distribution is symmetric

(C) Student's t-distribution is a generalization of Cauchy distribution

(D) Student's t-distribution has all moments finite

**54.** In a linear model, what does 'heteroscedastic' mean ?

(A) The error variances are same

(B) The error variances are different

(C) The errors have zero expectations

(D) The errors have zero variances

**55.** Let i be a state of a Markov chain and $a_i = \lim_{n\to\infty} p_{ii}^{(n)}$ where $p_{ii}^{(n)}$ is the transition probability of going to state i from state i in n-steps. Which of the following is a sufficient condition for existence of $a_i$ ?

(A) i is ergodic

(B) i is aperiodic

(C) i is recurrent null

(D) i is non-null

**56.** With reference to a Markov chain with states 1, 2, given that $p_{12} = \frac{2}{3}, p_{21} = \frac{1}{6}$, what is $\lim_{n\to\infty} p_{11}^{(n)}$ ?

(A) 1

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{5}$

**57.** A criterion for state k to be recurent in a Markov chain is which one of the following ?

(A) $\sum_{n=1}^{\infty} p_{kk}^{(n)}$ converges

(B) $\lim_{n\to\infty} p_{kk}^{(n)} = 0$

(C) $\sum_{n=1}^{\infty} p_{kk}^{(n)}$ is divergent

(D) $\sum_{n=1}^{\infty} p_{kk}^{(n)}$ converges to 0

**58.** If $\{B(t), t \geq 0\}$ denotes a standard Brownian motion, which of the following is a Brownian motion ?

(A) $\{B(t+2) - B(t), t \geq 0\}$

(B) $\{e^{B(t)}, t \geq 0\}$

(C) $\{|B(t)|, t \geq 0\}$

(D) $\left\{B\left(\frac{t}{4}\right), t \geq 0\right\}$

**59.** In an M/M/1 queue with arrival rate 3, service rate 2 and no waiting, what is the steady state probability that the system is idle ?

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{2}{3}$

(D) $\frac{1}{6}$

**60.** Let $\{N(t), t \geq 0\}$ be a renewal process with mean finite and mean inter-arrival time $\mu$. Which of the following is true ?

(A) $\lim_{t\to\infty} \frac{N(t)}{t} = 1$ a.s.

(B) $\lim_{t\to\infty} \frac{N(t)}{t} = 0$ a.s.

(C) $\lim_{t\to\infty} \frac{N(t)}{t} = \mu$ a.s

(D) $\lim_{t\to\infty} \frac{N(t)}{t} = \frac{1}{\mu}$ a.s.

**61.** If $T_n$ is the MLE of a parameter $\theta$ in a distribution $F(\cdot\,;\theta)$, which of the following is true ?

(A) $T_n$ is always unbiased for $\theta$

(B) $T_n$ is never consistent for $\theta$

(C) $T_n$ is consistent for $\theta$ under Cramer's regularity conditions

(D) $T_n$ is never unbiased for $\theta$

**62.** If $X_1, X_2, \ldots, X_n$ are independent with characteristic function

$\phi(t) = \begin{cases} 0 & \text{if } |t| > 1, \\ 1-|t| & \text{if } |t| \leq 1; \end{cases}$ what is the limit

of the characteristic function of

$\frac{X_1 + \ldots + X_n}{n}$ as $n \to \infty$ ?

(A) $e^{-t}$, $t \in \mathrm{IR}$

(B) $e^{-|t|}$, $t \in \mathrm{IR}$

(C) $e^{-t^2}$, $t \in \mathrm{IR}$

(D) $e^{t^2}$, $t \in \mathrm{IR}$

**63.** If correlation coefficient between X and Y is 0.3, what is the correlation coefficient between 1 + X and 1 – 2Y ?

(A) – 0.4 (B) 0.3

(C) 0.6 (D) – 0.3

**64.** Given the linear model $(Y, A\theta, \sigma^2 I_3)$ with

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

when is the linear parametric function $a'\theta = a_1\theta_1 + a_2\theta_2 + a_3\theta_3$ estimable

(A) $\theta_1 = \theta_2 + \theta_3$

(B) $\theta_2 = \theta_1 + \theta_3$

(C) $\theta_3 = \theta_1 + \theta_2$

(D) $\theta_1 = \theta_2 - \theta_3$

**65.** What is the bias of the ratio estimator $\frac{\bar{y}}{\bar{x}}$ in a simple random sampling ?

(A) $\frac{-\operatorname{Cov}\left(\frac{\bar{Y}}{\bar{X}}, \bar{X}\right)}{E(\bar{X})}$

(B) $\frac{-\operatorname{Cov}\left(\frac{\bar{Y}}{\bar{X}}, \bar{X}\right)}{E(\bar{Y})}$

(C) $\frac{-\operatorname{Cov}\left(\frac{\bar{Y}}{\bar{X}}, \bar{Y}\right)}{E(\bar{X})}$

(D) $\frac{-\operatorname{Cov}\left(\frac{\bar{Y}}{\bar{X}}, \bar{Y}\right)}{E(\bar{Y})}$

**66.** What is a necessary and sufficient condition for estimability of a linear parametric function $a'\alpha$ of treatment effects alone, in a general block design ?

(A) Vector a belongs to the column space of the design matrix

(B) Vector a belongs to the column space of the C-matrix of the design

(C) Vector a belongs to the row space of the design matrix

(D) $a'\alpha$ is an elementary treatment contrast

**67.** Find the value of the objective function at an optimal solution of the LPP. minimize x + y subject to x – y = – 5, $x \le 0, y \ge 0$

(A) – 5

(B) 0

(C) 10

(D) 5

**68.** Given the moment generating functions

$$M_X(t) = \left(\frac{3 + e^t}{4}\right)^3 \text{ and } M_Y(t) = e^{2(e^t - 1)}$$

defined for appropriate values of t, what is P (X + Y = 1) if X and Y are independent ?

(A) $\frac{27}{32e^2}$

(B) $\frac{11}{64e^2}$

(C) $\frac{81}{64e^2}$

(D) $\frac{27}{64e^2}$

**69.** Given that $P(X_n = 0) = \frac{1}{2^n} = 1 - P(X_n = 1)$, $n = 1, 2, \ldots$, what is $P(\{X_n = 0\}$ infinitely often) equal to ?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) 0

**70.** Let $\{X_1, \ldots, X_n\}$ be a random sample from a continuous distribution function F and $F_n$ be the corresponding empirical distribution function. Let

$D_n = \sup_x |F_n(x) - F(x)|$,

$D_n^+ = \sup_x (F_n(x) - F(x))$,

$D_n^- = \sup_x (F(x) - F_n(x))$, which of the following is true ?

(A) $D_n$, $D_n^+$ and $D_n^-$ are distribution free

(B) $D_n$ is distribution free but not $D_n^+$ and $D_n^-$

(C) $D_n^+$ and $D_n^-$ are distribution free but not $D_n$

(D) None of $D_n$, $D_n^+$, $D_n^-$ are distribution free

**71.** If $T_n$ is a CAN estimator of $\theta$ with variance $\sigma_n^2$ then $\log T_n$ is CAN for $\log \theta$ with variance

(A) $\frac{\sigma_n^2}{\theta^2}$

(B) $\sigma_n^2 \cdot \theta^2$

(C) $\sigma_n^2 \cdot \theta$

(D) $\frac{\sigma_n^2}{\theta}$

**72.** In a parallel system of k components, the system survival time is

(A) Minimum of the survival times of its components

(B) Maximum of the survival times of its components

(C) Mean survival time of its components

(D) Median survival time of its components

**73.** Let X be a random variable with $E|X|^k < \infty$. Then the Markov inequality states that

(A) $P[|X| \geq \epsilon] \leq \frac{E|X|^k}{\epsilon^k}$

(B) $P[|X| \geq \epsilon] \leq \frac{\epsilon^k}{E|X|^k}$

(C) $P[|X| \leq \epsilon] \leq \frac{E|X|^k}{\epsilon^k}$

(D) $P[|X| \leq \epsilon] \geq \frac{E|X|^k}{\epsilon^k}$

**74.** While estimating the parameters of a linear regression model, a ridge estimator is proposed when

(A) the errors are autocorrelated

(B) the dispersion matrix of the error vector is singular

(C) the columns of the regression matrix are linearly independent

(D) there is multicollinearity in the model

**75.** A sample of size n is drawn from a dichotomous population. If the sample has proportion p of items of category I and proportion q of category II then the variance of proportion p is

(A) $s_p^2 = \frac{pq}{n-1}$

(B) $s_p^2 = \frac{pq}{n}$

(C) $s_p^2 = \frac{npq}{n-1}$

(D) $s_p^2 = \frac{p^2q}{n-1}$

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**Space for Rough Work**

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**Space for Rough Work**