Test Paper : **III**

Test Subject : **MATHEMATICAL SCIENCE**

Test Subject Code : **K-2613**

Test Booklet Serial No. : ____________

OMR Sheet No. : ____________

Roll No. | | | | | | | | |

(Figures as per admission card)

Name & Signature of Invigilator/s

Signature: ____________ Signature : ____________

Name : ____________ Name : ____________

Paper : III

Subject : MATHEMATICAL SCIENCE

Time : 2 Hours 30 Minutes | Maximum Marks : 150

Number of Pages in this Booklet : **24** | Number of Questions in this Booklet : **75**

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ಎಪ್ಪತ್ತೈದು ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆ ಸ್ವೀಕರಿಸಬೇಡಿ. ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ, ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.
   ಉದಾಹರಣೆ: (A) (B) ● (D)
   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಗಳನ್ನು, **ಪತ್ರಿಕೆ III ಪುಸ್ತಿಕೆಯೊಳಗೆ ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಮಾತ್ರವೇ ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿನ ಅಂಡಾಕೃತಿ ಹೊರತುಪಡಿಸಿ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗಿರುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMR ನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯ ಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the question of Paper III are to be indicated in the **OMR Sheet kept inside the Booklet**. If you mark at any place other than in the ovals in OMR Answer Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet soon after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**

Total Number of Pages : 24

MATHEMATICAL SCIENCE
Paper – III

Note : This paper contains **seventy five** (**75**) objective type questions. **Each** question carries **two** (**2**) marks. **All** questions are **compulsory**.

1. If $0 < a < b$, then

$$\int_0^{\infty} \frac{\tan^{-1} ax - \tan^{-1} bx}{x} dx =$$

(A) $\frac{\pi}{2} \log \frac{a}{b}$ (B) $\frac{\pi}{4} \log \frac{a}{b}$

(C) $\frac{\pi}{6} \log \frac{a}{b}$ (D) $\frac{\pi}{2} \log \frac{b}{a}$

OR

$$\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{\tan x} =$$

(A) e (B) $\frac{1}{e}$

(C) 0 (D) 2

2. Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & \text{for } (x, y) \neq (0,0) \\ 0, & \text{for } (x, y) = (0,0) \end{cases}$$

Which one of the following statements is true ?

(A) f is not continuous at (0, 0)

(B) f is continuous at (0, 0)

(C) f has no partial derivatives at (0, 0)

(D) f is differentiable at (0, 0)

OR

Which of the following is necessarily true on the set $S \cap T$ if $S = \{ x : f(x) = 0\}$ and $T = \{x : g(x) = 0\}$?

(A) $\frac{f(x)}{g(x)} = 0$

(B) $\frac{g(x)}{f(x)} = 0$

(C) $(f(x))^2 + (g(x))^2 = 0$

(D) $f(x)\, g(x) = 1$

3. Let S be a subset of $\mathbb{R}$. Let C be the set of points $x \in \mathbb{R}$ with the property that $S_n (x - \delta, x + \delta)$ is uncountable for every $\delta > 0$. Then S – C is

(A) Uncountable

(B) Finite or countable

(C) Empty

(D) Always finite

OR

The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent for what values of p ?

(A) $p \geq 1$ (B) $p > 1$

(C) $p \leq 1$ (D) $p < 1$

4. If $a_n > 0$ for all n and $\sum a_n$ converges, then $\sum \sqrt{a_n a_{n+1}}$

(A) Converges

(B) Diverges

(C) Oscillates

(D) Converges to the sum of $\sum a_n$

OR

The function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ has a minimum at which point ?

(A) Nowhere

(B) (–1, 5)

(C) (5, –1)

(D) (3, 3)

5. Which one of the following improper integral diverges ?

(A) $\int_0^1 \frac{\log x}{\sqrt[4]{x}}\, dx$

(B) $\int_0^\infty e^{-x^2}\, dx$

(C) $\int_0^\infty \frac{7e^{-x} - 1}{\sqrt[3]{1 + 2x^2}}\, dx$

D) $\int_0^{1/2} \log\left(1/x\right) dx$

OR

If $x \in (0, 1)$ and

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational,} \\ \left[\frac{1}{x}\right]^{-1} & \text{if } x \text{ is irrational,} \end{cases}$$

with [x] equal to the integer part of x, what is $\int_0^1 f(x)dx$ equal to ?

(A) Not defined (B) Infinity

(C) 1 (D) 0

6. Let f be a real valued function defined for all $x \geq 1$ satisfying f (1) = 1 and $f'(x) = \frac{1}{x^2 + (f(x))^2}$. Then

(A) $\lim_{x \to \infty} f(x) < 1 + \frac{\pi}{4}$

(B) $\lim_{x \to \infty} f(x) > 1 + \frac{\pi}{4}$

(C) $\lim_{x \to \infty} f(x) < \frac{\pi}{4}$

(D) $\lim_{x \to \infty} f(x) = 1 + \frac{\pi}{4}$

OR

If $A_n = \begin{cases} \left(\frac{1}{n}, \frac{2}{3} - \frac{1}{n}\right) & \text{if } n \text{ is odd,} \\ \left(\frac{1}{3} - \frac{1}{n}, 1 + \frac{1}{n}\right) & \text{if } n \text{ is even,} \end{cases}$

$n \geq 1$, $\lim_n \sup A_n$ and $\lim_n \inf A_n$ are respectively equal to what ?

(A) [0, 1] and $\left[\frac{1}{3}, \frac{2}{3}\right]$

(B) (0, 1] and $\left[\frac{1}{3}, \frac{2}{3}\right)$

(C) (0, 1) and $\left(\frac{1}{3}, \frac{2}{3}\right)$

(D) [0, 1) and $\left(\frac{1}{3}, \frac{2}{3}\right]$

7. Suppose that f is twice differentiable on $(0,\infty)$, f'' is bounded on $(0,\infty)$ and $f(x) \to 0$ as $x \to \infty$. Then $\lim_{x\to\infty} f'(x)$ is equal to

(A) infinity

(B) 0

(C) 1

(D) $1/2$

OR

What is the index of the quadratic form $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_1x_3 + 12x_1x_2$?

(A) 3 (B) 2

(C) 0 (D) 1

8. $\lim_{n\to\infty} \frac{1}{n}\left(1 + \sqrt{2} + \sqrt[3]{3} + ... + \sqrt[n]{n}\right) =$

(A) 0 (B) 1

(C) ∞ (D) $1/2$

OR

What is the set of eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$?

(A) {2, 3, 6} (B) {2, 6, 7}

(C) {–2, 3, 6} (D) {1, 3, 6}

9. The sequence $\{x_n\}$ defined by $x_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$, n = 1, 2, ..., is

(A) Not a monotonic sequence

(B) A Cauchy sequence

(C) A bounded sequence

(D) Not a convergent sequence

OR

If [x] is the integer part of x, what is $\int_0^{11} [x]^3 \, dx$ equal to ?

(A) 1025 (B) 55

(C) 3025 (D) 125

10. The series $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n^x}$

(A) Converges for all $x > \frac{1}{2}$

(B) Converges for all $x < \frac{1}{2}$

(C) Converges for $x = \frac{1}{2}$

(D) Diverges for all x

OR

Which of the following functions is not of bounded variation on the interval [0, 1] ?

(A) $f(x) = x^2$

(B) $f(x) = x^{\frac{1}{3}}$

(C) $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$

(D) $f(x) = \begin{cases} x \cos\left(\frac{\pi}{2x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$

11. Which one of the following series is convergent ?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$ (B) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(C) $\sum_{n=1}^{\infty} \frac{3n+2}{4n+5}$ (D) $\sum_{n=1}^{\infty} \left(n - \frac{1}{n}\right)$

OR

Which of the following is a sufficient condition for $\int_{[a,b]} f\, d\alpha$ to exist ?

(A) f is continuous on [a, b] and α is continuous on [a, b]

(B) f is continuous on [a, b] and α is of bounded variation on [a, b]

(C) f is of bounded variation on [a, b] and α is continuous on [a, b]

(D) f is of bounded variation on [a, b] and α is of bounded variation on [a, b]

12. Which one of the following statements is not true ?

(A) A countable union of countable sets is countable

(B) A set A of all sequences whose elements are the digits 0 and 1 is uncountable

(C) Non empty perfect set in $\mathbb{R}^n$ is countable

(D) The set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable

OR

Let N = {1, 2, 3, . . . } and x = {A $\subset$ N : A is finite or $A^{\subset}$ is finite} which of the following is not true ?

(A) x is a class of subsets of N

(B) x is a field but not a sigma-field

(C) x is not finite

(D) x is a sigma-field

13. Which one of the following statements is not true ?

(A) If x, y $\in \mathbb{R}$ and x > 0, then there is a positive integer n such that n x > y

(B) Between any two real numbers there exists a rational number

(C) The set $\mathbb{R}$ of real numbers does not have the least upper bound property

(D) For every real x > 0 and every integer n > 0 there is one and only one real y such that $y^n = x$

OR

Which of the following is true for arbitrary non-null $n \times n$ matrices A, B, $n \geq 2$?

(A) Trace (AB – BA) = 0

(B) $B = T^{-1}AT$ with $|T| \neq 0$

(C) $(A + B)(A - B) = A^2 - B^2$

(D) $(A + B)' = A + B$

14. Which one of the following statements is not true ?

(A) Every convergent sequence in $\mathbb{R}$ is bounded

(B) Every bounded sequence in $\mathbb{R}$ contains a convergent subsequence

(C) The set of subsequential limits of a sequence in $\mathbb{R}$ is not closed

(D) A sequence in $\mathbb{R}$ is convergent if and only if it is a Cauchy sequence

OR

If S is the positively oriented circle $|z - 3i| = 2$, what is the value of $\int_S \frac{dz}{z^2 + 4}$?

(A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{2}$

(C) $-i\frac{\pi}{2}$ (D) $i\frac{\pi}{2}$

15. $\lim_{n\to\infty}\left(1-\frac{1}{2n}\right)^{n+1} =$

(A) $\sqrt{e}$

(B) $\frac{1}{\sqrt{e}}$

(C) e

(D) $\frac{1}{e}$

OR

At $z = 0$, the function $\frac{e^z}{z(1-e^{-z})}$ has which one of the following ?

(A) Removable singularity

(B) Pole of order 1

(C) Pole of order 2

(D) Essential singularity

16. If A is an $m \times n$ matrix and B is a non-singular matrix of order m, then

(A) rank (BA) $\neq$ rank (A)

(B) rank (BA) $\geq$ rank (A)

(C) rank (BA) $\leq$ rank (A)

(D) rank (BA) = rank (A)

OR

For what values of z does

$\sum_{n=0}^{\infty} 3^{-n}$ $(z-1)^{2n}$ converges ?

(A) $|z| \le 3$ (B) $|z| \le \sqrt{3}$

(C) $|z-1| < \sqrt{3}$ (D) $|z-1| \le \sqrt{3}$

17. Let V be the space of all $n \times n$ real skew symmetric matrices. Then dimension of V over $\mathbb{R}$ is

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n-1)}{2}$

(C) $n^2 - 1$ (D) $n^2 + n$

OR

Which of the following is not true for distribution functions F and G on the real line ?

(A) $\frac{F+G}{2}$ is a distribution function

(B) $\frac{F^2+2G^3}{3}$ is a distribution function

(C) $\frac{2F^3+G^2}{3}$ is a distribution function

(D) $\frac{F+3G}{3}$ is a distribution function

18. Let A be a 3×3 square matrix with eigen values 2, 3, –1 and $B = A^2 + A$. Then det B is equal to

(A) 0 (B) – 6

(C) 72 (D) 144

OR

If $\{X_1, X_2, \ldots, X_n\}$ is a random sample from a population with pdf

$f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$, what is the distribution of $-\sum_{i=1}^{n} \log X_i$

(A) Exponential (B) Chi-square

(C) Gamma (D) Lognormal

19. Which of the following mappings is a linear transformation ?

(A) $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (x^3, y^3)$

(B) $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $f(x, y, z) = (z, x + y)$

(C) $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = | x - y |$

(D) $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (x + 1, y + 1)$

OR

If X has moment generating function M_x, Y has moment generating function M_y, and X is distributed like – Y, which of the following is true ?

(A) $M_y(t) = M_x(-t)$

(B) $M_y(t) = 1 - M_x(t)$

(C) $M_y(t) = 1 - M_x(-t)$

(D) $M_y(t) = tM_x(t)$

20. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x, y) = (x+y, x - y, y)$. Then the dimension of the range space R(T) is

(A) 0

(B) 1

(C) 2

(D) 3

OR

The mean and variance of number of defective items from a lot are given as 10 and 6 respectively. What could be the distribution of the number of defective items ?

(A) Geometric with parameter 0.6

(B) Normal with mean 10 and variance 6

(C) Poisson with mean 10

(D) Binomial with parameters 25 and 0.4

21. If S and T are linear transformations on $\mathbb{R}^2$ defined by

$S(x, y) = (y, x)$ and $T(x, y) = (o, x)$ then

(A) $S^2 = I, T^2 = O$ (B) $S^2 = S, T^2 = O$

(C) $S^2 = S, T^2 = I$ (D) $S^2 = I, T^2 = I$

OR

If r is the correlation coefficient between X and Y, what is the correlation coefficient between (1 + X) and (1 – 2Y) ?

(A) r (B) 2r

(C) – r (D) 1 – 2r

22. Let A be a $n \times n$ square matrix. Then which one of the following assertions is correct ?

(A) det A = 0 implies rank A = 0

(B) det A = 0 if and only if rank $A \leq n - 1$

(C) det A = 0 implies rank A = n

(D) det A = 0 implies rank A = n^2

OR

A fair dice is rolled twice. What is the probability that the maximum in the two rolls is either 3 or 5?

(A) $\frac{1}{2}$ (B) $\frac{1}{9}$

(C) $\frac{2}{9}$ (D) $\frac{4}{9}$

23. A real symmetric matrix is positive definite if and only if all its eigen values are

(A) negative (B) imaginary

(C) zero (D) positive

OR

At a birthday party 10 children throw their caps into the center of a room and after mixing up, each one selects one cap randomly. What is the expected number of children who select their own caps ?

(A) 10 (B) 5

(C) 2 (D) 1

24. The equation $e^x = 1 + x + \frac{x^2}{2}$ has

(A) exactly one real root

(B) no real roots

(C) two real roots

(D) three real roots

OR

Sample mean of 16 items selected from a population having standard deviation 4 is given as 160. What is the standard error of the sample mean ?

(A) 1 (B) 4

(C) 10 (D) 40

25. Which one of the following is a subspace of $\mathbb{R}^n$?

(A) $\{(x_1, x_2, \ldots x_n) \mid x_1 + x_2 + \ldots + x_n = 1\}$

(B) $\{(x_1, x_2, \ldots x_n) \mid x_1 = x_2 = 0\}$

(C) $\{(x_1, x_2, \ldots x_n) \mid x_1 \neq 0\}$

(D) $\{(x_1, x_2, \ldots x_n) \mid 5x_1 - 9x_2 = 6\}$

OR

Given that 1% of a population suffers from a disease and a detection test has probability 0.99 of correct diagnosis. If a randomly chosen individual tests positive, what is the probability that the chosen individual really has the disease ?

(A) 0.01 (B) 0.05

(C) 0.5 (D) 0.99

26. In $\mathbb{R}^3$, 2 – dimensional subspaces can be geometrically described as

(A) All planes in $\mathbb{R}^3$

(B) All lines passing through (0, 0, 0)

(C) All planes passing through (0, 0, 0)

(D) The only planes $x = 0$, $y = 0$ and $z = 0$

OR

If U and V are independent uniform (0, 1) random variables, what is the variance of $Y = \frac{\log U}{\log U + \log(1 - V)}$

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

27. For a square matrix A, which one of the following statements is not true ?

(A) 0 is an eigen value of A if and only if A is non-singular

(B) A satisfies its characteristic equation

(C) A and A^T have the same eigen values

(D) A(adj A) = (det A) I

OR

If $V(\underline{X}) = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$, what is $V(X_1 - 2X_2 + X_3)$, where $\underline{X} = (X_1 \ X_2 \ X_3)$?

(A) 6 (B) 11

(C) 18 (D) 21

28. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (x+y, x+y)$. Then a basis and the dimension of the image of f and the dim Ker f are respectively

(A) $\{(1,1)\}$, 1 and 1

(B) $\{(0, 1)\}$, 1 and 2

(C) $\{(1, 0)\}$, 2 and 2

(D) $\{(1, 1)\}$, 2 and 1

OR

If $A_{n\times n}$ is a symmetric matrix and $\underline{X} = (X_1, ..., X_n)$ is a random vector with $E(\underline{X}) = 0$ and $V(\underline{X}) = \Sigma$, then what is $E(\underline{X}' A \underline{X})$ equal to ?

(A) Trace of $A\Sigma$ (B) $| A\Sigma |$

(C) 0 (D) 1

29. Let S be a non-empty set of real numbers which is bounded below. Then

(A) inf (–S) = Sup(+S)

(B) inf (S) = Sup(–S)

(C) inf (S) = –Sup(S)

(D) inf (S) = –Sup(–S)

OR

If $\overline{X}_n$ and S_n are, respectively, the sample mean vector and sample dispersion matrix based on a random sample of size n from Np (μ, Σ) population, which one of the following is not true ?

(A) Given μ, S_n is sufficient for Σ

(B) $\overline{X}_n$ and S_n are unbiased and sufficient for μ and Σ

(C) Given Σ, $\overline{X}_n$ is sufficient for μ

(D) $\overline{X}_n$ and S_n are consistent estimators.

30. The bilinear transformation which maps the points Z = 0, 1, –1 into $W = i, \infty, 0$ is

(A) $i\left(\frac{Z-1}{Z+1}\right)$

(B) $\frac{Z-1}{Z+1}$

(C) $\frac{Z+1}{Z-1}$

(D) $-i\left(\frac{Z+1}{Z-1}\right)$

OR

If A is a symmetric matrix, $Y \sim N_p(0, I_p)$, $Y' AY \sim \chi^2_k$, then what is the distribution of $Y'(I_p - A)Y$?

(A) χ^2_p

(B) χ^2_k

(C) χ^2_{pk}

(D) χ^2_{p-k}

31. Let $f(z)=\frac{1+2z}{z^2+z^3}$. Which one of the following statements is true ?

(A) The expansion of f(z) is $\frac{1}{z^3}+\frac{1}{z^2}-1+z-z^2+z^3+\ldots$ for $0<|z|<|$

(B) Residue of f(z) at z = 0 is 1

(C) f(z) is analytic at z = 0

(D) Residue of f(z) at z = 0 is –1

OR

What is the dispersion matrix of a bivariate normal random vector (Y_1, Y_2) with pdf $f(y_1, y_2)$ = constant $e^{-\frac{1}{2}\left\{2y_1^2+y_2^2+2y_1y_2-22y_1-14y_2+65\right\}}$ $(y_1, y_2)\in IR^2$?

(A) $\begin{pmatrix}1 & -1\\ -1 & 2\end{pmatrix}$ (B) $\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

(C) $\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}$ (D) $\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}$

32. The equation |z–2+3i| = 5 represents in the complex plane

(A) A circle with centre – 2 + 3i and radius 5

(B) A line passing through –2 + 3i and parallel to X = 5

(C) A line passes through 2 – 3i and parallel to Y = 5

(D) A circle with centre 2 –3i and radius 5

OR

What is the percentage of variance explained by the first principal component of the dispersion matrix $\Sigma=\begin{pmatrix}1 & 4\\ 4 & 100\end{pmatrix}$?

(A) 99.2 (B) 91.2

(C) 89.5 (D) 79.6

33. If $Z=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ then $(\bar{Z})^4$ is equal to

(A) $\frac{1-i\sqrt{3}}{2}$ (B) $\frac{1+i\sqrt{3}}{2}$

(C) $\frac{-1-i\sqrt{3}}{2}$ (D) $\frac{-1+i\sqrt{3}}{2}$

OR

What are all the values of p for which $\Sigma=\begin{pmatrix}1 & p & p\\ p & 1 & p\\ p & p & 1\end{pmatrix}$ is positive definite ?

(A) $p<\frac{1}{2}$ (B) $p>\frac{1}{2}$

(C) $p>-\frac{1}{2}$ (D) $p<-\frac{1}{2}$

34. If C is the circle |z| = 2 with counter clockwise orientation, then $\int_C (2z-1)e^{\frac{z}{z-1}}\,dz=$

(A) 4ei (B) $4\pi i$

(C) $4\pi ei$ (D) $4\pi e$

OR

In the Gauss-Markov model $(Y, A\theta, \sigma^2 I)$

with $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ what is a necessary and sufficient condition for estimability of $l_1 \theta_1 + l_2 \theta_2 + l_3 \theta_3$?

(A) $l_1 - l_2 = l_3$
(B) $l_2 - l_3 = l_1$
(C) $l_1 + l_2 = l_3$
(D) $l_1 + 2l_2 = l_3$

35. The set is $\{ Z \in \mathbb{C} : |Z-2| + |Z-1| < 3 \}$ is

(A) the interior of a disc
(B) the interior of an ellipse
(C) the null set φ
(D) the whole complex plane C

OR

For any two random variables X and Y, which of the following is true ?

(A) $E(V(X \mid Y)) = V(E(X \mid Y))$
(B) $E(X \mid Y) = E(Y \mid X)$
(C) $E(E(X \mid Y)) = E(E(Y \mid X))$
(D) $V(E(X \mid Y)) \geq -E(V(X \mid Y))$

36. The function $w(z) = -\left(\frac{1}{z} + bz\right), -1 < b < 1$ maps $|z| < 1$ on to

(A) a half plane
(B) exterior of a circle
(C) exterior of an ellipse
(D) interior of an ellipse

OR

If $Y_{n \times 1} \sim N_n(A\beta, \sigma^2 I_n)$ with Rank $(A_{n \times p}) = p$ and $\hat{\beta}$ is the least squares estimator of β, which of the following is not true ?

(A) $\hat{\beta}$ is unbiased for β
(B) $\hat{\beta}$ is MLE of β
(C) $\hat{\beta}$ and residual sum of squares are statistically dependent
(D) $\hat{\beta}$ has minimum variance in the entire class of linear unbiased estimators of β

37. Let α be a zero of f(z) of order m and pole of $\phi(z)$ of order n (m > n). Then for the function $f(z)\phi(z)$, α is

(A) a pole of order m + n
(B) a zero of order m + n
(C) a pole of order m – n
(D) a zero of order m – n

OR

What is the probability density function corresponding to the characteristic function $\varphi(t) = \frac{1}{1+t^2}, t \in \mathbb{R}$?

(A) $e^{-x}, x > 0$
(B) $\frac{1}{2} e^{-|x|}, x \in \mathbb{R}$
(C) $\frac{1}{\pi} \frac{1}{(1+x^2)}, x \in \mathbb{R}$
(D) $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, x \in \mathbb{R}$

38. Under the transformation $\omega = \frac{1}{z}$ the image of the circle $|z - 2i| = 2$ is

(A) a circle

(B) a straight line

(C) an ellipse

(D) a parabola

OR

Let $X \sim N(0, 1)$ and Y be independent of X with $P(Y = 0) = \frac{1}{2} = P(Y = 1)$. If $Z = \begin{cases} X & \text{if } Y = 1, \\ -X & \text{if } Y = 0, \end{cases}$ then what is the distribution of Z ?

(A) $N(0, 1)$ (B) $N(0, 2)$

(C) $N\left(0, \frac{1}{2}\right)$ (D) $N\left(\frac{1}{2}, 1\right)$

39. The series $\sum_{n=0}^{\infty}\left(\frac{z^n}{n!} + \frac{n^2}{z^n}\right)$ converges for

(A) $|z| > 1$

(B) $|z| < 1$

(C) $|z| = 1$

(D) $|z| \le 1$

OR

Which of the following is not a characteristic function ?

(A) 1

(B) $e^{it}, t \in \mathbb{R}$

(C) $\frac{1}{(1 - it)}, t \in \mathbb{R}$

(D) $e^{-|t|}, t \in \mathbb{R}$

40. If C is the circle $|z| = 1$, then the value of the integral $\oint_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$ is

(A) $4\pi i$ (B) $3\pi i$

(C) $2\pi i$ (D) πi

OR

If $\{X_1, ..., X_n\}$ is a random sample from a population having pdf

$f(x, \alpha, \beta) = \frac{1}{\Gamma\alpha\, \beta^\alpha} e^{-\frac{x}{\beta}} x^{\alpha - 1}, x > 0$, which of the following estimator is sufficient for α when β is known ?

(A) $\prod_{i=1}^{n} X_i$ (B) $\sum_{i=1}^{n} X_i$

(C) $\left(\prod_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i\right)$ (D) $\sum_{i=1}^{n} X_i^2$

41. For which value of n, primitive root modulo n does not exist ?

(A) n = 14 (B) n = 4

(C) n = 7 (D) n = 8

OR

If $\overline{X}_n$ is the sample mean of a random sample of size n from uniform $(0, \theta)$ population, which of the following is true ?

(A) $\overline{X}_n$ is unbiased for θ

(B) $\overline{X}_n$ is UMVUE for θ

(C) $\overline{X}_n$ is a biased estimator of θ

(D) $\overline{X}_n$ is MVB estimator of θ

42. Which one of the following statement is not true ?

(A) There exists a field with 5 elements

(B) There exists a field having 16 elements

(C) There exists a field with 36 elements

(D) There exists a field having 125 elements

OR

Which of the following distributions does not possess MLR property ?

(A) Exponential

(B) Uniform

(C) Power function

(D) Normal

43. The number of conjugacy classes in the symmetric group S_5 is

(A) 5 B) 25

(C) 10 (D) 7

OR

Which of the following is correct ?

(A) If the p-value of a test is 0.16 then the test is insignificant at 10% level

(B) If a test is significant at 5% level, then the probability of the null hypothesis being true is atleast 0.05

(C) If a test is significant at 1% level, then the value of the test statistic must be quite large

(D) If sample mean based on a random sample of size 1000 turns out to be 0.003, then the hypothesis that population mean is 0 should be rejected

44. Consider the following statements

Statement (i) : Every isomorphic image of a cyclic group is cyclic.

Statement (ii) : Every homomorphic image of a cyclic group is cyclic.

Then

(A) (i) holds and (ii) does not hold

(B) (ii) holds and (i) does not hold

(C) Neither (i) nor (ii) holds

(D) Both (i) and (ii) hold

OR

If $\{X_1, X_2, ..., X_n\}$ is a random sample from uniform $(0, \theta)$ and $M_n = \max\{X_1, ..., X_n\}$, which of the following gives the shortest $(1 - \alpha)$ level confidence interval for θ ?

(A) (M_n, M_n+1) (B) $\left(M_n, \alpha^{-\frac{1}{n}} M_n\right)$

(C) $\left(M_n, \alpha^{\frac{1}{n}} M_n\right)$ (D) $(\alpha M_n, M_n)$

45. The ring of all 2×2 real matrices with respect to usual matrix addition and multiplication, is

(A) a commutative ring without zero divisors

(B) a commutative ring with zero divisors

(C) a non-commutative ring without zero divisors

(D) a non-commutative ring with zero divisors

OR

Given that $\{X_1,...,X_n\}$ is a random sample from uniform $(0, \theta)$, which of the following is the MVUE of θ ?

(A) $\frac{X_1 + ... + X_n}{n}$

(B) $\frac{(n+1)}{n}$ min. $\{X_1,...,X_n\}$

(C) $\frac{(n+1)}{n}$ max. $\{X_1,...,X_n\}$

(D) $\frac{n}{(n+1)}$ max. $\{X_1,...,X_n\}$

46. The polynomial $f(x) = x^2 + x + 4$ over $\mathbb{Z}_{11}$

(A) is reducible

(B) has exactly one root in $\mathbb{Z}_{11}$

(C) is irreducible

(D) has two roots in $\mathbb{Z}_{11}$

OR

If X ~ B(1, p), $I_x(p)$ is the Fisher information about p, then a consistent estimator of $I_x(p)$ is which of the following ?

(A) $\overline{X}_n$ (B) $\frac{1}{\overline{X}_n}$

(C) $\overline{X}_n(1-\overline{X}_n)$ (D) $\frac{1}{\overline{X}_n(1-\overline{X}_n)}$

47. Which one of the following statements is not true ?

(A) The group $(\mathbb{Z}_*, +)$ is a cyclic group

(B) A cyclic group of order n has $\varphi(n)$ number of generators

(C) Any group of prime order is cyclic

(D) Any group of order 4 is cyclic

OR

If $\{X_1, X_2, X_3\}$ is a random sample from $N(\theta, 1)$, which of the following is a sufficient statistic for θ ?

(A) $X_1 + 2X_2 + 3X_3$

(B) $2X_1 + 2X_2 + 2X_3$

(C) $X_1 - X_2 + X_3$

(D) $X_1 + X_2 - X_3$

48. Let p be a prime number and let G be a group of order p^2. If G is not cyclic, then the number of elements in G of order p is

(A) p

(B) p^2

(C) $p^2 - 1$

(D) p –1

OR

If $X \sim N(\theta, 1)$ and $L(\theta, a) = (\theta - a)^2$ is the squared error loss, under which of the following is the decision rule $\delta_k(x) = kx$ inadmissible ?

(A) $k < 1$

(B) $k > 1$

(C) $0 \leq k \leq 1$

(D) $0 \leq k < \frac{1}{2}$

49. If ϕ denotes the Euler's totient function, then for any positive integer n, the sum $\sum_{d/n} \phi(d)$ equals

(A) 1

(B) 0

(C) n

(D) 2n

OR

If $\{X_1, X_2, ..., X_n\}$ is a random sample from uniform $\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$, what is the MLE of θ ?

(A) max. $\{X_1, ..., X_n\}$

(B) $\frac{1}{2}$ (min.$\{X_1, ..., X_n\}$ + max.$\{X_1, ..., X_n\}$)

(C) min. $\{X_1, ... X_n\}$

(D) $\frac{X_1 + \ldots + X_n}{n}$

50. Which one of the following statements is not true ?

(A) The ideal (X) is a prime ideal in $\mathbb{Z}$ [X]

(B) The ideal (X) is a maximal ideal in $\mathbb{Z}$ [X]

(C) The ideal (X) is a maximal ideal in $\mathbb{R}$[X]

(D) The ideal (X) is a prime ideal in $\mathbb{C}$[X]

OR

In simple random sampling, bias of the ratio estimator $R = \frac{\overline{Y}}{\overline{X}}$ is what ?

(A) Cov. $(\overline{Y}, \overline{X})$

(B) $-\frac{\text{Cov. }(R, \overline{X})}{E(\overline{Y})}$

(C) $-\frac{\text{Cov. }(R, \overline{Y})}{E(\overline{X})}$

(D) $-\frac{\text{Cov. }(R, \overline{X})}{E(\overline{X})}$

51. Which one of the following statements is not true ?

(A) The congruence equation $15x \equiv 6 \pmod{18}$ has 3 solutions mod 18

(B) The congruence equation $12x \equiv 5 \pmod{10}$ has 2 solutions mod 10

(C) The congruence equation $13x \equiv 1 \pmod{5}$ has a unique solution mod 5

(D) The congruence equation $10x \equiv 1 \pmod{17}$ has a unique solution mod 17

OR

A population was divided into groups and it was found that within group variation was less than between group variation. Which sampling procedure was used if a sample of units was selected from each group ?

(A) Cluster sampling

(B) Systematic sampling

(C) Probability proportional to size sampling

(D) Stratified sampling

52. Which one of the following statements is not true ?

(A) The map $f : (\overset{*}{\mathbb{R}}, X) \to (\overset{*}{\mathbb{R}}, X)$ given by $f(x) = x^3$ is a group isomorphism

(B) The map $g : (\overset{*}{\mathbb{R}}, X) \to (\overset{*}{\mathbb{R}}, X)$ given by $g(x) = x^2$ is a group isomorphism

(C) The permutations $\sigma = (12)\ (3456)$ and $\tau = (13)\ (2456)$ in S_6 are conjugates

(D) Product of an even permutation and an odd permutation is an odd permutation

OR

If a sample of size n is selected from a population of size N using SRSWR, what is $E\left(\frac{1}{D}\right)$ where D is the number of distinct units in the sample ?

(A) $\frac{N-n}{N-1}$ (B) $\frac{1}{N^n}\sum_{m=1}^{N} m^{m-1}$

(C) $\frac{1}{N^n}\sum_{m=1}^{N} m^{m}$ (D) $\frac{N^n-n^n}{N-1}$

53. Consider the ring

$R = \mathbb{Z}\left[\sqrt{-5}\right] = \left\{a+b\sqrt{-5} \mid a, b \in \mathbb{Z}\right\}$ and $\alpha = 1+\sqrt{-5}$ of $\mathbb{R}$. Then which one of the following statements is not true ?

(A) R is an integral domain

(B) R is not a unique factorization domain

(C) α is irreducible

(D) α is prime

OR

If $\bar{y}$ is the mean of a simple random sample of size n drawn from a population of size N, then what is the ratio $\frac{V(\bar{y})_{SRSWOR}}{V(\bar{y})_{SRSWR}}$ equal to ?

(A) $\frac{N-1}{N-n}$

(B) $\frac{N-n}{N-1}$

(C) $\frac{N-n}{Nn}$

(D) $\frac{N-1}{Nn}$

54. $\frac{\mathbb{Z}_2[X]}{(X^3+X^2+1)}$ is

(A) a field with 16 elements

(B) a field with 8 elements

(C) an infinite field

(D) not a field

OR

If the key block in a 2^5 factorial experiment consists of (1), bc, de, bcde, abd, acd, abe, ace, then what are the interactions that are confounded ?

(A) ABC, ADE, BCDE

(B) ABC, ACE, BCDE

(C) ADE, ABCD, BCE

(D) ACE, BCDE, ABD

55. Let $F \subseteq K \subseteq L$ be field extensions. If F is a field with 4 elements, which of the following is a possibility ?

(A) $|L| = 64$ and $|K| = 32$

(B) $|L| = 16$ and $|K| = 8$

(C) $|L| = 256$ and $|K| = 16$

(D) $|L| = 128$ and $|K| = 16$

OR

In Anova for one way classified data with 3 classes and 3 observations per class, F-value is 15 and total sum of squares is 12. What is the mean square between classes ?

(A) 2 (B) 3

(C) 4 (D) 5

56. Which one of the following statements is true ?

(A) Every Hausdorff space is regular

(B) Every regular space is normal

(C) Every normal space is metrizable

(D) Every closed subspace of a normal space is normal

OR

What are the respective interaction effects confounded in the two replications given below ?

$$\underline{\text{Replication I}} \qquad \underline{\text{Replication II}}$$
$$\begin{pmatrix} abc \\ bc \\ a \\ (1) \end{pmatrix}\begin{pmatrix} ab \\ ac \\ b \\ c \end{pmatrix} \qquad \begin{pmatrix} abc \\ ac \\ b \\ (1) \end{pmatrix}\begin{pmatrix} ab \\ bc \\ a \\ c \end{pmatrix}$$

(A) BC and AB (B) BC and AC
(C) ABC and AB (D) AB and BC

57. The boundary of $A = \{(x, 0) \mid 0 \le x < 1\}$ in $\mathbb{R}^2$ is

(A) $\{(x, 0) \mid 0 < x \le 1\}$

(B) $\{(0, 0), (1, 1)\}$

(C) $\{(x, 0) \mid 0 \le x \le 1\}$

(D) $\{(x, 0) \mid 0 \le x < 1\}$

OR

In a BIBD with v=b=4 and $\lambda = 2$, what are the values of r and k respectively ?

(A) 3 and 3 (B) 3 and 2
(C) 2 and 3 (D) 2 and 2

58. Let $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$

Then

(A) f is continuous only at rational points

(B) f is continuous only at irrational points

(C) f is nowhere continuous

(D) f is continuous everywhere

OR

If $P_{3\times3}$ is a doubly stochastic transition probability matrix of a Markov chain, then what is the stationary distribution ?

(A) $(1\ 0\ 0)$

(B) $\left(\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\right)$

(C) $\left(\frac{1}{3}\ \frac{2}{3}\ 0\right)$

(D) $\left(\frac{2}{3}\ \frac{1}{3}\ 0\right)$

59. Let τ be the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ and τ' be the product topology on $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ in the discrete topology. Then

(A) $\tau = \tau'$

(B) τ is strictly finer than τ'

(C) τ' is strictly finer than τ

(D) τ and τ' are not comparable

OR

If $\{N(t), t \ge 0\}$ is a Poisson process with mean value function $m(t) = 2t, t \ge 0$, then what is the variance of the number of events occurring by time 10 ?

(A) 20 (B) 40
(C) 10 (D) 05

60. Which one of the following statements is not true ?

(A) $\mathbb{R}^n$ in the product topology is metrizable

(B) $\mathbb{R}^w$ in the product topology is metrizable

(C) $\mathbb{R}^J$ in the product topology is metrizable

(D) $\mathbb{R}^w$ in the box topology is not metrizable

OR

If $\{N(t), t \geq 0\}$ is a homogenous Poisson process, what is the correlation coefficient between N(s) and N(t), s < t ?

(A) $\frac{s}{t}$

(B) $\sqrt{\frac{s}{t}}$

(C) $\sqrt{st}$

(D) st

61. Which one of the following statements is not true ?

(A) Every interval in $\mathbb{R}$ is both connected and locally connected

(B) The subspace $Y = [0, 1) \cup (1, 2]$ of $\mathbb{R}$ is not locally connected but connected

(C) The subspace Q of $\mathbb{R}$ is neither connected nor locally connected

(D) The deleted comb space is connected but not locally connected

OR

Men and women arrive in queue independently according to Poisson process with respective rates λ_1 and λ_2. What is the probability that the first to arrive in the queue is a woman ?

(A) $\frac{\lambda_1}{\lambda_2}$ (B) $\frac{\lambda_2}{\lambda_1}$

(C) $\frac{\lambda_1}{\lambda_1+\lambda_2}$ (D) $\frac{\lambda_2}{\lambda_1+\lambda_2}$

62. Which one of the following spaces is connected but not path connected ?

(A) $I \times I$, where $I = [0, 1]$ in the dictionary order topology

(B) The space $Y = [0, 1) \cup (1, 2]$ of $\mathbb{R}$

(C) $\mathbb{R}$ in the standard topology

(D) The subspace Q of $\mathbb{R}$

OR

A system consists of four identical units with independent lifetimes. The system consists of part A and part B which are connected in series and part A has 2 units connected in parallel and part B has 2 units connected in parallel. Given that the probability of failure of a unit is $\frac{1}{2}$, what is the reliability of the system ?

(A) $\frac{1}{16}$ (B) $\frac{9}{16}$

(C) $\frac{11}{32}$ (D) $\frac{21}{32}$

63. Let a be a complex number with $|a| < 1$

Then $\int\limits_{|z|=1} \frac{|dz|}{|z-a|^2} =$

(A) $\frac{2\pi}{1+|a|^2}$ (B) $\frac{2\pi}{|a|^2}$

(C) $\frac{2\pi}{1-|a|^2}$ (D) $\frac{\pi}{1-|a|^2}$

OR

Lifetimes of two components connected in parallel are standard exponential. What is the expected lifetime of the system ?

(A) $\frac{3}{2}$ (B) 1

(C) $\frac{1}{2}$ (D) 2

64. $\int_0^{2\pi} e^{e^{i\theta}}\, d\theta =$

(A) 2π (B) $2\pi i$

(C) πi (D) π

OR

If $P\left(X_n = e^n\right) = \frac{1}{n} = 1 - P\left(X_n = 0\right), n \geq 1$, which of the following is true ?

(A) $X_n \to 0$ in r^{th} mean

(B) $X_n \to 0$ a.s.

(C) X_n does not converge in probability

(D) $X_n \to 0$ probability

65. Which one of the following transformations reduces the differential equation $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ into the form $\frac{du}{dx} + P(x)u = Q(x)$?

(A) $u = \log z$

(B) $u = \frac{1}{\log z}$

(C) $u = e^z$

(D) $u = (\log z)^2$

OR

Which of the following is not correct ?

(A) $X_n \to X$ a.s. $\Rightarrow X_n \xrightarrow{p} X$

(B) $X_n \xrightarrow{p} X$ and $\{X_n, n \geq 1\}$ monotone $\Rightarrow X_n \to X$ a.s.

(C) $X_n \xrightarrow{p} X \Rightarrow E\,(X_n - X)^2 \to 0$

(D) $X_n \xrightarrow{p} X$ and g(.) real valued and continuous $\Rightarrow g(X_n) \xrightarrow{p} g(X)$

66. The initial value problem $x(1+x)\frac{dy}{dx} = (2x+1)y;\ y(x_0) = y_0$ has unique solution if

(A) $x_0 = y_0 = 0$

(B) $x_0 = -1\ y_0 = 1$

(C) $x_0 = 0\ y_0 = 1$

(D) $x_0 = 2\ y_0 = 1$

OR

Which of the following is not true, given that $\{F_n, n \geq 1\}$ is a sequence of dfs, F is a df and $\xrightarrow{w}$ denotes weak convergence ?

(A) $F_n \xrightarrow{w} F$ and F continuous $\Rightarrow \sup_{x\in\mathbb{R}} F_n(x) \to \sup_{x\in\mathbb{R}} F(x)$, as $n \to \infty$

(B) $F_n \xrightarrow{w} F$ and F continuous $\Rightarrow \sup_{x\in\mathbb{R}} |F_n(x) - F(x)| \to 0$ as $n \to \infty$

(C) $F_n(x) \to F(x)$ as $n \to \infty$ for all $x \in \mathbb{R} \Rightarrow F_n \xrightarrow{w} F$

(D) $F_n \xrightarrow{w} F \Rightarrow \int_a^b g\, dF_n \to \int_a^b g\, dF$ for all continuity points a < b of F and continuous functions g on $\mathbb{R}$

67. The particular integral of the differential equation $\left(\text{where } D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}\right)$

$(D - D' - 1)(D - D' - 2)u = e^{2x-y} + x$ is

(A) $\frac{1}{2}e^{2x-y} + \frac{1}{2}\left(x + \frac{1}{2}\right)$

(B) $\frac{1}{2}e^{2x-y} + \frac{1}{2}\left(x^2 + \frac{1}{2}\right)$

(C) $\frac{1}{2}e^{2x-y} + \frac{1}{2}\left(x + 3/2\right)$

(D) $e^{2x-y} + x + 3/2$

OR

What is the mean time spent by a customer in a stable M/M/1 queueing system with arrival rate λ and service rate μ ?

(A) $\frac{1}{\mu - \lambda}$ (B) $\frac{\lambda}{\mu - \lambda}$

(C) $\frac{\lambda}{\mu}$ (D) $\frac{\lambda}{\mu(\mu - \lambda)}$

68. The solution u(x, t) of the Dirichlet problem

$u_t = \alpha u_{xx},\ 0 \leq x \leq l,\ 0 \leq t \leq T$

$u(x, 0) = f(x),\ 0 \leq x \leq l$

$u(0, t) = g(t),\ 0 \leq t \leq T$

$u(l, t) = h(t)$ depends on

(A) x and t (B) f(x) and g(t)

(C) f(x), g(t) and h(t) (D) α only

OR

If X(t) is the number of customers in an M/M/1 queueing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$, then what is the process $\{X(t), t \geq 0\}$?

(A) Markov process

(B) Poisson process with rate $\lambda - \mu$

(C) Birth process with rate $\lambda - \mu$

(D) Birth and death process with birth rate $\frac{1}{\lambda}$ and death rate $\frac{1}{\mu}$

69. The general solution of $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$ is of the form

(A) $\theta = c\, f(x + iy) + g(iy)$

(B) $\theta = f(x + y) + g(x - y)$

(C) $\theta = f(x + iy) + g(x - iy)$

(D) $\theta = g(x + iy) + f(ix)$

OR

In a Markov chain, what is a sufficient condition for existence of $\lim_{n\to\infty} p_{ij}^{(n)}$?

(A) All states communicate

(B) The chain is null recurrent

(C) All states communicate and the chain is positive recurrent

(D) The chain is positive recurrent

70. In Newtons - cotes formula, if f(x) is interpolated at equally spaced nodes by a polynomial of degree 3, then it represents

(A) Trapezoidal rule

(B) Simpson 1/3 rule

(C) Simpson Three – Eighth rule

(D) Booles rule

OR

In a homogeneous Poisson process, if S_n denotes the time of occurrence of n^{th} event, n = 1,2,..., then, given $S_n = t$, what is $(S_1,...,S_{n-1})$ distributed as ?

(A) Multivariate normal

(B) Multivariate exponential

(C) A set of (n – 1) independent Poisson random variables

(D) A set of (n – 1) independent uniform random variables over (0, t)

71. Which of the following states that "if f(x) is three-times differentiable and f', f'' are not zero at a solution S of f(x) = 0 then x_0 sufficiently close to S" ?

(A) First order Newton's method

(B) Second order Newton's method

(C) Third order Newton's method

(D) Fourth order Newton's method

OR

Which of the following is not a hazard function ?

(A) t, t > 0

(B) – ln t, t > 1

(C) ln t, t > 1

(D) t^2, t > 0

72. Consider the following statements :

i) The convergence rate of secant method is 1.61

ii) Convergence of Regula-falsi method is linear

iii) Bisection method is based upon the repeated application of intermediate theorem

Which of the following is true ?

(A) only (i) and (ii) are true

(B) only (i) and (iii) are true

(C) only (ii) and (iii) are true

(D) (i), (ii) and (iii) are all true

OR

With reference to a BIBD, which of the following need not be true ?

(A) BIBD is orthogonal
(B) BIBD is connected
(C) BIBD is variance balanced
(D) BIBD is binary

73. Consider a holonomic dynamical system with 3 degrees of freedom. The Hamilton's Cannonical equations of motion for the dynamical system constitutes

(A) 6 first order differential equations
(B) 6 second order differential equations
(C) 3 second order differential equations
(D) 3 first order differential equations

OR

In a Gauss-Markov model the rank of the design matrix $A_{8\times 5}$ is 4. What is the dimension of the estimation space of this model ?

(A) 4
(B) 5
(C) 16
(D) 20

74. Consider the integral equation

$y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt$. Then which of the following K(x, t) is not degenerate Kernel ?

(A) $K(x, t) = \cos(x - t)$
(B) $K(x, t) = \sin xt$
(C) $K(x, t) = e^{x-t}$
(D) $K(x, t) = e^{-(x-t)}$

OR

If X and Y are independent exponential random variables ith respective means $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$, what is P(X < Y) ?

(A) $\frac{\lambda_1}{\lambda_1+\lambda_2}$
(B) $\frac{\lambda_1}{\lambda_2}$
(C) $\frac{\lambda_2}{\lambda_1}$
(D) $\frac{\lambda_2}{\lambda_1+\lambda_2}$

75. Let the curve C be the external of the functional $I[y(x)] = \int_{x^1}^{x^2} F(x, y, y')dx$ with suitable boundary conditions. Then the Legendre condition for testing for a weak maximum is

(A) $F_{Y'Y'} < 0$ on C
(B) $F_{Y'Y'} > 0$ on C
(C) $F_{Y'Y'}$ is constant on C
(D) $F_{Y'Y'}$ should be identically zero on C

OR

What is the MLF of the median of a lifetime random variable following an exponential distribution with mean $\frac{1}{\lambda}$ based on the sample mean $\overline{X}_n$?

(A) $\overline{X}_n$
(B) $\frac{1}{\overline{X}_n}$
(C) $\overline{X}_n$ .log 2
(D) $\frac{\log 2}{\overline{X}_n}$

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Space for Rough Work

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Space for Rough Work